IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

A SYSTEMATIC REVIEW ON THE 'BALANCE MODEL' FOR TEACHING LINEAR EQUATIONS

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Abstract: The balancing model is among the most frequently used approaches to teaching and learning of linear equations, and this research offers a literature consideration of the systematic application of the framework. Finding out why this model is employed was the main goal of the review. After reviewing twenty peer-reviewed journal publications, we have a comprehensive summary of the presented justifications for utilizing the balancing model, its appearances, its applications, and the learning results. In order to make well-informed judgments on the successful use of balancing models for teaching linear equation solving, more systematic research is required.

***Index Terms* - Teaching linear equations, algebra, and the balance model.**

I. INTRODUCTION

The ability to solve algebraic problems is a crucial skill for every algebra graduate. One of the first steps in learning algebra is solving linear equations, which allows pupils to go from using numbers to reasoning with unknowns (Sanders, Y. (2016), Filloy & Rojano, 1989). Furthermore, Carraher et al. (2017) defined early algebra as the transition 'from thinking about relations between specific numbers and measures to thinking about relations between sets of numbers and measures, from finding numerical solutions to explaining and modeling relations between variables.' Middle school mathematics curricula place a heavy emphasis on teaching students how to solve linear equations (Otten et. al., 2019; Garfield et. al., 2022). According to Al-Mutawah et. al. (2019) and Prismana et. al. (2018), a large number of students struggle to grasp the fundamentals of solving equations and struggle to understand the ideas and abilities in this area.

I must explain that if you need to find out how to solve linear equation , then you have to be aware of the fact that in order to get the values of the unknown quantities, it is necessary to compare the two mathematical expressions which are located in between the sign of equality. For solving linear equations, there is a need to understand equality (Indrzejczak, A. 2021) and one of the most fundamental requirements for solving equations that involves a mathematical symbol is that the symbol represents the same thing (Crooks, N. M., & Alibali, M. W. (2014)). It is important to realize that they are solving an equation: each symbol on the left and right half of the equivalent sign ought to be of a similar worth (Kieran et al., 2016). The possibility of balance in addressing straight equations is a topic that has long been the subject of misunderstanding. One area where pupils' misunderstandings of the equal sign are most apparent is in their mathematical understanding. In contrast to its relational meaning of "is the same as," pupils often interpret the equal sign as an operational signal to "do something" or "calculate the answer" (Ardiansari et. al., 2020). By way of illustration, according to Lipscombe (2021), a typical error while attempting to solve the equation 8 + 4 =

__ + 5 is to add the two integers on the left side and then substitute a 12.

According to Fyfe et al., this idea of the equal sign may begin in elementary school and continue until middle school. 2020).This is especially so because models can be described as “tools for formulating theories and analyzing information” (Otten et.al., 2019) and thus, one of the ways of helping the students gain conceptual understanding as they solve equations. When examined this way, these instructional Models can be thought of as representations of mathematical problems. They sum up the problem's key mathematical ideas and components and show how the real-life scenario relates to the more abstract form of mathematics (Vergnaud, G. (2016). The model may be utilized to solve numerous new issues since it starts as a model of a particular circumstance and eventually adapts to different contexts where it can be used to comparable problems (Rebello et.al;.2017).

Various didactical models are used in mathematics education to help students understand certain mathematical topics. For example, the number line and bar model are used to teach fractions. Among the many models employed in the field of education, the balancing model stands out. When teaching pupils to solve linear equations, this model is a common tool to utilize. These stages coordinate to form the balancing model where the shape resembles the function of balancing, equal liquid in two different beakers that we indicated that the equations are balanced because the expressions on both sides are equal. Leibniz, the philosopher and mathematician, previously discussed a connection between mathematical equality and equal material objects placed at both sides: Thus, with regard to matter, the equality is on the one side and the equal objects on the other side. In a more extensive study of algebraic approaches, we were in fact hoping to find hints to indicate an organization for a lesson plan in solving linear equations. Concerning the use of the balancing model for teaching mathematical algorithms, we considered whether it could assist students in mastering techniques for solving linear equations.Our first search yielded a diversified and fragmented image, so we decided to study this more methodically. As a result, we intended to conduct a literature study to see where the balancing model appears in the many academic and professional works on the topic of teaching students to solve linear equations. We set out to address the following research topic with this review:In research on how to best instruct students to solve linear equations, what part does the balancing model play?It is crucial to get insight into many significant components of a didactical model in order to understand more about it and how it may be used to teach students (mathematical) ideas. To get a full view of the model's potential applications, it is necessary to consider not only the model's exact representation but also data pertaining to probable reasons for selecting this model and the time of its usage in a learning and teaching trajectory. Finally, students' learning outcomes should be considered when evaluating the efficacy of a didactical approach in improving students' conceptual comprehension. In order to ascertain the function of balance-models in research on the instruction of linear equation solving, we reviewed the authors' descriptions of the situations in which these models are used, the kinds of models utilized, the timing of their application, and the results of the related learning outcomes. Teachers, researchers, and curriculum creators may use this information to make better judgments when selecting a balance model to teach students how to solve linear equations.

II. METHOD:

1. ARTICLE SEARCH AND SELECTION

We combed through fifty peer-reviewed research publications covering mathematical education, educational sciences, pedagogy, special education, and technology in education to find the papers we wanted to include in the review. Articles written in English were sought for using Springer, Eric, and Google Scholar.

Line equation, equal sign, equality, equivalence, balance, algebra, math, unknown and solve can be considered as the possible search terms.

We found articles with very recent publication dates. Scopus, ERIC, and Google Scholar were among the more than 600 results returned by the search in June 2024. Afterwards, items that were not written in English were also eliminated, and duplicates were found. Titles and abstracts were vetted using a six-step process.

2. DATA EXTRACTION

Data was culled from each of the twenty articles with respect to the following: the benefits and drawbacks of the balance model, the model's outward look, and the results of student learning. From a single article, many justifications for using the model might be derived from the search results. Specifically, to detail the contexts when the balancing model was put into play.

3. RESULTS

Why was the balance model used?

The papers that were looked for provide justifications for employing the balancing model. Depending on the details of the balancing model's setting, three distinct categories of justifications emerged. Also retrieved were the constraints of the balancing model as a tool for instructing students in the solution of linear equations.

III. RATIONALES RELATED TO THE EQUALITY CONCEPT

The idea of equality was central to the justifications given for using the balancing model in most of the publications. According to several sources (Law, D. 2015, Theodora, et. al., 2018, Tan et. al., 2023), the notion of equality may be better understood by referring to the model of a balance. One reason why balancing models are so useful for illustrating concepts like quantitative sameness (Mainaly B. 2021, Bobis et.al., 2018) and equality (Lehtonen, D. (2022)) is because their two halves are interchangeable and of equal worth. Several writers have made reference to the balancing model in support of this idea, arguing that it helps clarify the meaning of the equal sign (Law, D. 2015). As a result, the balance model is often said to be a good way to show students how to do exactly the same thing on the two sides of a situation, which is a significant strategy (Skovsmose, O. 2020; Marschall & Andrews, 2015), and it also helps students visualize the operations they need to apply (Vlassis et. al., 2022). Another of the other advantages of the balance model is that it is possible to use the model to demonstrate how terms on both sides of the equation can be cancelled because “it is possible to monitor the equation's entire numerical relationship while it is being processed for transformations” ((Konopelchenko, B. G;, 2013).

IV. RATIONALES RELATED TO THE PHYSICAL EXPERIENCES

A second group of justifications was found to be associated with physical learning experiences. Some of the papers touched on the importance of prior physical experiences in balancing. Keeping one's equilibrium is something that every human being is familiar with because of its fundamental biological base, according to Araya et al. (2010).

One way to link this first-hand biological information to the more abstract concept of keeping an equation equal is by use the balancing model. Bajwa et al. (2021) noted that some people saw parallels between the model and a teeter-totter, which is a kind of see-saw, and made reference to the fact that children often play with such toys. Additional research has shown that learning linear equations in conjunction with contemporaneous physical experiences using the balancing model is an effective strategy. Moving one's body (to demonstrate a balance, for instance) and making expressive gestures are crucial components of the learning trajectory for conceptualizing mathematical concepts (Norman, D. A. 2014). They said that thinking back on these events at a later point in time might help with learning. The significance of hands-on activities using real-world items in fostering comprehension of linear equations was also highlighted in several papers. Donavan et al. (2022) found that giving young children hands-on experience with balance scale manipulation helped them better comprehend the notion of equality since it allowed them to see, define, construct, and sustain it. Using physical items that can be handled has a sense-making function, according to Scheiner (2016). This is achieved by bridging the gap between procedural knowledge (the things themselves) as well as a conceptual comprehension of algebraic equations. However, these authors did suggest that teachers should exercise care when utilizing manipulatives to teach formal equation solving, as not all students make the immediate connection between manipulating the manipulatives and manipulating an abstract symbol. According to Goldin, G. A. (2020), the balancing model may serve as a physical tool for developing abstract mathematical reasoning, as it stands as a kind of intermediary between concrete sensory input and mathematical abstraction. Simultaneously, (Fyfe et al. 2019) supported a sequence-based on fading concreteness approach, in which the introduction of abstract mathematical symbols is gradual after the introduction of concrete material. Some models also provide students real-time feedback on their balance, which is crucial because it lets them check the outcomes of their manipulations and reasoning processes, which in turn helps them build knowledge (Booth, et. al., 2013). Examples of how physical experiences, when coupled with social ones, may benefit in knowledge formation are Kelly et.al. (2013) and LenkauskaitĎ et.al. (2020), both of which establish shared meaning between the instructor and the students.

V. RATIONALES RELATED TO LEARNING THROUGH MODELS AND REPRESENTATIONS

A more broad-ranging argument including learning via models and representations was part of the third category of rationales, which was brought up in eight publications (four of which were part of the same study project). Sanders (2016) states that comparable models, such as the balancing model, provide a chance to define and linguistically establish a framework for solving linear equations. Here, students may first deduce the context-based meaning of algebraic operations and equality, and then, after abstracting, they can establish a connection between this context-based meaning and the syntactic level meaning. An argument put forth by researchers from the Australian Early Algebraic Thinking Project (Mainaly B 2021, Bobis et.al., 2018) is that models serve to externalize mathematical concepts through language, symbols, or iconic representations, and that understanding these concepts takes place internally, in mental models or internal cognitive representations of the underlying mathematical ideas. According to this theory, a student's capacity to grasp mathematical concepts is dependent on the density and quality of the connections inside their own internal representational network. Additionally, it was suggested that when teaching abstract mathematical concepts or strategies, it would be beneficial to use multiple representations. This is because, as Schneider (2016) points out, students can gain a deeper understanding of mathematics by experiencing and connecting various modes of representation.

Scheiner (2016) built on the idea that representations serve as sensemakers by arguing that students may understand By connecting the symbolic equation to its representation, one can abstract symbolic equations. Justifications for the utilization of balance model representations were also advanced. For instance, it is believed to reduce students' cognitive burden when solving equations (Araya et al., 2010) or that it may provide a common linguistic basis that students can use to describe their results (Law, D. 2015).

VI. LIMITATIONS OF THE BALANCE MODEL

According to Covington et al. (2019), middle school students learned formal linear equation solving using the balance model. The authors came to the conclusion that while the model had its benefits—such as giving students a "operative mental image" of the strategies that would be used to solve the equations—it also had its drawbacks. As an example, equations with negative integers or those that are no longer related to the model in any way were not helped by the model. According to a different study by Ngu B. H. and Phan H. P. (2022), eleventh graders had difficulty solving linear equations (such as $5x - 3 = 4x + 7$) that they should have learnt in middle school, even when using the balancing model.

VII.DISCUSSION OF THE FINDINGS REGARDING WHY THE MODEL WAS USED

All three types of rationales are distinct from one another, yet they are also linked and have certain commonalities. A common thread that emerged was a need to comprehend equality, which is considered a fundamental need for solving linear equations (Kieran et al., 2016). The notion of equality and the tactics that might be implemented to preserve balance were linked to the inherent features of the balance. Fewer people brought up the other two justifications. In these justifications, the use of the balancing model was hintted to in a roundabout way, using allusions to physical experience and representational learning, as a means to improve students' comprehension of equality in an equation. The articles in this class of reasonings about actual encounters either examined the natural premise of equilibrium or related actual encounters (like adjusting on a seesaw) that could be connected to the idea of keeping up with balance in a situation through the equilibrium model.It is possible that pupils' prior physical experiences with balancing may help them better grasp the concept of equality in equations. This makes sense when seen through the lens of embodied cognition theory, which posits that creating conceptual knowledge and cognitive learning processes relies on the integration of our perceptual and bodily experiences in our interactions with the environment

(Barsalou, L. W., 2020). According to Abrahamson et al. (2021) and Abrahamson et al. (2017), perceptuo-motor experiences are important for the growth of mathematical concepts and reasoning is seen as closely related to embodied activities. The underlying assumption of embodied cognition theory in the context of teaching and learning linear equations is that students must first acquire perceptuo-motor knowledge of the balancing action before they can gain a comprehension of the mathematical idea of equality (Barsalou, L. W. (2020).

Class articles on learning from models and representations contained broader justifications for helping students grasp the concept of equality in equations. Having said that, there is some repetition here with the rationales pertaining to the tangible senses. They both have something to do with perceptuo-motor experiences involving balance. This experience is more relevant to the course's focus on models and representations and how balance is conceptualized. You may think of the balance as a central fulcrum supporting two equal-value expressions on each side of the equal sign, much like an equation. Introduced in 2001 in Indonesia by the Realistic Mathematical Education of Indonesia (also known as Pendidikan Matematik Realistik Indonesia or PMRI), Realistic Mathematics Education (RME) is one such method of instruction and assessment. The purpose of PMRI is to enhance and transform mathematics instruction, according to Sembiring RK et al. (2008). Using didactical models to connect less formal, context-based solution approaches with more formal ones is a central tenet of RME training; the goal is to help students make the transition to the latter and improve their knowledge (Menanti et. al., 2008).

VIII. DISCUSSION OF THE LEARNING OUTCOMES

In general, it seems that the balancing model has a greater beneficial impact on learning outcomes connected to solving linear equations for (younger) pupils who do not have any previous understanding of this task. This could be because the balancing model is more often used to refresh the conceptual foundation of solving linear equations for older students, as opposed to younger children who are just starting to learn the idea, or to build on it. The balancing model is an excellent tool for introducing younger pupils to the idea of equality and helping them solve linear equations. Students in higher grades often use the balancing model to model, convert, or solve problems. That is to say, when they have a solid foundation in solving linear equations, they may apply the balancing model to a wide variety of new problems. An example of how students might use the balancing model to solve problems involving subtraction was given by Borenson, H. (2023). First in their lesson plan was a physical model that students could use to learn about equality as "balance" and the technique of mirroring one another's actions. In subsequent lessons, students may use this method to solve problems on paper that included symbolically notated equations and subtraction.

IX. CONCLUSIONS

When it comes to the balancing model's usefulness as a tool for teaching linear equations, our systematic study paints a pretty patchwork picture. Research on the factors that led to the usage of balancing models, the models themselves, the contexts in which they were implemented, and the resulting learning outcomes is very variable. Authors' justifications for writing these pieces were also more transparent when they used the balancing model; the majority of these justifications included the equality feature and the students' actual physical sensations. Studies generally found that the balancing model improved students' learning results when it came to solving linear equations, and the equations taught to these students mostly included positive values and addition. For pupils who have taken algebra before, drawn balance models were a more common tool. To make these models more versatile, they would frequently have other elements added to them, allowing them to represent a broader variety of issues, including those involving negative numbers and subtraction. The articles that claimed to have employed drew balancing models to teach students how to solve linear equations were vague on why they did it, and they often found that the model had both positive and bad impacts. Though these tendencies were present, several studies nevertheless differed from one another, especially with regard to the length of the intervention and the specifics of the students' lessons.

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