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Triangular Decomposition Of Graphs

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ABSTRACT. Let G = (V, E) be a simple connected graph of order p and size q. If $\{G_1, G_2,\ldots,G_n\}$ are edge disjoint subgraphs of G such that $E(G) = E(G_1) \cup E(G_2) \cup E(G_3) \cup \ldots \cup E(G_n)$ then $\{G_1, G_2,\ldots,G_n\}$ is said to be a Decomposition of a graph G. A graph of size $q = \binom{n+2}{3}$ is said to have a Triangular decomposition (TD) if G can be decomposed into n - subgraphs $\{G_1, G_2,\ldots,G_n\}$ such that each subgraphs G_i is connected and $|E(G_i)| = \binom{i+1}{2}$ for $1 \le i \le n$. In this paper we investigate Triangular decomposition of graphs.

Keywords – Triangular Decomposition, Octopus Graph, Wheel Graph, Crown Graph.

1.Introduction

Let G = (V, E) be a simple connected graph with p vertices and q edges. A Decomposition of a graph G is a collection of edge disjoint subgraphs $\{G_1, G_2, G_3,\ldots,G_n\}$ of G such that every edge of G belongs to exactly one of the subgraph G_i. A graph G of size $q = \binom{n+2}{3}$ is said to have a Triangular decomposition (TD) if G can be decomposed into n - subgraphs $\{G_1, G_2, G_3,\ldots,G_n\}$ such that each subgraph G_i is connected and $|E(G_i)| = \binom{i+1}{2}$ for $1 \le i \le n$.

An Octopus graph O_m ($m\geq2$) can be constructed by joining a Fan graph F_m ($m \geq 2$) with a Star graph $K_{1,m}$ by designating the centre vertex of Star graph and the centre vertex of Fan graph as the common vertex. A Wheel Graph W_m is defined to be the join K_1+C_m. The vertex corresponding to K_1 is known as the apex and the vertices corresponding to cycle are known as rim vertices while the edges corresponding to cycle are known as rim edges. The Crown $C_m \odot K_1$ is obtained by joining a pendent edge to each vertex of cycle C_m. In this paper we investigate Triangular Decomposition of graphs.

2.Triangular Decomposition of Graphs

Lemma 2.1. If k=3r and r ≡ 2(mod3) then every graph admits a Triangular Decomposition $\{G_1, G_2, G_3,\ldots,G_k\}$ where k is the number of decomposition.

Proof. We have k = 3r and r ≡ 2(mod3) where r∈N. We prove this theorem by using induction method. When r = 2, k = 6. Now $\frac{k(k+1)(k+2)}{6} = \frac{6x7x8}{6} = 56$ can be decomposed into $\{G_1, G_2,\ldots,G_6\}$. Hence the result is true for r = 2.

Assume that the result is true for 3r-1. Then k = 3(3r-1) = 9r-3 and q' = $\frac{(9r-3)(9r-2)(9r-1)}{6}$ can be decomposed into $\{G_1, G_2, G_3,\ldots,G_{3(3r-1)}\}$. Now to prove the result is true for 3r+2. Then k = 9r + 6 and q = $\frac{(9r+6)(9r+7)(9r+8)}{6}$. We have to prove that q = $\frac{(9r+6)(9r+7)(9r+8)}{6}$ can be decomposed into $\{G_1, G_2, G_3,\ldots,G_{9r+6}\}$.

Now q = $\frac{(9r+6)(9r+7)(9r+8)}{6}$

$$= \frac{(729r^3+1701r^2+1314r+336)}{6}$$

$$= \frac{(9r-3)(81r^2-27r+2)}{6} + \{\frac{(162r^2-36r+2)}{2} + \frac{(162r^2+36r+2)}{2} + \frac{(162r^2+108r+18)}{2} + \frac{(162r^2+180r+50)}{2} + \frac{(81r^2+117s+42)}{2}\}$$

$$= \frac{(9r-3)(9r-2)(9r-1)}{6} + \{\frac{(9r-1)(18r-2)}{2} + \frac{(9r+1)(18r+2)}{2} + \frac{(9r+3)(18r+6)}{2} + \frac{(9r+5)(18r+10)}{2} + \frac{(9r+6)(9r+7)}{2}\}$$

$$= q' + \{\frac{(9r-2)(9r-1)}{2} + \frac{(9r-1)(9r)}{2} + \frac{(9r)(9r+1)}{2} + \frac{(9r+1)(9r+2)}{2} + \frac{(9r+2)(9r+3)}{2} + \frac{(9r+3)(9r+4)}{2} + \frac{(9r+4)(9r+5)}{2} + \frac{(9r+5)(9r+6)}{2} + \frac{(9r+6)(9r+7)}{2}\}.$$

Therefore q = $\frac{(9r+6)(9r+7)(9r+8)}{6}$ can be decomposed into $\{G_1, G_2, G_3,\ldots,G_{9r+6}\}$. Hence by induction hypothesis if k ≡ 3r and r ≡ 2(mod3) then every graph admits a Triangular Decomposition $\{G_1, G_2, G_3,\ldots,G_k\}$.

This complete the proof

Lemma 2.2. If k+1=3r and r ≡ 2(mod3) then every graph admits a Triangular Decomposition $\{G_1, G_2, G_3,\ldots,G_k\}$ where K is the number of decomposition.

Proof. We have k+1 = 3r and r ≡ 2(mod3) where r∈N. We prove this theorem by using induction method. When r = 2, k = 5. Now $\frac{k(k+1)(k+2)}{6} = \frac{5x6x7}{6} = 35$ can be decomposed into $\{G_1, G_2,\ldots,G_5\}$. Hence the result is true for r = 2.

Assume that the result is true for 3r-1. Then k = 3(3r-1)-1 = 9r-4 and q' = $\frac{(9r-4)(9r-3)(9r-2)}{6}$ can be decomposed into $\{G_1, G_2, G_3,\ldots,G_{9r-4}\}$. Now to prove the result is true for 3r+2. Then k = 9r+5 and q = $\frac{(9r+5)(9r+6)(9r+7)}{6}$. We have to prove that q = $\frac{(9r+5)(9r+6)(9r+7)}{6}$ can be decomposed into $\{G_1, G_2, G_3,\ldots,G_{9r+5}\}$.

Now q = $\frac{(9r+5)(9r+6)(9r+7)}{6}$

$$= \frac{(729r^3+1458r^2+963r+210)}{6}$$

$$= \frac{(9r-4)(81r^2-45r+6)}{6} + \{\frac{(162r^2-72r+8)}{2} + \frac{(162r^2)}{2} + \frac{(162r^2+72r+8)}{2} + \frac{(162r^2+144r+32)}{2} + \frac{(81r^2+99r+30)}{2}\}$$

$$= \frac{(9r-4)(9r-3)(9r-2)}{6} + \{\frac{(9r-2)(18r-4)}{2} + \frac{(9r)(18r)}{2} + \frac{(9r+2)(18r+4)}{2} + \frac{(9r+4)(18r+8)}{2} + \frac{(9r+5)(9r+6)}{2}\}$$

$$= q' + \{\frac{(9r-3)(9r-2)}{2} + \frac{(9r-2)(9r-1)}{2} + \frac{(9r-1)(9r)}{2} + \frac{(9r)(9r+1)}{2} + \frac{(9r+1)(9r+2)}{2} + \frac{(9r+2)(9r+3)}{2} + \frac{(9r+3)(9r+4)}{2} + \frac{(9r+4)(9r+5)}{2} + \frac{(9r+5)(9r+6)}{2}\}.$$

Therefore $q = \frac{(9r+5)(9r+6)(9r+7)}{6}$ can be decomposed into $\{G_1, G_2, G_3, \ldots, G_{9r+5}\}$. Hence by induction hypothesis if k+1= 3r and r ≡ 2(mod3) then every graph admits a Triangular Decomposition $\{G_1, G_2, G_3, \ldots, G_k\}$. This complete the proof.

Lemma 2.3. If k+2=3r and r ≡ 2(mod3) then every graph admits a Triangular Decomposition $\{G_1, G_2, G_3, \ldots, G_k\}$ where k is the number of decomposition.

Proof. We have k+2 = 3r and r ≡ 2(mod3) where r∈N. We prove this theorem by using induction method. When r = 2, k = 4. Now $\frac{k(k+1)(k+2)}{6} = \frac{4x5x6}{6} = 20$ can be decomposed into $\{G_1, G_2, G_3, G_4\}$. Hence the result is true for r = 2.

Assume that the result is true for 3r-1. Then k = 3(3r-1)-2 = 9r-5 and $q' = \frac{(9r-5)(9r-4)(9r-3)}{6}$ can be decomposed into $\{G_1, G_2, G_3, \ldots, G_{9r-5}\}$. Now to prove the result is true for 3r+2. Then k = 9r+4 and $q = \frac{(9r+4)(9r+5)(9r+6)}{6}$. We have to prove that $q = \frac{(9r+4)(9r+5)(9r+6)}{6}$ can be decomposed into $\{G_1, G_2, G_3, \ldots, G_{9r+4}\}$.

$$\text{Now } q = \frac{(9r+4)(9r+5)(9r+6)}{6}$$

$$= \frac{(729r^3+1215r^2+666r+120)}{6}$$

$$= \frac{(9r-5)(81r^2-63r+12)}{6} + \{\frac{(162r^2-108r+18)}{2} + \frac{(162r^2-36r+2)}{2} + \frac{(162r^2+36r+2)}{2} + \frac{(162r^2+108r+18)}{2} + \frac{(81r^2+81r+20)}{2}\}$$

$$= \frac{(9r-5)(9r-4)(9r-3)}{6} + \{\frac{(9r-3)(18r-6)}{2} + \frac{(9r-1)(18r-2)}{2} + \frac{(9r+1)(18r+2)}{2} + \frac{(9r+3)(18r+6)}{2} + \frac{(9r+4)(9r+5)}{2}\}$$

$$= q' + \{\frac{(9r-4)(9r-3)}{2} + \frac{(9r-3)(9r-2)}{2} + \frac{(9r-2)(9r-1)}{2} + \frac{(9r-1)(9r)}{2} + \frac{(9r)(9r+1)}{2} + \frac{(9r+1)(9r+2)}{2} + \frac{(9r+2)(9r+3)}{2} + \frac{(9r+3)(9r+4)}{2} + \frac{(9r+4)(9r+5)}{2}\}.$$

Therefore $q = \frac{(9r+4)(9r+5)(9r+6)}{6}$ can be decomposed into $\{G_1, G_2, G_3, \ldots, G_{9r+4}\}$. Hence by induction hypothesis if k + 2 = 3r and r ≡ 2(mod3) then every graph admits a Triangular Decomposition $\{G_1, G_2, G_3, \ldots, G_k\}$.

This complete the proof.

Theorem 2.4. An Octopus Graph O_m, m ≥ 2 admits Triangular Decomposition $\{G_1, G_2, \ldots, G_k\}$ if and only if there exists an integer k satisfying the following properties:

(i) $m = \frac{k^3+3k^2+2k+6}{18}$, m ∈ N.

(ii) $k = \begin{cases} 3r-2, r \equiv 2(\text{mod } 3) & r \in N \\ 3r-1,\ r \equiv 2(\text{mod } 3) & r \in N \\ 3r, r \equiv 2(\text{mod } 3) & r \in N \end{cases}$

where k denotes the number of decompositions.

Proof. Let G = O_m, m ≥ 2 and m is an integer. Then q(G) = 3m-1. Assume G has a Triangular Decomposition. By the definition of Triangular Decomposition, $q(G) = \frac{k(k+1)(k+2)}{6}$

Hence $3m-1 = \frac{k(k+1)(k+2)}{6}$

⇨ $3m = \frac{k(k+1)(k+2)}{6} + 1$

⇨ $m = \frac{k^3+3k^2+2k+6}{18}$

Since m is an integer, $k = \begin{cases} 3r-2, r \equiv 2(\text{mod } 3) & r \in N \\ 3r-1,\ r \equiv 2(\text{mod } 3) & r \in N \\ 3r, r \equiv 2(\text{mod } 3) & r \in N \end{cases}$

⇨ Conversely assume (i) $m = \frac{k^3+3k^2+2k+6}{18}$ (ii) $k = \begin{cases} 3r-2, r \equiv 2(\text{mod } 3) & r \in N \\ 3r-1, \ r \equiv 2(\text{mod } 3) & r \in N \\ 3r, r \equiv 2(\text{mod } 3) & r \in N \end{cases}$

Consider G = O_m and E(G) = $\{x_1x_j/2\leq j\leq 2m+1\}\cup\{x_jx_{j+1}/2\leq j\leq m\}$. Then q(G) = 3m-1. By lemma 2.1, 2.2 and 2.3, G can be decomposed into $\{G_1, G_2, G_3,\ldots\ldots\ldots,G_k\}$. Thus G admits Triangular Decomposition.

Table 2.5: List of first 10, TD of Octopus Graph O_m.

m	q(G)	Triangular Decomposition
7	20	G_1, G_2, G_3, G_4
12	35	$G_1, G_2, G_3,\ldots\ldots\ldots,G_5$
19	56	$G_1, G_2, G_3,\ldots\ldots\ldots,G_6$
152	455	$G_1, G_2, G_3,\ldots\ldots\ldots,G_{13}$
187	560	$G_1, G_2, G_3,\ldots\ldots\ldots,G_{14}$
227	680	$G_1, G_2, G_3,\ldots\ldots\ldots,G_{15}$
675	2024	$G_1, G_2, G_3,\ldots\ldots\ldots,G_{22}$
767	2300	$G_1, G_2, G_3,\ldots\ldots\ldots,G_{23}$
867	2600	$G_1, G_2, G_3,\ldots\ldots\ldots,G_{24}$
1819	5456	$G_1, G_2, G_3,\ldots\ldots\ldots,G_{31}$

Illustration 2.6.

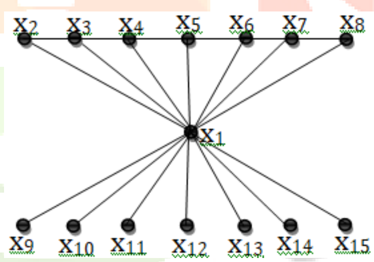

Figure 1. Octopus Graph O_7

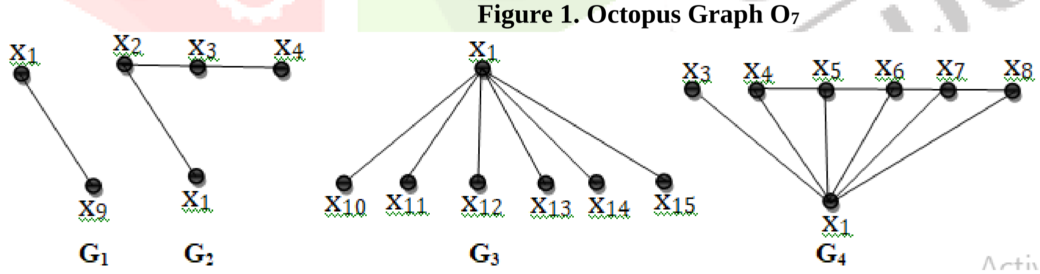

Figure 2.TD $\{G_1, G_2, G_3, G_4\}$ of O_7

Lemma 2.7. If $k \equiv 0(\text{mod}4)$, then every graph admits a Triangular Decomposition $\{G_1, G_2, G_3,\ldots\ldots\ldots,G_k\}$ where k is the number of decomposition.

Proof. We have k = 4r, r∈N. We prove this theorem by using induction method. When r = 1, k = 4. Now $\frac{k(k+1)(k+2)}{6} = \frac{4x5x\ 6}{6}$ = 20 can be decomposed into $\{G_1, G_2, G_3, G_4\}$. Hence the result is true for r = 1.

Assume that the result is true for r-1. Then k = 4(r-1) = 4r-4 and q' = $\frac{(4r-4)(4r-3)(4r-2)}{6}$ can be decomposed into $\{G_1, G_2, G_3,\ldots\ldots,G_{4(r-1)}\}$. Now to prove the result is true for r. Then k = 4r and q = $\frac{4r(4r+1)(4r+2)}{6}$. We have to prove that $\frac{4r(4r+1)(4r+2)}{6}$ can be decomposed into $\{G_1, G_2, G_3,\ldots\ldots\ldots,G_{4r}\}$.

$$\text{Now } q = \frac{4r(4r+1)(4r+2)}{6}$$

$$= \frac{64r^3+48\,r^2+8r}{6}$$

$= \frac{(4r-4)(16r^2-20r+6)}{6} + \{\frac{(4r-2)(8r-4)}{2} + \frac{(4r)(8r)}{2}\}$

$= \frac{(4r-4)(16r^2-8r-12r+6)}{6} + \{\frac{(4r-2)(4r-3+4r-1)}{2} + \frac{(4r)(4r-1+4r+1)}{2}\}$

$= q' + \{\frac{(4r-3)(4r-2)}{2} + \frac{(4r-2)(4r-1)}{2} + \frac{(4r-1)4r}{2} + \frac{4r(4r+1)}{2}\}$.

Therefore $q = \frac{4r(4r+1)(4r+2)}{6}$ can be decomposed into $\{G_1, G_2, G_3, \ldots\ldots\ldots, G_{4r}\}$. Hence by induction hypothesis if $k \equiv 0(\mathrm{mod}\,4)$, then every graph admits a Triangular Decomposition $\{G_1, G_2, G_3, \ldots\ldots\ldots, G_k\}$.

This complete the proof.

Lemma 2.8. If $k+1 \equiv 0(\mathrm{mod}\,4)$, then every graph admits a Triangular Decomposition $\{G_1, G_2, G_3, \ldots\ldots\ldots, G_k\}$ where k is the number of decomposition.

Proof. We have k = 4r–1, r∈N. We prove this theorem by using induction method. When r = 1, k = 3. Now $\frac{k(k+1)(k+2)}{6} = \frac{3x4x5}{6} = 10$ can be decomposed into $\{G_1, G_2, G_3\}$. Hence the result is true for r = 1.

Assume that the result is true for r-1. Then k = 4(r-1)-1 = 4r-5 and $q' = \frac{(4r-5)(4r-4)(4r-3)}{6}$ can be decomposed into $\{G_1, G_2, G_3, \ldots\ldots\ldots, G_{4r-5}\}$. Now to prove the result is true for r. Then k = 4r-1 and $q = \frac{(4r-1)(4r)(4r+1)}{6}$. We have to prove that $q = \frac{(4r-1)(4r)(4r+1)}{6}$ can be decomposed into $\{G_1, G_2, G_3, \ldots\ldots, G_{4r-1}\}$.

Now $q = \frac{(4r-1)(4r)(4r+1)}{6}$

$= \frac{64r^3-4r}{6}$

$= \frac{(4r-5)(16r^2-28r+12)}{6} + \{\frac{(4r-3)(8r-6)}{2} + \frac{(4r-1)(8r-2)}{2}\}$

$= \frac{(4r-5)(16r^2-12r-16r+12)}{6} + \{\frac{(4r-3)(4r-4+4r-2)}{2} + \frac{(4r-1)(4r-2+4r)}{2}\}$

$= q' + \{\frac{(4r-4)(4r-3)}{2} + \frac{(4r-3)(4r-2)}{2} + \frac{(4r-2)(4r-1)}{2} + \frac{(4r-1)4r}{2}\}$.

Therefore $q = \frac{(4r-1)(4r)(4r+1)}{6}$ can be decomposed into $\{G_1, G_2, G_3, \ldots\ldots\ldots, G_{4r-1}\}$. Hence by induction hypothesis if $k+1 \equiv 0(\mathrm{mod}\,4)$, then every graph admits a Triangular Decomposition $\{G_1, G_2, G_3, \ldots\ldots\ldots, G_k\}$.

This complete the proof.

Lemma 2.9. If $k+2 \equiv 0(\mathrm{mod}\,4)$, then every graph admits a Triangular Decomposition $\{G_1, G_2, G_3, \ldots\ldots\ldots, G_k\}$ where k is the number of decomposition.

Proof. We have k = 4r-2, r ∈ N. We prove this theorem by using induction method. When r = 1, k = 2. Now $\frac{k(k+1)(k+2)}{6} = \frac{2x3x4}{6} = 4$ can be decomposed into $\{G_1, G_2\}$. Hence the result is true for r = 1.

Assume that the result is true for r-1. Then k = 4(r-1)-2 = 4r-6 and $q' = \frac{(4r-6)(4r-5)(4r-4)}{6}$ can be decomposed into $\{G_1, G_2, G_3, \ldots\ldots\ldots, G_{4r-6}\}$. Now to prove the result is true for r. Then k = 4r-2 and $q = \frac{(4r-2)(4r-1)(4r)}{6}$. We have to prove that $q = \frac{(4r-2)(4r-1)(4r)}{6}$ can be decomposed into $\{G_1, G_2, G_3, \ldots\ldots, G_{4r-2}\}$.

Now $q = \frac{(4r-2)(4r-1)(4r)}{6}$

$= \frac{64r^3 - 48r^2 + 8r}{6}$

$= \frac{(4r-6)(16r^2-16r-20r+20)}{6} + \{\frac{(4r-4)(8r-8)}{2} + \frac{(4r-2)(8r-4)}{2}\}$

$= q' + \{\frac{(4r-5)(4r-4)}{2} + \frac{(4r-4)(4r-3)}{2} + \frac{(4r-3)(4r-2)}{2} + \frac{(4r-2)(4r-1)}{2}\}$ Therefore $q = \frac{(4r-2)(4r-1)(4r)}{6}$ can be decomposed into $\{G_1, G_2, G_3, \ldots\ldots\ldots, G_{4r-2}\}$. Hence by induction hypothesis if $k+2 \equiv 0(\mathrm{mod}\,4)$ then every graph admits a Triangular Decomposition $\{G_1, G_2, G_3, \ldots\ldots\ldots, G_k\}$.

This complete the proof

Theorem 2.10. For any integer m, the Wheel graph W_m ($m \geq 3$) admits a Triangular Decomposition $\{G_1, G_2, G_3, \ldots\ldots\ldots, G_k\}$ if and only if thereexists an integer k satisfying the following properties:

(i) $m = \frac{k(k+1)(k+2)}{12}$

(ii) $k = \begin{cases} 4r-2 & r \in N - \{1\} \\ 4r-1 & r \in N \\ 4r & r \in N \end{cases}$

where k denotes the number of decompositions.

Proof. Let G = W_m. Then q(G) = 2m. Assume G has a Triangular Decomposition. By the definition of Triangular Decomposition, q(G) = $\frac{k(k+1)(k+2)}{6}$.

Hence $2m = \frac{k(k+1)(k+2)}{6}$.

⇨ $m = \frac{k(k+1)(k+2)}{12}$.

Since m is an integer, $k = \begin{cases} 4r-2 & r \in N-\{1\} \\ 4r-1 & r \in N \\ 4r & r \in N \end{cases}$

Conversely assume (i) $m = \frac{k(k+1)(k+2)}{12}$ (ii) $k = \begin{cases} 4r-2 & r \in N-\{1\} \\ 4r-1 & r \in N \\ 4r & r \in N \end{cases}$

Consider G = W_m. Let $V(W_m) = \{z, x_1, x_2, x_3, \ldots\ldots, x_m\}$ be the vertex set and $E(W_m) = \{zx_j/1\leq j \leq m\}\cup\{x_jx_{j+1}/1\leq j \leq m-1\}\cup\{x_mx_1\}$ be the edge set. Then q(G) = 2m. By lemma 2.7, 2.8 and 2.9, G can be decomposed into $\{G_1, G_2, G_3,\ldots\ldots\ldots,G_k\}$. Thus G admits Triangular Decomposition.

Table 2.11: List of first 10, TD of W_m.

m	q(G)	Triangular Decomposition
5	10	G_1, G_2, G_3
10	20	G_1, G_2, G_3, G_4
28	56	$G_1, G_2, G_3,\ldots\ldots\ldots,G_6$
42	84	$G_1, G_2, G_3,\ldots\ldots\ldots,G_7$
60	120	$G_1, G_2, G_3,\ldots\ldots\ldots,G_8$
110	220	$G_1, G_2, G_3,\ldots\ldots\ldots,G_{10}$
143	286	$G_1, G_2, G_3,\ldots\ldots\ldots,G_{11}$
182	364	$G_1, G_2, G_3,\ldots\ldots\ldots,G_{12}$
280	560	$G_1, G_2, G_3,\ldots\ldots\ldots,G_{14}$
340	680	$G_1, G_2, G_3,\ldots\ldots\ldots,G_{15}$

Illustration 2.12.

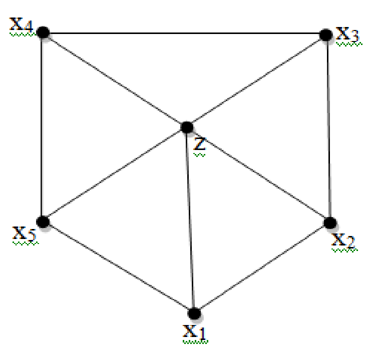

Figure 3. Wheel graph W_5

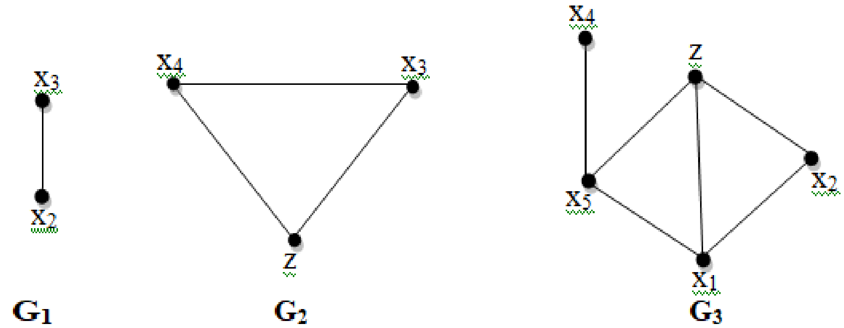

Figure 4. TD {G_1, G_2, G_3} of W_5.

Theorem 2.13. The Crown graph $C_m \odot K_1$ admits a Triangular Decomposition {G_1, G_2, G_3,……,G_k} if and only if there exists an integer k satisfying the following properties:

(i) $m = \frac{k(k+1)(k+2)}{12}$

(ii) $k = \begin{cases} 4r-2 & r \in N-\{1\} \\ 4r-1 & r \in N \\ 4r & r \in N \end{cases}$

where k denotes the number of decompositions.

Proof. Let $G = C_m \odot K_1$. Then q(G) = 2m. Assume G has a Triangular Decomposition. By the definition of Triangular Decomposition, $q(G) = \frac{k(k+1)(k+2)}{6}$.

Hence $2m = \frac{k(k+1)(k+2)}{6}$.

⇨ $m = \frac{k(k+1)(k+2)}{12}$.

Since m is an integer, $k = \begin{cases} 4r-2 & r \in N-\{1\} \\ 4r-1 & r \in N \\ 4r & r \in N \end{cases}$

Conversely assume (i) $m = \frac{k(k+1)(k+2)}{12}$ (ii) $k = \begin{cases} 4r-2 & r \in N-\{1\} \\ 4r-1 & r \in N \\ 4r & r \in N \end{cases}$

Consider $G = C_m \odot K_1$. Let $V(C_m \odot K_1)$= {x_1, x_2, x_3,……,x_m, y_1, y_2, y_3,……,y_m} be the vertex set and $E(C_m \odot K_1)$ = $\{x_jy_j/1 \le j \le m\} \cup \{y_jy_{j+1}/1 \le j \le m-1\} \cup \{y_1y_m\}$. By lemma 2.7, 2.8 and 2.9, G can be decomposed into {G_1,G_2,G_3,………,G_k}. Thus G admits Triangular Decomposition.

Table 2.14: List of first 10, TD of $C_m \odot K_1$

m	q(G)	Triangular Decomposition
5	10	G_1, G_2, G_3
10	20	G_1, G_2, G_3,G_4
28	56	G_1, G_2, G_3,………,G_6
42	84	G_1, G_2, G_3,………,G_7
60	120	G_1, G_2, G_3,………,G_8
110	220	G_1, G_2, G_3,………,G_{10}
143	286	G_1, G_2, G_3,………,G_{11}
182	364	G_1, G_2, G_3,………,G_{12}
280	560	G_1, G_2, G_3,………,G_{14}
340	680	G_1, G_2, G_3,………,G_{15}

Illustration 2.15.

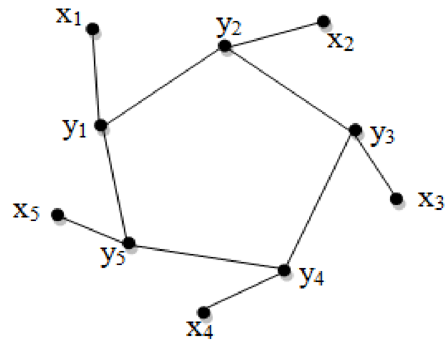

Figure 5. The Crown graph $C_5 \Theta K_1$

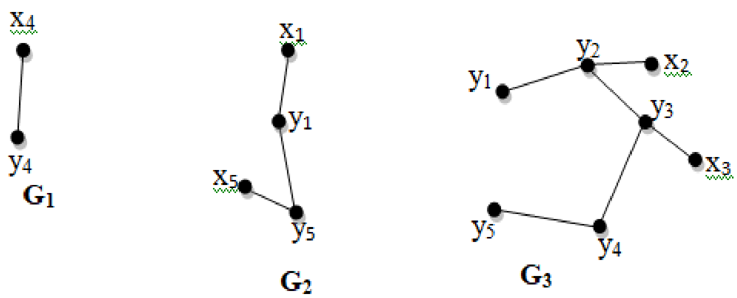

Figure 6. TD $\{G_1, G_2, G_3\}$ of $C_5 \Theta K_1$.

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