Solution Of Fuzzy Game Problem Using Octagonal, Pentagonal And Hexagonal Fuzzy Numbers Processed By Artificial Intelligence

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ABSTRACT

We have used Artificial Intelligence process for solving the fuzzy games containing Octagonal, Pentagonal and Hexagonal fuzzy numbers. This process is decision making process in the logic forms. We have made optimum solution of fuzzy game theory problems having, octagonal, pentagonal and hexagonal fuzzy numbers. In this case new ranking was used to order any fuzzy numbers. This is logical decision makers game theory for the study of mathematical models. In this solution a game situation involving a set of players who have a set of possible choices and in which outcome for any individual player depends partially on the choices made by the other players.

Keywords: Artificial Intelligence, Fuzzy game, octagonal, pentagonal, hexagonal, fuzzy number, optimum solution.

1. INTRODUCTION

Buckland[1] and Millington[2] used artificial intelligence as technique for injecting intelligence into video games for many different aspects of a video games for animation control, steering, flocking, path finding, procedural generation, tactical and straitening learning. The problems whose efficient solution required artificial intelligence algorithmic. This brought the non payable characters to present in the game. Johnson and Wiles[3] studied artificial intelligence for recreation of human like or human level cognitive processes and in the capability of these processes to learn. Scott[4] studied the puzzle of defining artificial intelligence was connected to game for actual existence of intelligence but it was just a illusion of it. Pinto and Alvares[5] studied the use of artificial intelligence in robotics and control system and same was used in fuzzy logic in response to games. Tozour[6] used the methods for artificial intelligence in different parallel direction for detour development and advanced for fuzzy games. Woodcock[7-8] presented CPU time given at

each game frame to the AI more than enough for most applications and some programmers. Chan etal[9] presented computation power of player nodes with peer to peer architectures and live migration techniques of central grid for game and implementation. Lebres etal[10] showed game drop out because of latency when a multiple regression was performed and for fine different predictors. Mohanty etal[11] studied back propagation of network and widely used for pattern classification and prediction. Lee and Chang[12] showed that it was suitable for regression for learner and were utilized in concealment of packet loss occurrences. Sangeetha and parimala[13] presented the solution of a fuzzy game using hexagonal fuzzy number. Thirucheran etal[14] presented a new approach for solving fuzzy game problem. Selva Kumari and Lavanya[15] solved a fuzzy game problem using octagonal fuzzy numbers. Thamariselvi etal[16] studied dthe optimal solution of fuzzy transportation problem using hexagonal fuzzy numbers. Kumar and Kumaraghuru[17] made solution of fuzzy game using triangular fuzzy number. Selva Kumari and Lavanya[18] made an approach to solve fuzzy game problem. Xu etal[19] studied the fuzzy sets to obtain the uncertainty fits with several situations. The fuzzy games strategic form in which the pay off of an agent is generally a fuzzy set. Such fuzzy sets arise when where is uncertainty on the worth that an agent got under certain strategy profiles. Gerasimou[20] studied the dominance of strategies in the class of interval fuzzy games and resulted a weakly higher interval pay off when facing any set of opponents profile. It was found that crisp games, a loosely dominant solution was a Nash equilibrium and was generalized to the class of multi-criteria games. Chakeri and Sheikholeslam[21] studied on triangular two person zero sum games which defined a graded representation of Nash equilibria in crips and fuzzy games. The fuzzy[22] equilibrium definition based on a binary fuzzy ordering relation is presented and applied into a traffic flow problem. Mallozzi and Puga[23] considered the Hurwicz criterion in fuzzy interval games in strategic form and allowed equilibrium definitions by choosing elements of the α- cuts depending on the pessimistic or optimistic attitude of the players. Wagnmare etal[24] studied the structure of group using first order partial differential equations. Mhaske and Bondar[25] studied fuzzy transportation problem by using Triangular Pentagonal and Heptagonal fuzzy numbers with Lagrange's polynomial to approximate fuzzy cost for Nonagon and Hendecagon. Mahaske[26] presented difference between fuzzy and crisp transportation problem using pentagonal fuzzy numbers with ranking by α-cut method. Deshmukh etal[27] presented Fuzzy database and fuzy logic using triangular and trapezoidal fuzzy number of coronavirus disease- 2019 diagnosis.

2. METHOD

Fuzzificaiton, interference and defuzzificaiton are the three stages of assembling fuzzy logic to determine dynamic problem solutions.

2.1 Fuzzification

The fuzzification stage involves the creation of a membership graph to determine the degree to which variable is a member[28]. The equation has been used to solve fuzzy trigonometric functions. The fuzzification stage maps the imuts based on the input variables degree of membership. Two input variables are present. This refers to the rate and duration of explorations[29].

2.2 Interference

Rules of determining rewards based on exploration percentage and time are established at this stage. During the fuzzification stage, this is computd. The rules that were developed are shown below.

IF		
Exploratory Presentation	Time	Reward
Low	Fast	Small
Low	Medium	Small
Low	Slow	Enough
Normal	Fast	Enough
Normal	Medium	Enough
Normal	Slow	Big
High	Fast	Enough
High	Medium	Big
High	Low	Big

Comparison of Algorithms A and B

2.3 Defuzzification

The players to explore extensively and thoroughly. The highest rewards take a long time to obtain, whereas the lowest rewards require less searching and a shorter time to obtain. The discovery stage's goal is to achieve this.

2.4 Fuzzy Method

Fuzzy Mathematics and fuzzy logic are used to make decision in Artificial Intelligence. Game theory, analysis is used in economic competition, economic conditions such as negotiation, auctions, voting theory etc. In real life situations the information available for decision making to select an optimum strategy. The crisp game theory problem is transformed into a fuzzy game theory problem by using triangular and trapezoidal fuzzy numbers. Any two fuzzy numbers, a new and simple method invented which is based on the area of membership function. There are three input classes and one output class in fuzzy method. For the output there is not a deportation Boolean class as in Naïve-Bayes classifier. The worst-condition class is used to determine these criteria. For downstream power, the Gaussian membership function has been selected. It is the perfect match as smoothness is desired and conditions start from worst, worse, good to best and then goes down the hill symmetrically. Optimal connection has the steepest curve as it demands a little bit closer range to zero millivolts. Bandwidth is important to some degree. A triangular member function is employed to show the direct proportional relation. After the threshold value, the result is found constant. If the game publishers share bounding values, every player calculated their network quality index without the need of any programming. A simulation within Matlab has been drawn in triangular member functions are selected for four distinct classes. Values in the range are equally divided.

2.5 Fuzzy Intervals

A payoff $x_i \in R$ of an agent i in a game is either possible or not possible to achieve. In the fuzzy logic, this possibility is not binary but uses a degree of membership that varied in the closed real-valued interval [0,1]. If the fuzzy number is 0, this payoff is not possible to achieve. If the fuzzy number is 1, this payoff is possible to achieve. All the intermediate situations are also possible. A fuzzy set in R is a real valued function $F : R \rightarrow [0,1]$ which associates with each $x_i \in R$ the grade of achievement $F(x_i)$ of x_i. This $F(x_i)$ is then called a fuzzy number[30].

2.6 Fuzzy Sets

In an ordinary set the characteristic function assigns a value of either 0 or 1 to each individual in the universal set. But this function is generalized is the case of fuzzy concept in such a way that the values assigned to the elements of universal set fall within a specified range in [0,1] a closed interval of real numbers.

2.7 Fuzzy Mathematical Approach

In classical game theory it is assumed that all data of a game are known exactly by players. In real games, the players are often not able to evaluate exactly the game due to lack of information and precision in the available information of the situation. In reality imprecision is of different types has been modeled by fuzzy sets. The notion of fuzzy sets shows the degree an element is a member of a set. It helps to fuzzify each component in a game such as: set of players, set of strategies, set of payoffs, stable sets. These notions lead to the proposition of fuzzy games.

2.8 Fuzzy Number

A fuzzy number $\tilde{A}$ is a fuzzy set on the real line R, must satisfy the following conditions.

1- There exist at least one $x_0 \in R$ with $\mu_{\tilde{A}} = 1$.

2- $\mu_{\tilde{A}(x)}$ is piecewise continuous.

3- $\tilde{A}$ must be normal and convex.

2.9 Trapezoidal Fuzzy Number

A fuzzy number $\tilde{A} = (a, b, c, d; 1)$ is said to be a fuzzy trapezoidal fuzzy number if its membership function is given by:

$$\mu_{\tilde{A}(x)} = \begin{cases} \dfrac{(x-a)}{b-a} & , \quad a \leq x \leq b \\ 1 & , \quad b \leq x \leq c \\ \dfrac{(d-x)}{(d-c)} & , \quad c \leq x \leq d \\ 0 & , \quad \text{otherwise} \end{cases}$$

2.10 Ranking Function

If $F(R)$ is the set of all fuzzy numbers defined on R the set of real numbers then a ranking function $R: F(R) \to R$ maps each fuzzy number into a real ordinary number where there is a natural order, the order rules are as

$\tilde{A} > \tilde{B}$ if and only if $R(\tilde{A}) > R(\tilde{B})$.

$\tilde{A} = \tilde{B}$ if and only if $R(\tilde{A}) = R(\tilde{B})$.

$\tilde{A} < \tilde{B}$ if and only if $R(\tilde{A}) < R(\tilde{B})$.

Where A and B are two fuzzy numbers belong to $F(R)$.

2.11 α-cut of Fuzzy set

The α-cut of α-level set of fuzzy set $\tilde{A}$ is a set consisting of those elements of the universe X whose membership values exceed the threshold level. That is $\tilde{A}_\alpha = \left\{ \dfrac{x}{\mu_{\tilde{A}}(x)} \geq \alpha \right\}$.

2.12 Convex Fuzzy Set

A fuzzy set $\tilde{A}$ is a convex fuzzy set if any and only if each of its α-cut A_α is a convex set.

2.13 Triangular Fuzzy Number

It is a fuzzy number represented with three points as follows: $\tilde{A} = (a_1, a_2, a_3)$. This representation is interpreted as membership functions and holds the following conditions

(i) a_1 to a_2 is increasing function

(ii) a_2 to a_3 is decreasing function

(iii) $a_1 \leq a_2 \leq a_3$.

Its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \leq x \leq a_2 \\ 1 & \text{for } x < a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{for } x > a_3 \end{cases}$$

2.14 Octagonal Fuzzy Number

A fuzzy number $\tilde{A}$ is a normal octagonal fuzzy number denoted by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ where $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6 \leq a_7 \leq a_8$ are real numbers of its membership function $\mu_A(x)$.

2.15 Pentagonal Fuzzy Number

A fuzzy number $\tilde{A}_{w_1,w_2} = (a_1, a_2, a_3, a_4, a_5)$ is called a pentagonal fuzzy number when the membership function has the form. The middle point a_3 has grade of membership 1 and r, s are the grades of points a_2, a_4.

2.16 Hexagonal Fuzzy Number

A fuzzy number $\tilde{A}_{w_1,w_2} = (a_1, a_2, a_3, a_4, a_5)$ is called a hexagonal fuzzy number when the membership function has the form. Where the middle point a_3 has the grade membership 1 and r, s are the grades of points a_2, a_4.

2.17 Support of Fuzzy Set

The support of a fuzzy set $\tilde{A}$ of the set X is a classical set defined as

$$\text{Sup}(\tilde{A}) = \{x \in X : \mu_{\tilde{A}}(x) > 0\}.$$

2.18 Value of Game

This is the expected pay off at the end of the game, when each player uses his optimal strategy.

2.19 Crisp Set

A crisp set is a special case of a fuzzy set in which the membership function only takes two values, commonly defined as 0 and 1.

2.20 Pure Strategy

Pure strategy is a decision making rule in which particular course of action is selected.

2.21 Mixed Strategy

A set of strategies that a player chooses a particular move of the game some fixed probability are called mixed strategies.

2.22 Saddle Point

If the maximum value equals to the minimum value, then the game is said to have a saddle point and the corresponding strategies which give the saddle point are called optimal strategies. The amount of payoff at an equilibrium point is called the crisp game value of the game matrix.

3. RESULTS AND DISCUSSION

A game is a decision making situation with many players, each having objectives that conflict with each other. The player involved the game usually make their decision under conditions of risk or uncertainty. A fuzzy approach is to solve the strategic game problem in which the pure strategy set for each player is defined. Based on the concepts of fuzzy set theory, the approach uses a multi-criteria decision making method to obtain the optical strategy in the game, a method which shows more advantages than the classical strategy and shows better results. To apply fuzzy logic to determine the priority of a pay off to other payoffs. In this method the fuzzy payoffs, fuzzy satisfaction functions and satisfaction degree from each payoff are defined. The calculation of similarity between satisfaction functions enables making crips from the fuzzy one.

3.1 Octagonal Fuzzy Number Definition

A fuzzy number $\tilde{A}$ is a normal octagonal number denoted by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ where $a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6 \le a_7 \le a_8$ are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given as.

$$
\mu_{\tilde{A}}(X) \begin{cases}
0 & \text{for } x < a_1 \\
k\left(\dfrac{x-a_1}{a_2-a_1}\right) & \text{for } a_1 \le x \le a_2 \\
k & \text{for } a_2 \le x \le a_3 \\
k + (1-k)\left(\dfrac{x-a_3}{a_4-a_2}\right) & \text{for } a_3 \le x \le a_4 \\
1 & \text{for } a_4 \le x \le a_5 \\
k + (1-k)\left(\dfrac{a_6-x}{a_6-a_5}\right) & \text{for } a_5 \le x \le a_6 \\
k & \text{for } a_6 \le x \le a_7 \\
k\left(\dfrac{a_6-k}{a_6-a_7}\right) & \text{for } a_7 \le x \le a_8 \\
0 & \text{for } x < a_8
\end{cases}
$$

Where $0 < k < 1$

Suppose $A = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$ be two octagonal fuzzy numbers then we have the coefficient of fuzzy numbers from the triangle of Pascal and applying the approach
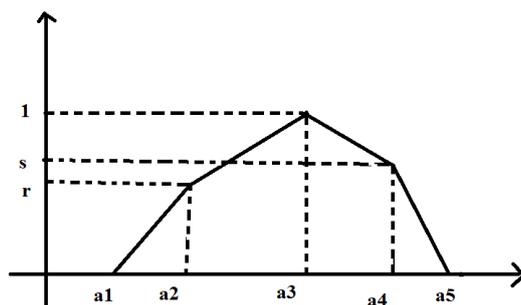
$$
p(A) = \frac{x_1 + 7x_2 + 21x_3 + 35x_4 + 35x_5 + 21x_6 + 7x_7 + x_8}{128}
$$

The coefficient $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$ are 1,7,21,35,35,21,7,1. This method is applied to the octagonal fuzzy order of n-dimensional Pascal. By using Pascal's triangle graded mean approach, we can convert given octagonal fuzzy numbers to a crisp value.

3.2 Pentagonal Fuzzy Number

A fuzzy number $\tilde{A}_{w_1, w_2} = (a_1, a_2, a_3, a_4, a_5)$ is called a pentagonal fuzzy number when the membership function has the form. The middle point a_3 has grade of membership 1 and r, s are the grades of points a_2, a_4. Its membership function is defined as

$$
\mu_{\tilde{A}}(x; r, s) =
\begin{cases}
0 & , \ x < a_1 \\[2mm]
r\dfrac{(x - a_1)}{a_2 - a_1} & , \ a_1 \le x \le a_2 \\[2mm]
1 - \dfrac{(1 - r)(x - a_2)}{a_3 - a_2} & , \ a_2 \le x \le a_3 \\[2mm]
1 & , \ x = a_3 \\[2mm]
1 - \dfrac{(1 - s)(x - a_3)}{a_4 - a_3} & , \ a_3 \le x \le a_4 \\[2mm]
\dfrac{s(x - a_5)}{a_4 - a_5} & , \ a_4 \le x \le a_5 \\[2mm]
0 & , \ x > a_5
\end{cases}
$$

Pentagonal Fuzzy number a_1, a_2, a_3, a_4, a_5

3.3 Ranking of Pentagonal Fuzzy Number

Suppose $\tilde{A}_{r,s} = (a_1, a_2, a_3, a_4, a_5)$ be a Pentagonal fuzzy number with $\mu_{\tilde{A}}(x)$ is membership function and $\sup(\tilde{A}) = (a_1, a_5)$.

Area of membership function $\mu_{\tilde{A}}(x)$ over (a_1, a_5)

$$
= \frac{1}{2}(a_2 - a_1)r + (a_3 - a_2)r + \frac{1}{2}(a_3 - a_2) \times (1 - r)
$$
$$
+ \frac{1}{2}(a_4 - a_3)(1 - s) + s(a_4 - a_3)
$$
$$
+ \frac{1}{2}(a_5 - a_4)s.
$$

$\therefore$ Area of membership function $\mu_{\tilde{A}}(x)$ over

$$
[a_1, a_5] = \frac{1}{2}\{a_3(r - s) + a_1(-r) + a_5 s - a_2 + a_4\}
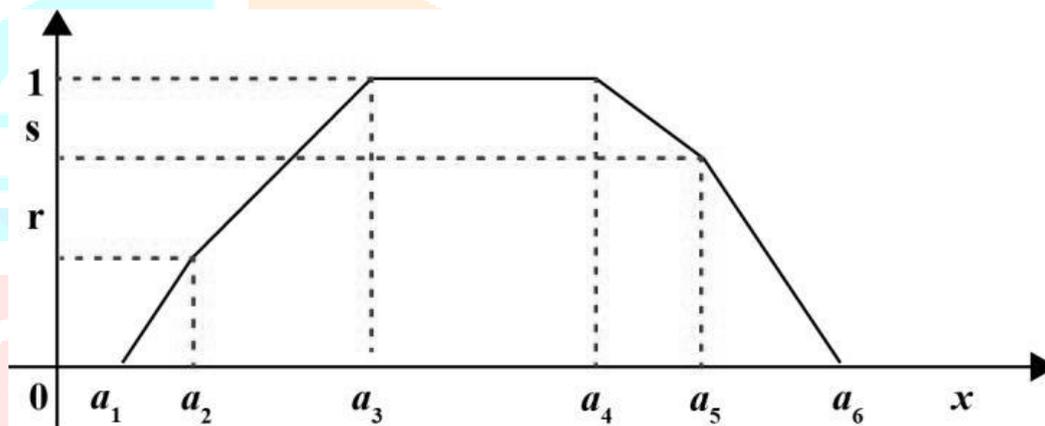$$

$$
\therefore \ R\tilde{A} = (a_1 + a_5)\left[\frac{1}{a_5 - a_1} \times \frac{1}{2}\{a_3(r - s) + a_1(-r) + a_5 s - a_2 + a_4\}\right]
$$

$$
\therefore \ R(\tilde{A}) = \frac{a_1 + a_5}{2(a_5 - a_1)}\{a_3(r - s) + a_1(-r) + a_5 s - a_2 + a_4\}
$$

3.4 Hexagonal Fuzzy Number

A fuzzy number $\tilde{A}_{w_1,w_2} = (a_1, a_2, a_3, a_4, a_5)$ is called a hexagonal fuzzy number when the membership function has the form. Where the middle point a_3 has the grade membership 1 and r, s are the grades of points a_2, a_4. Its membership function is defined as

$$\mu_{\tilde{A}}(x; r, s) \begin{cases} 0 & , \ x < a_1 \\[2mm] \dfrac{r(x - a_1)}{(a_2 - a_1)} & , \ a_1 \le x \le a_2 \\[3mm] 1 - \dfrac{(1-s)(x - a_3)}{a_3 - a_2} & , \ a_2 \le x \le a_3 \\[3mm] 1 & , \ a_3 \le x \le a_4 \\[2mm] 1 - \dfrac{(1-s)(x - a_3)}{a_4 - a_3} & , \ a_4 \le x \le a_5 \\[3mm] \dfrac{s(x - a_5)}{(a_4 - a_5)} & , a_5 \le x \le a_6 \\[3mm] 0 & , \ x > a_6 \end{cases}$$

Hexagonal Fuzzy Number $(a_1, a_2, a_3, a_4, a_5, a_6)$

3.5 Ranking of Hexagonal Fuzzy Number

Suppose $\tilde{A}_{r,s} = (a_1, a_2, a_3, a_4, a_5, a_6)$ be Hexagonal number with $\mu_{\tilde{A}}(x)$ is membership function and $\sup(\tilde{A}) = (a_1, a_5)$. Area of membership function $\mu_{\tilde{A}}(x)$ over $(a_1, a_2) =$

$$= \frac{1}{2}(a_2 - a_1)r + r(a_3 - a_2) + \frac{1}{2}(a_3 - a_2)(1 - r) + (a_4 - a_3)$$
$$\quad + \frac{1}{2}(a_5 - a_4)(1 - s) + s(a_5 - a_4) + (a_6 - a_5)s.$$

$\therefore$ Area of membership function $\mu_{\tilde{A}}(x)$ over $[a_1, a_5]$.

$$= \frac{1}{2}\{a_1(-r) + a_3(r-1) - a_4(s-1) + a_6 s - a_2 + a_5\}$$

$$\therefore \ R(\tilde{A}) = (a_1 + a_2)\left[\frac{1}{a_6 - a_1} \times \frac{1}{2}\{a_1(-w_1) + a_3(w_1 - 1) - a_4(w_2 - 1) + a_6 w_2 - a_2 + a_5\}\right]$$

$$\therefore R(\tilde{A}) = \frac{a_1 + a_6}{2(a_6 - a_1)}\{a_1(-r) + a_3(r-1) - a_4(s-1) + a_6 s - a_2 + a_5\}$$

1. **Example**

Considering the following game problem

Player B

$$\text{Player A} \begin{bmatrix} (1,2,3,4,6,9) & (8,9,11,12,14) \\ (-2,-1,0,3,5) & (-5,-3,-1,0,1) \end{bmatrix}$$

Taking $r = 0.3, s = 0.6$

Solution by definition of Ranking of Pentagonal fuzzy number suppose $\tilde{A}_{w_1,w_2} = (a_1, a_2, a_3, a_4, a_5)$ be a triangular fuzzy number with $\mu_{\tilde{A}}(x)$ is a membership function and $\sup(\tilde{A}) = (a_1 - a_5)$

$$R(\tilde{A}) = \frac{a_1 + a_5}{2(a_5 - a_1)} \{ a_3(r-s) + a_1(-r) + a_5 s - a_2 + a_4 \}$$

The Pay off Matrix is

Player B

$$\text{Player A} \begin{bmatrix} 4.9375 & 10.45 \\ \dfrac{57}{55} & -1.8 \end{bmatrix}$$

Minimum of 1st row =4.9375 and minimum of 2nd row =-1.8. Maximum of 1st column =4.9375 and maximum of 2nd column = 10.45

$\therefore$ Maximin =4.9375 and Minimax = 4.9375

Where crisp values are, $R(a_1) = 4.9375, R(a_{12}) = 10.45, R(a_{21}) = \dfrac{57}{35}$ and $R(a_{22}) = -1.8$.

It has saddle point

$\therefore$ Strategy for player $A = A_1$ and Strategy for player $B = B_1$.

Value of the game $V = 4.9375$

2. **Example**

Considering the following fuzzy game problem

Player B

$$\text{Player A} \begin{bmatrix} (0,2,4,5,6,9) & (-5,-3,-2,1,0,1) \\ (-4,-3,-1,0,1,2) & (5,8,9,11,12,13) \end{bmatrix}$$

Taking $r = 0.4, s = 0.5$

Solution from definition of ranking of Hexagonal fuzzy number suppose $\tilde{A}_{r,s} = (a_1, a_2, a_3, a_4, a_5, a_6)$ be a Hexagonal fuzzy number with $\mu_{\tilde{A}}(x)$ is a membership function and $\sup(\tilde{A}) = (a_1, a_6)$

$$R(\tilde{A}) = \frac{a_1 + a_6}{2(a_6 - a_1)} \{ a_1(-r) + a_3(r-1) - a_4(s-1) + a_6 s - a_2 + a_5 \}$$

The Pay off Matrix is

$$\text{Player A} \begin{array}{c} \text{Player B} \\ \begin{bmatrix} 4.3 & -2.4 \\ -1.2 & 9.675 \end{bmatrix} \end{array}$$

Minimum of 1st row =-2.4 and minimum of 2nd row =-1.2. Maximum of 1st column =4.3 and maximum of 2nd column = 9.675

$$\therefore \text{Maximin } = -1.2 \text{ and Minimax} = 4.3$$

$$\therefore -1.2 \neq 4.3$$

It has no saddle point

Where crisp values are, $R(a_{11}) = 4.3, R(a_{12}) = -2.4, R(a_{21}) = -1.2$ and $R(a_{22}) = 9.675$.

A crisp set is special case of fuzzy set in which he membership function only takes two values, and defined as 0 and 1.

3.6 CONCLUSION

We have made solution of fuzzy games problems using octagonal, pentagonal and hexagonal fuzzy numbers processed by artificial intelligence. For this purpose crisp fuzzy games problems were converted into fuzzy game in response to octagonal, pentagonal and hexagonal fuzzy names. The new ranking procedure was utilized to obtained favorable results. The Pascal's triangular graded mean integration was used to obtain the solution. For solution of the problem the pay offs were changed. Game theory field is the decision making theory. The conversion of fuzzy valued problem into a crisp valued game problem by changing the pay offs was made in the case octagonal fuzzy number. The obtained results were found in good agreement with previously found results.

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