IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

CHANNEL REPLACEMENT USING ENTROPY CONCEPTS UNDER TYPE-2 FUZZY ENVIRONMENT

1 INTRODUCTION

Operations research might be defined as a scientific approach to decision-making that involves the operations of organizational system. The success of organization is based on making optimal decision at the appropriate time. Optimal decisions could be taken depending upon the degree of information decision maker knows in that particular situation (or) problem. The degree of information is widely divided into four categories: Certainty, Risk, Uncertainty, Ignorance.

In most of the real life situations or problems, the degree of information is uncertain and so it is wiser to use fuzzy theory to arrive at the optimal solution for any problem under consideration.

In particular, the problem of replacement arises when the job performing units become less effective or useless all of a sudden. Obviously, the degree of information the decision maker knows in replacement problem would be uncertain in most of the situations. Hence, Replacement models can use fuzzy theory (especially, type-2 fuzzy sets) to arrive at the optimal solution for the replacement problem.

In information theory, the information transmitted from a channel need not be exactly same as it was sent by the source (i.e) a very small arbitrary error can be accepted. But when the capacity of that particular channel becomes less effective or useless all of a sudden, then it is advisable to replace the channel at the appropriate time using fuzzy theory.

The channel through which the information is transmitted would be considered as a hypercube I^3 and the replacement of the channel would be decided by the decision maker in the optimal manner using the type-2 fuzzy sets.

2 PRELIMINARIES

2.1 BASIC DEFINITIONS

DEFINITION 2.1.1- FUZZY SET:

If R is a collection of objects denoted generically by r, then a fuzzy set $\tilde{s}$ *in R is a set of ordered pair*:

$$\tilde{s} = \{(r, \mu_{\tilde{s}}(r)) | r \in R\}$$

Where $\mu_{\tilde{s}}(r)$ is called the membership function or grade of membership or degree of compatibility or degree of truth of r in $\tilde{s}$ that maps R to the membership space M.

DEFINITION 2.1.2-TYPE-2 FUZZY SET:

A type-2 fuzzy set is a fuzzy set where membership values are type-1 fuzzy sets on the closed interval [0,1].

DEFINITION 2.1.3-TYPE-M FUZZY SET:

A type-m fuzzy set is a fuzzy set whose membership values are type-m-1, m>1 fuzzy sets on[0,1].

DEFINITION 2.1.4 - TRIANGULAR- NORM INEQUALITIES:

For any two elements $r, s \in R$, the triangular-norm inequalities is defined as

$$T(r,s) \leq \min(r,s) \leq \max(r,s) \leq c(r,s)$$

Where $T(r,s)$ is the triangular-norm and

$c(r,s)$ is the dual triangular norm

2.2 TRANSMISSION OF INFORMATION PROCESSES

Sharing any information among people or organization is a very common-thing in everyday life for eras. But when the sender and the receiver of the information are very far apart then the information is transmitted through a proper communication channel. The information which is to be transmitted can be of any form such as verbal, written, non-verbal.

To carry the information from any sender a to any receiver, there are five essential parts of a communication system, which are as follow: Source, Encoder, Communication channel, Decoder, Destination

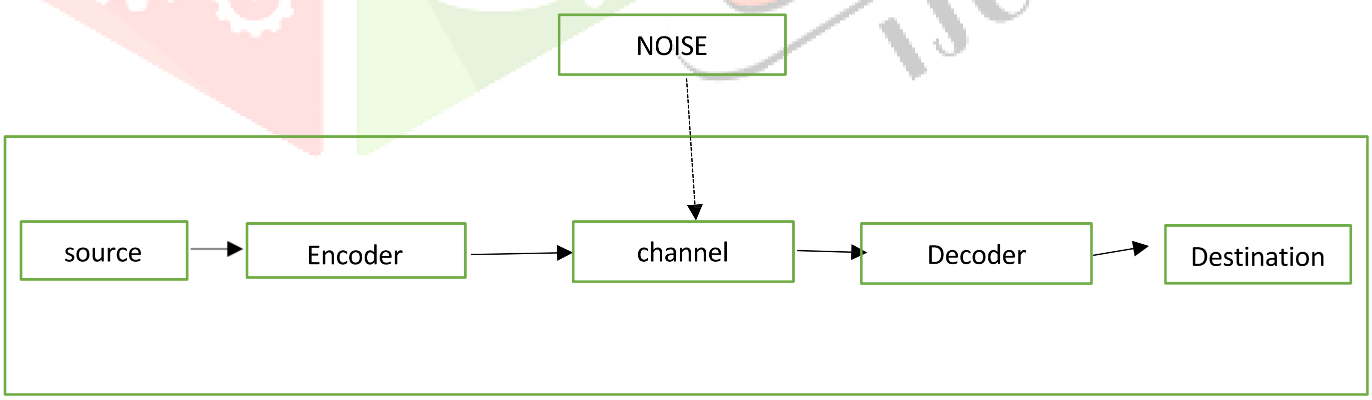


Fig.2.2.1. COMMUNICATION SYSTEM

2.3 COMMUNICATION CHANNEL

The communication system described in the above diagram is statistical in nature. This is because the source selects and transmits sequence of symbols to the channel through the encoded based on some statistical rule, then, the channel also transmits this symbolic information to the destination through the decoder based on some statistical rule.

The study of information theory is based upon a fundamental theorem which states that; "It is possible to transmit information through a noisy channel at any rate less than the channel capacity with an arbitrarily small probability of error".

DEFINITION 2.3.1-ENTROPY FUNCTION:

The expected value of information can also be interpreted as the expected amount of information needed to determine which event of set X has occurred.

Shannon and Wiener have suggested the following expression as the measure of Expected Amount of Information:

$$H(p_1, p_2, \dots, p_n) = -\sum_{i=1}^{n} p_i \log pi \,.$$

The function H is known as Entropy function.

2.4 MEASURES OF INFORMATION QUALITIES:

Let two finite discrete sets R $=\{r_1, r_2, \dots\dots, r_m\}$ *and* S $=\{s_1, s_2, \dots\dots, s_n\}$ exist, where $r_i's$ *and* $s_i's$

Denote the messages transmitted and received respectively. Thus, if:

(i) $r_i = s_j$, then the message received is correct, and

(ii) *if* $r_i \neq s_j$, *then the message received is not correct, whatever be reason.*

Let $P(r)$ *and* $P(s_j)$ *be the probability* of message r_i transmitted and message s_j received, respectively;

DEFINITION 2.4.1 - MARGINAL ENTROPY FUNCTION:

The marginal entropy functions are defined as

$$H(R) = -\sum_{i=1}^{m} P\{r_i\} \log P\{r_i\}$$

$$H(S) = -\sum_{j=1}^{n} P\{s_j\} \log P\{s_j\}$$

The entropy $H(R)$ and $H(S)$ measures the uncertainty of the message transmitted aand received, irrespective of the message received and transmitted, respectively.

2.5 GEOMETRY OF TYPE-2 FUZZY SETS:

The membership function of any fuzzy set can be visualized as two-dimensional graphs with the domain R represented as a one-dimensional axis. The membership function of any type-2 fuzzy set can be visualized as three-dimensional object with the domain R represented as a one- dimensional axix. The geometry of fuzzy sets involves the domain R=$\{r_1, r_2, \dots, r_n\}$ $and\ the\ range\ closed\ interval[0,1]$ of mappings.$\mu_{\tilde{r}}(r_i): R \rightarrow [0,1] \forall\, i = 1,2, \dots n$ $in\ the\ domain\ R, while$ the geometry of type-2 fuzzy sets involves the domain $R = \{r_1, r_2, \dots, r_n\}$ and the range would be an interval valued function(which is the membership value of the elements in the domain) which is again lies in the closed interval [0,1].

Visualizing this geometry is an extremely powerful to handle fuzziness in replacement models. Clearly, the set of all fuzzy subsets of any fuzzy set R would appear like an cube in 3- dimensional space. Any particular fuzzy set would appear like a particular point in a cube in 3- dimensionalspace

2.6 UNIT HYPERCUBE:

In geometry, a hypercube is defined as an n-dimensional analogue of a square and a cube. In particular, a unit hypercube is a hypercube whose side has length one unit. In fuzzy theory, the set of all fuzzy subsets equals to the unit hypercube, which is defined by I^n and is defined as $I^n = [0,1]^n$. Clearly, the vertices of the unit hypercube $I^n\ is\ non-fuzzy\ in\ nature,\ (i.e),\ non-fuzzy\ sets.\ Also,$ the midpoint of the unit hypercube I^n is maximally fuzzy as all the membership values of the midpoint of the unit hypercube I^n is exactly equal to ½.

It is to be noted that the midpoint is the only point in the unit-hypercube $I^n,\ which\ is\ equidistant\ to\ each$ of the 2^n vertices of the hypercube I^n, it is obvious that the midpoint of the unit hypercube is unique.

2.6.1 ILLUSTRATION:

The concept of unit-hypercube can be easily explained using the following illustration. Here, each type-fuzzy set would be viewed as a point in the unit fuzzy hypercube I^n. Let us consider a particular point, say, p in the unit hypercube, which is defined as p=(1/3,3/4)

$$p^c = \left(1 - \frac{1}{3}, 1 - \frac{3}{4}\right)$$

$$p^c = \left(\frac{2}{3}, \frac{1}{4}\right)$$

Also $p \cap p^c = min\{\mu(p, p^c)\}$, p

=min{(1/3,2/3),(3/4,1/4)}

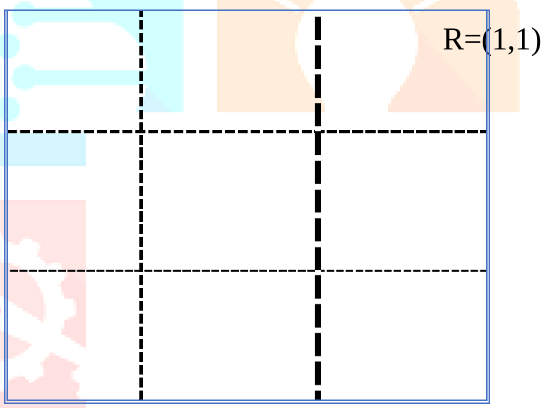
Therefore, $p \cap p^c = \left(\frac{1}{3}, \frac{1}{4}\right)$

$p \cup p^c = max\{\mu(p, p^c)\}$

=max{(1/3,2/3),(3/4,1/4)}

Therefore, $p \cup p^c = \left(\frac{2}{3}, \frac{3}{4}\right)$

The set-as-points view shows that these four points, namely p, p^c , $p \cap p^c$, $p \cup p^c$ in the unit square hang together and move together in the natural manner which is as shown in the diagram



It is evident that the fuzziness of the set p(which is in sets-as-point view) contract to the midpoint as P becomes maximally fuzzy and expand out to the corners of the hyper-cube as P becomes minimally fuzzy. The same contraction and expansion occurs in "n-dimensions" for the 2^n $fuzzy\ sets\ defined\ by\ its\ membership\ functions\ respectively.$

3 RESULT

THEOREM 3.1 TYPE-2 FUZZY ENTROPY THEOREM:

Let us denote the fuzzy entropy of A by E(A), then

$$E(A) = \frac{\mu(A \cap A^c)}{\mu(A \cup A^c)}$$

DEFINITION 3.1 - ENTROPY FUNCTION FOR TYPE-2 FUZZY SET:

For a type-2 fuzzy set whose membership function is type-1 fuzzy set, whose membership function is denoted by μ_{r_i} for every $r_i \in R$ is defined as:

$$H(R) = -\sum_{i=1}^{n}(\mu_{r_i}).(\log \mu_{r_i}), where\ r_i \in R$$

EXAMPLE

1. A message is passed through a particular channel whose transmitter has a character consisting of five letters, say, x_1, x_2, x_3, x_4, x_5 and receiver has a character consisting of four letters ,say, y_1, y_2, y_3, y_4. The following is the data set which has been taken in the type-2 fuzzy set which has been transmitted through the channel for a specific time period. Check whether the channel has to be replaced at this point of time.

$$M = \{(x_1/y_1, y_1/0.25), (x_1/y_2, y_2/0), (x_1/y_3, y_3/0), (x_1/y_4, y_4/0)\ (x_2/y_1, y_1/0.10), (x_2/y_2, y_2/0.30), (x_2/y_3, y_3/0), (x_2/y_4, y_4/0)\ (x_3/y_1, y_1/0), (x_3/y_2, y_2/0.05), (x_3/y_3, y_3/0.10), (x_3/y_4, y_4/0)\ (x_4/y_1, y_1/0), (x_4/y_2, y_2/0), (x_4/y_3, y_3/0.05), (x_4/y_4, y_4/0.10)\ (x_5/y_1, y_1/0), (x_5/y_2, y_2/0), (x_5/y_3, y_3/0.05), (x_5/y_4, y_4/0)\}$$

Solution:

From the given data,

1. $(x_1/\mu_{x_1}) = (x_1/\sum_{j=1}^{4}\mu(\frac{x_1}{y_j}))$

$$= x_1/(0.25 + 0 + 0 + 0)$$

$\Rightarrow (x_1/\mu_{x_1}) = (x_1/0.25)$

2. $x_2/\mu_{x_2} = x_2/\sum_{j=1}^{4}\mu(\frac{x_2}{y_j})$

$= x_2/(0.10+0.30+0+0)$

$\Rightarrow x_2 \,/\mu_{x_2} = x_2/0.40$

3. $x_3/\mu_{x_3} = x_3/\sum_{j=1}^{4} \mu\,(\frac{x_3}{y_j})$

$= x_3/(0+0.05+0.10+0)$

$\Rightarrow x_3 \,/\mu_{x_3} = x_3/0.15$

4. $x_4/\mu_{x_4} = x_4/\sum_{j=1}^{4} \mu\,(\frac{x_4}{y_j})$

$= x_4/(0+0+0.05+0.10)$

$\Rightarrow x_4 / \mu_{x_4} = x_4/0.15$

5. $x_5/\mu_{x_5} = x_5 / \sum_{j=1}^{4} \mu\left(\frac{x_5}{y_j}\right)$

$= x_5/(0+0+0.05+0)$

$\Rightarrow x_5 / \mu_{x_5} = x_5/0.05$

SIMILARLY,

1. $(y_1/\mu_{y_1}) = y_1 / \sum_{i=1}^{5} \left(\frac{y_1}{x_i}\right)$

$= y_1 \; / (0.25 + 0.10 + 0 + 0 + 0)$

$(y_1/\mu_{y_1}) = y_1/0.35$

2. $(y_2/\mu_{y_2}) = y_2/0.35$
3. $(y_3/\mu_{y_3}) = y_3/0.20$
4. $(y_4/\mu_{y_4}) = y_4/0.10$

USING ENTROPY FUNCTION

$H(X) = -\sum_{i=1}^{5} P(x_i).\, log\, P(x_i)$

$H(X) = -\sum_{i=1}^{5} (\mu_{x_i}).\, log\, (\mu_{x_i})$

$= -[0.25 * log(0.25) + 0.40 * log(0.40) + 0.15 * log(0.15) + 0.15 * log(0.15) + 0.05 * log(0.05)]$

$= [0.0156 + 0.1591 + 0.1235 + 0.1235 + 0.0650]$

$= +0.4867$

$\Rightarrow H(X) = +0.4867$

USING ENTROPY FUNCTION,

$H(Y) = -\sum_{j=1}^{5} (\mu_{y_j}).\, log\, (\mu_{y_j})$

$= -[0.35 * \log(0.35) + 0.35 * \log(0.35) + 0.20 * \log(0.20) + 0.10 * \log(0.10)]$

$= -[0.35 * (-0.4559) + 0.35 * (-0.4559) + 0.20 * (-0.6989) + 0.10 * (-1)]$

$= -[-0.1595 - 0.1595 - 0.1397 - 0.10]$

$= 0.5587$

$\Rightarrow H(Y) = 0.5587$

Assume that the channel through which the message is transmitted as a unit hypercube. It has been proved that each type-2 fuzzy set would be viewed as a point in the fuzzy hypercube.

Hence,

$X = (\mu_{x_1}, \mu_{x_2}, \mu_{x_3}, \mu_{x_4}, \mu_{x_5})$

$\Rightarrow X = (0.25, 0.40, 0.15, 0.15, 0.05)$..(1)

Then,

$X^c = (1 - 0.25, 1 - 0.40, 1 - 0.15, 1 - 0.15, 1 - 0.05)$

$X^c = (0.75, 0.60, 0.85, 0.85, 0.95)$ …………………………………………… (2) Then,

$X \cap X^c = \{\min(0.25, 0.75), \min(0.40, 0.60) \min(0.15, 0.85), \min(0.15, 0.85), \min(0.05, 0.95)\}$

$X \cap X^c = \{0.25, 0.40, 0.15, 0.15, 0.05\}$………………………………………………… (3)

Then,

$X \cup X^c =$
$\{\max(0.25, 0.75), \max(0.40, 0.60), \max(0.15, 0.85), \max(0.15, 0.85), \max(0.05, 0.95)\}$

$X \cup X^c = \{0.75, 0.60, 0.85, 0.85, 0.95\}$.. (4)

Similarly,

$y = (\mu_{y_1}, \mu_{y_2}, \mu_{y_3}, \mu_{y_4})$

$\Rightarrow y = (0.35, 0.35, 0.20, 0.10)$ ……………………………………………… (5) Then,

$Y^c = (1 - 0.35, 1 - 0.35, 1 - 0.20, 1 - 0.10)$

$\Rightarrow Y^c = (0.65, 0.65, 0.80, 0.90)$ ……………………………………….... (6) Then,

$Y \cap Y^c = (0.35, 0.35, 0.80, 0.90)$ …………………………………………… (7)

$Y \cup Y^c = (0.65, 0.65, 0.80, 0.90)$……………………………………………….. (8)

Since,

$X \cap X^c \neq \emptyset$ *and* $X \cup X^c \neq M$ *and*

$Y \cap Y^c \neq \emptyset$ *and* $Y \cup Y^c \neq M$, there exist proper fuzzing Clearly,

from (1) and (3)

X= $X \cap X^c$

From (2) and (4)

X^c= $X \cup X^c$

Similarly, from (5) and (7) Y= $Y \cap$

Y^c

From (6) and (8)

Y^c= $Y \cup Y^c$

From triangular-norm inequality

$$T(x,y) \leq \min(x,y) \leq \max(x,y) \leq c(x,y)$$

$$\Rightarrow \; X \cap X^c \leq X \cup X^c$$

$$\Rightarrow X = \; X \cap X^c \leq X \cup X^c = X^c$$

Similarly,

$$Y = Y \cap Y^c \leq Y \cup Y^c = Y^c$$

Using entropy theorem,

$$E(A) = \frac{\mu(A \cap A^c)}{\mu(A \cup A^c)}$$

We have,

$H(X) = +0.4867$ and

$H(Y) = 0.5587$

∴ both the values are closer to 0.5.

$\Rightarrow X \cap X^c and\ X \cup X^c\ contracts\ to\ the\ midpoint\ of\ the\ hypercube.$

$\Rightarrow X\ and\ X^c\ contracts\ to\ the\ midpoint\ of\ the\ hypercube.$

Similarly,

$\Rightarrow \quad \mathrm{Y} \cap Y^c\ and\ \mathrm{Y} \cup Y^c\ contracts\ to\ the\ midpoint\ of\ the\ hypercube.$

$\Rightarrow Y\ and\ Y^c\ contracts\ to\ the\ midpoint\ of\ the\ hypercube.$

We know that the midpoint of the hypercube is unique.

Hence, the message transmitted from the source and the message received in the destination are almost (approximately) same.

⇒ The message has been almost properly been transmitted through the given particular channel. W.K.T,

"It is possible to transmit information through a noisy channel at any rate less than the channel capacity with an arbitrarily small probability of error."

The message has been successfully transmitted through the given channel and hence the channel which has been used for transmission of the message need not be replaced at the given point of time.

4 CONCLUSION:

Replacement is an important field in operations research in which uncertainty privileges in most of the real-life situations. The uncertainty which privileges in the data of the replacement problem can be handled by fuzzy theory. In this paper the data of the replacement problem are considered as type-2 fuzzy sets and the channel which is used to transmit the message is considered as hypercube. Thus the channel replacement problem is solved effectively, which is illustrated using an example.

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