



Power Dominator Edge Coloring Of Certain Classes Of Graphs

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Abstract: A dominator edge coloring of G , is a proper edge coloring of G , in which every edge of some color class (possibly its own class) are dominated by every edge in G . The dominator edge chromatic number of G , is the least number of color classes among all dominator edge colorings of G , denoted by $\chi'_d(G)$. We introduce a new variate known as power dominator edge coloring based on the notion of power edge domination and determine the power dominator edge chromatic number for certain classes of graph. **Power Dominator Edge Coloring** of a graph G , refers to the proper edge coloring where each edge in the edge set power dominates every edge of at least one-color class. The Power Dominator Edge Chromatic number (PDEC-number) $\chi'_{pd}(G)$, is the minimum number of colors that a power dominator edge coloring of the graph requires.

Index Terms: Dominator edge coloring, Power dominating set, Power dominator edge coloring

1. Introduction:

A graph is a data structure used to represent a set of objects, known as vertices or nodes, and the connections or relationships between them, known as edges. In this paper we consider only finite, undirected graphs. Two edges have a common vertex are called adjacent edges also we can say that an edge dominates all its adjacent edges. A *tadpole graph* $T(m, n)$ is the graph, [3] obtained by joining by an edge, a vertex of the cycle C_m , $m \geq 3$ and an end vertex of the path P_n ; $n \geq 1$. The *m-book graph* B_m is the graph [3] $S_{m+1} \times P_2$, the Cartesian product of S_{m+1} and P_2 where S_{m+1} is the star graph and P_2 is the path on two vertices. The *n-sunlet graph* S_n is a graph on $2n$ vertices with a cycle C_n and each vertex of the cycle being joined to a new pendant vertex [4].

Let $G(V, E)$ be a graph. A subset $D' \subset E$ is called an edge dominating set if every edge of not in D' is adjacent to an element of D' . The number of elements of a minimum edge dominating set is called an edge dominating set and is denoted by $\gamma'(G)$. A *proper edge coloring* of a graph G is a function $c: E(G) \rightarrow \{1, \dots, k\}$ ($k \in \mathbb{N}$) ensuring that no adjacent edges have the same color. The minimum number of colors needed for a proper coloring of the graph is called its chromatic number and is represented as $\chi'(G)$. [3] "A dominator edge coloring of G , is a proper edge coloring of G in which each edge of G dominates every edge of some color class (possibly its own class). The dominator edge chromatic number of G , is the minimum number of color classes among all dominator edge colorings of G , denoted by $\chi'_d(G)$."

A subset $S \subseteq V$ is a power dominating set of G if all vertices of V can be observed recursively by the following rules: (i) all vertices in $N[S]$ are observed initially, and (ii) if an observed vertex u has all neighbours observed except one neighbor v , then v is observed (by u). The minimum cardinality of a power dominating set of G is called the power domination number of G and is denoted by $\gamma_p(G)$.

Sathish [4], studied the concept of Power dominator coloring of a graph and the chromatic number is denoted by $\gamma_{pd}(G)$.

2. PDEC- Number of Graph

Informed by the power dominator coloring concept for graphs, we have merged the two concepts namely, dominator edge coloring and power domination of graphs to introduce a new variation of coloring known as a power dominator edge coloring. This technique mandates that each edge in a color class power dominates every other edge. Prior to this, we revisit the concept of monitoring set for edges in graphs.

Definition 2.1 In a graph G , we correlate a set of monitoring set $M(e)$ with an edge e , as follows:

Step: 1 $M(e) = N[e]$; the closed neighbourhood of e

Step: 2 If g is the neighbour of f and $M(e)$ already contains all of g 's neighbours except for f , then add edge f to $M(e)$.

Step: 3 Once another edge can no longer be added to $M(e)$, repeat step 2:

The edges of $M(e)$ are therefore said to be Power Dominated by the e .

Definition 2.2

When speaking of the proper edge colouring of a graph G , the term "Power Dominator Edge Colouring" refers to the situation when each edge in the edge set powers over every edge in at least one colour class. The minimum number of colours needed for a power dominator edge colouring of the graph is known as the Power Dominator Edge Chromatic number (PDEC-number) and is denoted by $\chi'_{pd}(G)$.

Example 2.1 A power dominator edge coloring of a graph G in fig. 2.1 is discussed now.

For the graph G , $\chi'_{pd}(G) = 3$. In fact, color 1 is assigned to the edge e_5 while color 2 can be allocated to the edges e_1, e_4, e_7 . The edges e_2, e_3, e_6 can have color 3. For the edge e_1 ; we see the monitoring set, $M(e_1) = \{e_1, e_2, e_3, e_4\} = N[e_1]$. Now the only edge e_5 , not in $M(e_1)$; but is adjacent to e_2 and so we add e_5 to $M(e_1)$ using Step: 2 in the definition of monitoring set. No other edge could be added to $M(e_1)$. Thus $M(e_1) = \{e_1, e_2, e_3, e_4, e_5\}$. Indeed, every edge e_2, e_3, e_4 has the same monitoring set as that of e_1 . The color class-1 is power dominated by the edges e_1, e_2, e_3, e_4 . In the same procedure, the edges e_6, e_7 also power dominate the color class-1. The edge e_5 power dominates itself. Hence every edge of at least one-color class is power dominated by each edge. Hence $\chi'_{pd}(G) = 3$. It is clear that two colors are not enough to get a power dominator edge coloring for this graph. Note also that the dominator edge chromatic number $\chi'_d(G)$ for this graph is 5.

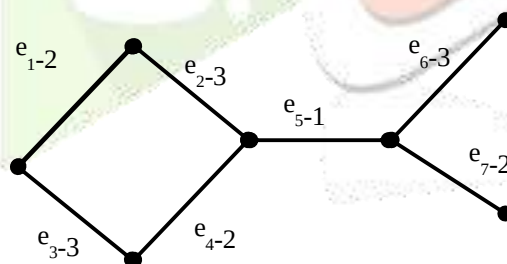


fig:1

Theorem 2.1

For a Path $P_n, n \geq 3, \chi'_{pd}(P_n) = 2$.

Proof: Let $a_1, a_2, \dots, a_n$ be the vertices of the path P_n and let $f_1, f_2, \dots, f_{n-1}$, where $f_i = v_i v_{i+1}, 1 \leq i \leq n-1$ be the edges. Clearly each edge $f_i, 1 \leq i \leq n-1$, power dominates every edge of the path P_n . Assign color 1 for f_i , when i is odd and assign color 2 for f_i for when i is even for proper coloring. Hence every edge power dominates all edges of both color class and so $\chi'_{pd}(P_n) = 2$

Remarks: For $n=2, \chi'_{pd}(P_n) = 1$

Theorem 2.2

For a cycle $C_n, n \geq 3, \chi'_{pd}(C_n) = \begin{cases} 2, & \text{if } n \text{ is even} \\ 3, & \text{if } n \text{ is odd} \end{cases}$

Proof: Let $a_1, a_2, \dots, a_n$ and $f_1, f_2, \dots, f_n$, where $f_i = a_i a_{i+1}, 1 \leq i \leq n-1$ and $f_n = a_n a_1$ be the vertices and edges of the cycle C_n respectively. Clearly every edge $f_i, 1 \leq i \leq n$, power dominates all edges of cycle C_n , since all edges having degree 2.

Case:1 n is even. Allocate the color-1 for edge f_i , when i is odd and assign color-2 to f_i when i is even. This becomes proper edge coloring of C_n . Thus, every edge power dominates all the edges of both color class and so $\chi'_{pd}(C_n) = 2$.

Case:2 n is odd. Allocate the color-1 for edge f_i , when i is odd, $1 \leq i \leq n-2$ and assign color-2 to f_i when i is even, $1 \leq i \leq n-1$. Assign color-3 to the remaining edge f_n , since f_n is adjacent to both f_1 and f_{n-1} . This becomes proper edge coloring of C_n . Hence every edge power dominates all edges of both color class and so $\chi'_{pd}(C_n) = 3$.

Theorem 2.3

For the Star graph $K_{1,a}, n \geq 2, \chi'_{pd}(K_{1,a}) = a$.

Proof: Let $K_{1,a}$ be the Star graph on $a+1$ vertices for which v_0 is the common vertex for all n -edges of $K_{1,a}$. Since each edges have a common vertex v_0 , all the edges are adjacent to each other and so we color all edges with distinct n colors for a proper coloring. Since all edges have distinct color and all are adjacent to each other, so every edge dominates all color classes. Hence $\chi'_{pd}(K_{1,a}) = a$.

Theorem 2.4

For Wheel graph, $W_{1,n}, n \geq 3, \chi'_{pd}(W_{1,n}) = n$.

Proof: Let $a_1, a_2, \dots, a_n$ and $f_1, f_2, \dots, f_n$ be the vertices and the edges, respectively in the rim of the wheel graph and let v_0 be the center vertex of $W_{1,n}$. We assign distinct colors to the spokes that are incident with v_0 as they are adjacent to each other. Also assign colors to the rim of the wheel with the same n colors like as in the Fig.2

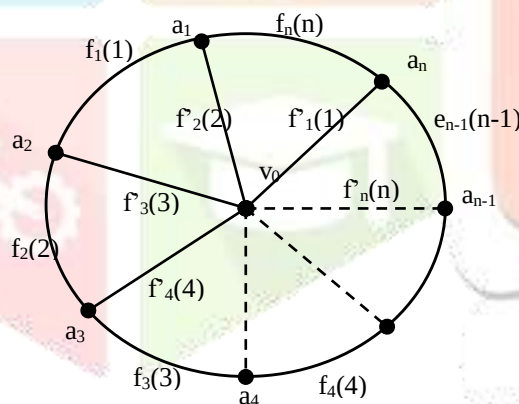


fig: 2

Theorem 2.5

For the Sunlet graph $S_n, \chi'_{pd}(S_n) = \begin{cases} \left\lfloor \frac{n}{2} \right\rfloor + 3, & \text{if } n \text{ is odd} \\ \frac{n}{2} + 2, & \text{if } n \text{ is even} \end{cases}$

Proof: Let $a_1, a_2, \dots, a_n$ be the vertices of the n -cycle in the Sunlet graph S_n and let $a'_1, a'_2, \dots, a'_n$ be the pendant vertices of S_n .

Case:1 n is odd

We grouping the pendant vertices as distinct pair for which they share a common adjacent edge in the cycle. Since n is odd, the number of pendant edge pairs are $\left\lfloor \frac{n}{2} \right\rfloor + 1$. Assign distinct $\left\lfloor \frac{n}{2} \right\rfloor + 1$ colors to the edges in the cycle which was adjacent to distinct pendant edge pairs. Now we assign a new color c_1 , to the remaining edges in the cycle and assign color c_2 to the pendant edges in S_n . Since each pendant edge adjacent to a non-repeated color and this way of coloring is edge proper coloring, all edges of at least one-color class are dominated by each edge of S_n .

Therefore $\chi'_{pd}(S_n) = \left\lfloor \frac{n}{2} \right\rfloor + 3$

Case:2 n is even

As we done in Case:1, we grouping the pendant vertices $\frac{n}{2}$. Assign distinct $\frac{n}{2}$ colors to the edges in the cycle which was adjacent to distinct pendant edge pairs. Now we assign a new color c_1 , to the remaining edges in the cycle and assign color c_2 to the pendant edges in S_n . Since each pendant edge adjacent to a non-repeated color and this way of coloring is edge proper coloring, all edges of at least one-color class are dominated by each edge of S_n . Therefore $\chi'_{pd}(S_n) = \frac{n}{2}$.

Theorem 2.6

For the Book Graph B_m , $\chi'_{pd}(B_m) = m + 1, m \geq 3$

Proof:

The m -book graph B_m is the rectangular product $S_{m+1} \times P_2$ where S_{m+1} is the star graph with a central vertex v_1 and the m pendant vertices $a_i, 2 \leq i \leq m$, and P_2 is the path on two vertices b_1, b_2 . The vertex set $V(B_m)$ of the m -book graph B_m consists of vertices $(a_i, b_j), (1 \leq i \leq m; 1 \leq j \leq 2)$, such that (a_1, b_1) and (a_1, b_2) are adjacent in B_m and are of degree $m+1$ while the degree of remaining $2m$ vertices is 2. We assign color 1 to the edges connecting the vertices (a_i, b_1) and $(a_i, b_2), 1 \leq i \leq m$. Assign m distinct colors to the edges which are incident with the central vertex (a_1, b_1) . Now color the remaining edges which are joined with another central vertex (a_1, b_2) with the same m colors as shown in the Fig:2.3

This coloring is a proper coloring, this is because, and the same-colored edges are parallel to each other. This proper edge coloring has $m+1$ colors. Also edges having degree 2 in B_m dominates all edges of at least one-color class (parallel color classes) and the edge with central vertex as its one end also dominates all edges of some color class. Hence $\chi'_{pd}(B_m) = m + 1$.

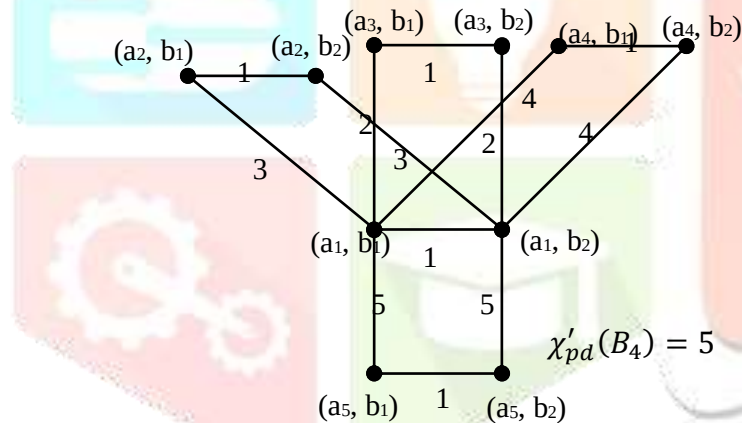


fig.3

Theorem 2.7

For a Tadpole graph $T(m, n), n \geq 2$

$$\chi'_{pd}(T(m, n)) = \begin{cases} 3, & \text{if } m \text{ is even} \\ 4, & \text{if } m \text{ is odd} \end{cases}$$

Proof:

Let $a_1, a_2, \dots, a_m$ be the vertices of the cycle and let $b_1, b_2, \dots, b_n$ be the vertices in the path of the Tadpole $T(m, n)$.

Case:1 m is even. Since m is even, we can color the edges of the cycle C_m properly only by 2 colors and the also we can color the edges of the path P_n by same two colors. Now we assign the edge which connect the cycle and path by color 3. Each edge of $T(m, n)$, power dominates the color class 3. Hence $\chi'_{pd}(G) = 3$.

Case:2 m is odd and $n \geq 2$. Since m is odd, we can color the edges of the cycle C_m properly by 3 colors $\{1, 2, 3\}$ and also, we can color edges of the path $P_n, n \geq 2$ by the colors $\{1, 2\}$. Now we assign the edge which connect the cycle and the path by the color 4. Therefore, each edge of $T(m, n)$ power dominates the color class 4. Hence $\chi'_{pd}(G) = 4$.

Remark:

For $n=1$, $\chi'_{pd}(T(m, n)) = 3$

Theorem 2.8 For the Centipete graph, $\chi'_{pd}(Ct_n) = \begin{cases} \frac{n}{2} + 2, & \text{if } n \text{ is even} \\ \left\lfloor \frac{n}{2} \right\rfloor + 3, & \text{if } n \text{ is odd} \end{cases}, n \geq 4$

Proof: Let $a_1, a_2, \dots, a_m$ be the vertices of the path and let $b_1, b_2, \dots, b_n$ be the pendant vertices for which b_i 's is adjacent to a_i 's, $1 \leq i \leq n$. Let $f_1, f_2, \dots, f_{n-1}$ be the edges in path. Now we color the pendant edges with the color c_1 .

Case:1 if n is even, then assign distinct colors to odd f_i 's. Since n is even, the number of odd f_i 's are $\frac{n}{2}$ and hence we need $\frac{n}{2}$ distinct color. And color the remaining edges by color c_2 . Therefore, all the edges in at least one-color class are power dominated by each edge of Ct_n and so $\chi'_{pd}(Ct_n) = \frac{n}{2} + 2$.

Case:2 if n is odd, then we assign distinct colors to odd f_i 's. since n is odd, the number of odd f_i 's are $\left\lfloor \frac{n}{2} \right\rfloor$ and hence we need $\frac{n}{2} + 2$ distinct colors and color the uncolored edges by the color c_2 color the last edge in the path with color c_3 , since the last pendant edge adjacent to only on edge. Hence all the edges in at least one-color class are power dominated by each edge of Ct_n and so $\chi'_{pd}(Ct_n) = \left\lfloor \frac{n}{2} \right\rfloor + 3$

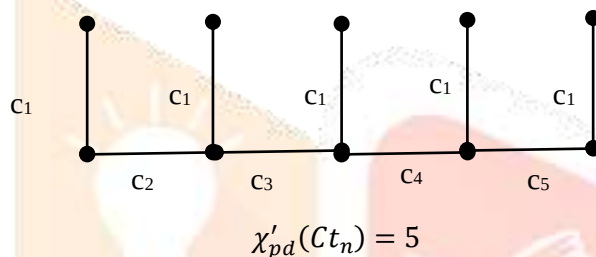


Fig.3

3. Conclusion

The notion Power Dominator Edge Coloring relates coloring problem with Power Dominating set in graphs. The primary aim of this work is to compute the Power Dominator Edge Chromatic number of certain classes of graphs.

Classes of Graphs	PDE Chromatic number
$P_n, n \geq 3$	2
$C_n, n \geq 3$	$\begin{cases} 2, & \text{if } n \text{ is even} \\ 3, & \text{if } n \text{ is odd} \end{cases}$
$K_{1,n}, n \geq 3$	n
$W_{1,n}, n \geq 3$	n
$S_n, n \geq 3$	$\begin{cases} \left\lfloor \frac{n}{2} \right\rfloor + 3, & \text{if } n \text{ is even} \\ \frac{n}{2} + 2, & \text{if } n \text{ is odd} \end{cases}$
$B_m, m \geq 3$	$m+1$
$T(m, n), n \geq 2$	$\begin{cases} 3, & \text{if } m \text{ is even} \\ 4, & \text{if } m \text{ is odd} \end{cases}$

$Ct_n, n \geq 4$	$\begin{cases} \frac{n}{2} + 2, \text{ if } n \text{ is even} \\ \left\lceil \frac{n}{2} \right\rceil + 3, \text{ if } n \text{ is odd} \end{cases}$
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The certain classes of graphs with PDE - Chromatic number are mentioned above. For general graphs there is an opportunity to determine the power dominator edge chromatic number.

4. Acknowledgement:

I would like to express my gratitude to Dr. S. Banupriya and Dr. N. Srinivasan, who served as my supervisor in realizing this work. Their guidance and insights were crucial in all aspects of my project writing. Furthermore, I extend my thanks to my family for their unwavering support and understanding throughout my research and writing process. Ultimately, I give thanks to the divine for the numerous blessings bestowed upon my life.

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