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Homotopy Perturbation Method For Thermal Analysis Of Rectangular Fin

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Abstract

This study utilizes the Homotopy Perturbation Method (HPM), a straightforward and precise semi-analytical approach, to address the nonlinear energy equation governing a straight rectangular fin. The thermal conductivity and surface emissivity are treated as temperature-dependent, along with a constant source of internal heat generation. The results include temperature distribution, efficiency, and optimal dimensionless parameters, all of which are practical and relevant for real-world fin applications. Additionally, the study demonstrates that this method yields more accurate results compared to other techniques reported in the literature

Keywords: sink temperature, Homotopy perturbation method, efficiency, variable surface emissivity.

Nomenclature

N_r Radiation-conduction parameter

C Constant which represents the temperature

k Temperature dependent thermal conductivity, $W/(mK)$

k_a Thermal conductivity corresponding to ambient condition, $W/(mK)$

ε_s The surface emissivity corresponding to radiation sinks temperature, T_s

T Temperature, K

P Fin perimeter, m

T_b Fin's base temperature, K

T_a Convection sink temperature, K

T_s Sink temperature for radiation, K

b Length of the fin, m

x Axial co-ordinate of the entire fin, m

A_c Cross-sectional area of the entire fin, m^2

X Dimensionless axial co-ordinate

A Thermal conductivity parameters

B The surface emissivity parameters

Greek symbols

α Slope of the thermal conductivity-temperature curve, K^{-1}

β Slope of the surface emissivity-temperature curve, K^{-1}

θ Dimensionless temperature of the fin,

θ_a Dimensionless convection sinks temperature,

θ_s Dimensionless radiation sinks temperature,

σ Stefan-Boltzmann constant, $W/\left(m^4K^4\right)$

ε Emissivity

1. Introduction

Fins are extended surfaces designed to transfer heat from the surface of primary to the atmosphere [1]. Transfer of heat through longitudinal fins is usual due to their low producing costs and simplicity. These fins are often exposed to both environment of convection and radiation [2]. In scenarios where convection and radiation heat transfer, the present fin may generate internal heat from the electric current passage, as seen in electric filaments, or from atomic or chemical reactions, as in an atomic reactor [3-6]. Internal heat generation is assumed to occur at a constant rate relative to the fin's volume. To utilize the fin materials effectively, the model incorporates thermal conductivity that varies linearly with temperature, reflecting that the actual surface properties of the fin material often do the same. The energy equation for a convective-radiative fin with heat generation and variable thermal properties leads to highly nonlinear terms. This equation does not have an exact solution; therefore, the nonlinear energy equation is typically solved either numerically or through various approximate analytical methods. Various methods have been developed to solve nonlinear problems, including the Homotopy Analysis Method (HAM), Galerkin Method (GM), Spectral Collocation Method (SCM), and Adomian Decomposition Method (ADM). Among these, the Homotopy Perturbation Method (HPM) is a semi-analytical approach introduced by He for tackling nonlinear boundary value problems [7]. The Homotopy Perturbation Method (HPM) retains all the advantages of perturbation methods while being independent of the assumption of a small parameter. Unlike the Adomian Decomposition Method, HPM does not require the calculation of Adomian polynomials; it only necessitates an initial approximation [8-10]. Additionally, the Homotopy Perturbation Method does not require the determination of 'h-curves,' distinguishing it from the Homotopy Analysis Method. Arka Bhowmik et al. [11]

predicted the dimensions of rectangular and hyperbolic fins with variable thermal properties. Saheera Azmi Hazarika et al. [12] developed an analytical method to assess the performance and design parameters of T-shaped fins with simultaneous heat and mass transfer. Several researchers have applied the Collocation Method (CM) to obtain analytical solutions for the unsteady motion of fluid particles in conjunction with other numerical techniques [13, 14]. M. Rahimi-Gorji et al. [15] utilized the Galerkin Method to analyze the heat transfer characteristics of microchannel heat sinks cooled by various nanofluids in porous media. O. Pourmehran et al. [16] investigated the thermal performance of fin-shaped microchannel heat sinks cooled by different nanofluids using the least squares method. The Spectral Collocation Method has also been employed to study the performance parameters of simple and complex cross-sections of moving rods in the thermal processing of continuous casting and rolling [17, 18]. Yasong Sun et al. [19] used Lagrange interpolation polynomials to approximate temperature distributions at the spectral collocation points in nonlinear heat transfer problems. Jing Ma et al. [20] presented the Spectral Collocation Method to predict the thermal performance of porous fins with temperature-dependent heat transfer coefficients, surface emissivity, and internal heat generation.

The discussion above highlights the importance of selecting the appropriate conductive-convective parameter in fin design applications for practicing engineers. Practical fins typically operate at low values of this parameter. The Homotopy Perturbation Method (HPM) is employed to examine the effects of surface emissivity, thermal conductivity parameters, and heat generation numbers on the temperature distribution and efficiency of fins under practical operating conditions.

2. Mathematical Formulations

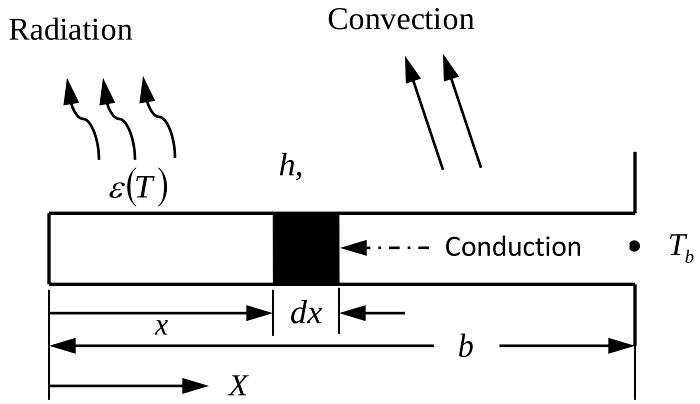
Consider a rectangular fin geometry with various specifications, as illustrated in Figure 1. The thermal conductivity k and surface emissivity ε of the fin materials are assumed to be temperature-dependent. Internal heat generation within the fin material is considered to occur at a constant rate q. The fin is assumed to be insulated at the free end, and the effects of heat transfer in the vertical direction are neglected. The steady-state, one-dimensional energy equation is expressed as follows.

$$\frac{d}{dx}\left[k(T)\frac{dT}{dx}\right]-\frac{hP(T-T_a)}{A_c}-\frac{\varepsilon(T)\sigma P\left(T^4-T_s^{\,4}\right)}{A_c}+q=0 \qquad (1)$$

With the insulated boundary condition

$$\frac{dT}{dX}=0 \text{ at } x=0 \qquad (2a)$$

$$T=T_b \text{ at } x=b \qquad (2b)$$



Radiation sinks temperature, Ts

Convection sinks temperature, T_a

Figure 1. The geometry of straight rectangular fin.

For the better utilization of the fin materials the thermal conductivity of fin material is assumed to be linear function of temperature. Also the emissivity of all real surfaces are not constant, it sometimes varies linearly with temperature. Therefore both of the parameter can written as below

$$k(T) = k_a[1+\alpha(T-T_a)] \tag{3}$$

$$\varepsilon(T) = \varepsilon_s[1+\beta(T-T_s)] \tag{4}$$

In order to express the equations (1) in non dimensional forms, the following dimensionless parameters are defined as

$$\theta = \frac{T}{T_b} \qquad \theta_a = \frac{T_a}{T_b} \qquad \theta_s = \frac{T_s}{T_b} \qquad X = \frac{x}{b}$$
$$A = \alpha T_b \qquad B = \beta T_b \qquad N_c^{\,2} = \frac{hPb^2}{k_a A_c} \qquad N_r = \frac{\varepsilon_s \sigma T_b^{\,3} P b^2}{k_a A_c} \qquad Q = \frac{b^2 q}{k_a T_b} \tag{5}$$

The formulation of the fin problem reduces to the following equations:

$$\frac{d^2\theta}{dX^2} + A\theta\frac{d^2\theta}{dX^2} + A\left(\frac{d\theta}{dX}\right)^2 - A\theta_a\frac{d^2\theta}{dX^2} - N_c^{\,2}(\theta-\theta_a) - N_r[1+B(\theta-\theta_s)](\theta^4-\theta_s^{\,4}) + Q = 0 \tag{6}$$

With the following boundary conditions:

$$\frac{d\theta}{dX} = 0 \;\; at \;\; X = 0 \tag{7a}$$

$$\theta = 1 \;\; at \;\; X = 1 \tag{7b}$$

3. Homotopy Perturbation Method (HPM)

The Homotopy Perturbation Method (HPM) is a semi-numerical technique for solving linear or nonlinear, homogeneous or inhomogeneous boundary value problems, first introduced by He [7]. Unlike the Adomian

Decomposition Method, HPM does not require the calculation of Adomian polynomials and tends to converge to a solution more quickly. Additionally, this method requires only the initial conditions as input for its solution

To illustrate the basic idea of HPM according to He [7], consider the following nonlinear differential equation,

$$A(\theta) - f(r) = 0, \quad r \in \Omega. \tag{8}$$

With the boundary conditions

$$B\left(\theta, \frac{\partial \theta}{\partial X}\right) = 0 \quad r \in \Gamma, \tag{9}$$

Where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytic function, and Γ is the boundary of the domain Ω.

The operator A can be generally divided into linear and nonlinear parts say $L(\theta)$ and $N(\theta)$. Therefore the equation (8) can be written as

$$L(\theta) + N(\theta) - f(r) = 0 \tag{10}$$

Define the Homotopy $v(r, p): \Omega \times [0,1] \to \Re$ that satisfies

$$H(\theta, p) = (1-p)L(\theta - \theta_0) + p[L(\theta) + N(\theta) - f(r)] = 0. \tag{11}$$

This can be rearranged as

$$H(\theta, p) = L(\theta) - L(\theta_0) + pL(\theta_0) + p[N(\theta) - f(r)] = 0. \tag{12}$$

Where $L = \frac{d^2}{dX^2}$ and θ_0 is the initial approximation. Here $\theta = f(X)$ and p is the embedding parameter such that $p \in [0,1]$ .It is clear that

$$H(\theta, 0) = [L(\theta) - L(\theta_0)] \tag{13}$$

For $p = 1$ we obtain

$$H(\theta, 1) = [L(\theta) + N(\theta) - f(r)] \tag{14}$$

Using the perturbation technique by taking by taking into account of small values of p, then the solution of the equation (10) can be written as

$$\theta = \theta_0 + p\theta + p^2\theta_2 + p^3\theta_3 + p^4\theta_4 + \cdots\cdots \tag{15}$$

The series converges for $p = 1$ the solution for θ can be given by

$$\theta = \theta_0 + \theta + \theta_2 + \theta_3 + \theta_4 + \cdots\cdots \tag{16}$$

4. HPM Formulation

Using equation (12), the equation (6),can be written as in HPM form

$$H(\theta, p) = (1-p)L(\theta - \theta_0) + p\begin{bmatrix} \frac{d^2\theta}{dX^2} + A\theta\frac{d^2\theta}{dX^2} + A\left(\frac{d\theta}{dX}\right)^2 - A\theta_a\frac{d^2\theta}{dX^2} - N_c^{\,2}(\theta - \theta_a) \\ -N_r\left(\theta^4 - \theta_s^{\,4} + B\theta^5 - B\theta_s^{\,4}\theta - B\theta_s\theta^4 + B\theta_s^{\,4}\right) \end{bmatrix} = 0 \tag{17}$$

This can be arranged as

$$H(\theta,p)=L(\theta)-L(\theta_0)+p(\theta_0)+p\begin{bmatrix} A\theta\dfrac{d^2\theta}{dX^2}+A\left(\dfrac{d\theta}{dX}\right)^2-A\theta_a\dfrac{d^2\theta}{dX^2}-N_c^{\,2}(\theta-\theta_a) \\ -N_r\left(\theta^4-\theta_s^{\,4}+B\theta^5-B\theta_s^{\,4}\theta-B\theta_s\theta^4+B\theta_s^{\,4}\right)\end{bmatrix}=0 \tag{18}$$

Substituting θ, from equation (15), to the equation (18), and separating the variables of identical power of p.

$$p^0: \qquad \theta=\theta_0 \tag{19}$$

From the boundary condition (7) it is clear that the solution is becoming meaningless. Therefore in order to predict the solution physically meaningful the $\theta(0)$ must be an arbitrary constant. This $\theta_0=C$ is taken as initial input for HPM.

And

p^1:

$$\frac{d^2\theta_1}{dX^2}+\begin{bmatrix} A\theta_0\dfrac{d^2\theta_0}{dX^2}+A\left(\dfrac{d\theta_0}{dX}\right)^2-A\theta_a\dfrac{d^2\theta_0}{dX^2}-N_c^{\,2}(\theta-\theta_a) \\ -N_r\left(\theta_0^{\,4}-\theta_s^{\,4}+B\theta_0^{\,5}-B\theta_s^{\,4}\theta_0-B\theta_s\theta^4+B\theta_s^{\,5}\right)\end{bmatrix}=0 \tag{20}$$

$$\frac{d\theta_1}{dX}=0 \ \ at \ \ X=0, \quad \theta_1=0 \ \ at \ \ X=0 \tag{21}$$

And

p^2:

$$\frac{d^2\theta_2}{dX^2}+\begin{bmatrix} A\left(\theta_0\dfrac{d^2\theta_1}{dX^2}+\theta_1\dfrac{d^2\theta_0}{dX^2}\right)+A\left(2\dfrac{d\theta_0}{dX}\dfrac{d\theta_1}{dX}\right)-A\theta_a\dfrac{d^2\theta_1}{dX^2}-N_c^{\,2}(\theta-\theta_a) \\ -N_r\left(4\theta_0^{\,3}\theta_1+5B\theta_0^{\,4}\theta_1-B\theta_s^{\,4}\theta_1-B\theta_s 4\theta_0^{\,3}\theta_1\right)\end{bmatrix}=0 \tag{22}$$

$$\frac{d\theta_2}{dX}=0 \ \ at \ \ X=0, \quad \theta_2=0 \ \ at \ \ X=0 \tag{23}$$

p^3:

$$\frac{d^2\theta_3}{dX^2}+\begin{bmatrix} A\left(\theta_0\dfrac{d^2\theta_2}{dX^2}+\theta_1\dfrac{d^2\theta_1}{dX^2}+\theta_2\dfrac{d^2\theta_0}{dX^2}\right)+A\left(2\dfrac{d\theta_0}{dX}\dfrac{d\theta_2}{dX}+\left(\dfrac{d\theta_1}{dX}\right)^2\right) \\ -A\theta_a\dfrac{d^2\theta_2}{dX^2}-N_c^{\,2}(\theta-\theta_a)-N_r\begin{pmatrix}\left(4\theta_0^{\,3}\theta_2+6\theta_0^{\,2}\theta_1^{\,2}\right)+B\left(5\theta_0^{\,4}\theta_2+10\theta_0^{\,3}\theta_1^{\,2}\right) \\ -B\theta_s^{\,4}\theta_2-B\theta_s\left(4\theta_0^{\,3}\theta_1+6\theta_0^{\,2}\theta_1^{\,2}\right)\end{pmatrix}\end{bmatrix}=0 \tag{24}$$

$$\frac{d\theta_3}{dX}=0 \ \ at \ \ X=0, \quad \theta_3=0 \ \ at \ \ X=0 \tag{25}$$

p^4 :

$$\frac{d^2\theta_4}{dX^2}+\left[\begin{array}{l} A\left(\theta_0\frac{d^2\theta_3}{dX^2}+\theta_1\frac{d^2\theta_2}{dX^2}+\theta_2\frac{d^2\theta_1}{dX^2}+\theta_3\frac{d^2\theta_0}{dX^2}\right) \\ +A\left(2\frac{d\theta_0}{dX}\frac{d\theta_3}{dX}+2\left(\frac{d\theta_1}{dX}\right)\left(\frac{d\theta_2}{dX}\right)\right)-A\theta_a\frac{d^2\theta_3}{dX^2}-N_c^{\ 2}(\theta-\theta_a) \\ -N_r\left(\begin{array}{l}\left(4\theta_0^{\ 3}\theta_3+12\theta_0^{\ 2}\theta_1\theta_2+4\theta_0\theta_1^{\ 3}\right)+B\left(5\theta_0^{\ 4}\theta_3+20\theta_0^{\ 3}\theta_1\theta_2+10\theta_1^{\ 3}\theta_0^{\ 2}\right) \\ -B\theta_s^{\ 4}\theta_3-B\theta_s\left(4\theta_0^{\ 3}\theta_3+12\theta_0^{\ 2}\theta_1\theta_2+4\theta_0\theta_1^{\ 3}\right)\end{array}\right)\end{array}\right]=0 \tag{26}$$

$$\frac{d\theta_4}{dX}=0 \ \ at \ \ X=0, \quad \theta_4=0 \ \ at \ \ X=0 \tag{27}$$

By increasing number of terms in the solution higher accuracy will be obtained. Solving equation (20), to (27) results $\theta_1,\theta_2,\theta_3,\theta_4\cdots$ can be obtained as below

$$\theta_1=-\frac{QX^2}{2}+\frac{N_rC^4X^2}{2}-\frac{N_r\theta_s^{\ 4}X^2}{2}+\frac{N_rBC^5X^2}{2}-\frac{N_rB\theta_s^{\ 4}CX^2}{2}-\frac{N_rB\theta_sC^4X^2}{2}$$
$$+\frac{N_rB\theta_s^{\ 5}X^2}{2}-\frac{N_c^{\ 2}\theta_aX^2}{2}+\frac{N_c^{\ 2}CX^2}{2}$$

Substituting the values of $\theta_{0,}\theta_1,\theta_2,\theta_3 and\theta_4\cdots$, in equation (13) the formulation of non dimensional temperature can be obtained as

$$\theta=\sum_0^{\alpha}\theta_m=\theta_0+\theta_1+\theta_2+\theta_3+\theta_4+\cdots\cdots \tag{28}$$

Now the temperature field, θ can be evaluated if the fin tip temperature C is known whose value lies in the interval (0, 1). Using an arbitrary initial guess value for C, for the temperature field θ computed from the above equation (26) and applying Newton-Rapson method satisfying the boundary conditions (6) the actual temperature field can be obtained.

4. Fin efficiency (η): The fin efficiency is defined as the ratio of actual heat transfer rate to the ideal heat transfer rate. The ideal heat transfer rate takes place in the fin if the whole fin is subjected to base temperature.

The actual rate of heat transfer can be obtained by computing the heat losses from the base of the fin

$$Q_{actual}=K(T)A_c\left[\frac{dT}{dx}\right]_{x=b} \tag{29}$$

The ideal heat transfer rate Q_{ideal} obtained if the entire fin is kept at the base temperature and can be expressed as

$$Q_{ideal}=\left[hPb(T-T_a)+\varepsilon_s\sigma Pb[1+\beta(T-T_s)]\left(T^4-T_s^{\ 4}\right)\right] \tag{30}$$

The fin efficiency can be expressed using non dimensional terms by the Eq (5)

$$\eta = \frac{Q_{actual}}{Q_{ideal}} = \frac{\frac{k_a A_c T_b}{b}\left[1 + A_c(\theta - \theta_a)\right]\left.\frac{d\theta}{dX}\right|_{X=1}}{\left[hPb(T - T_a) + \varepsilon_s \sigma Pb\left[1 + \beta(T - T_s)\right]\left(T^4 - T_s{}^4\right)\right]}$$
$$= \frac{\left[1 + A(\theta - \theta_a)\right]\left.\frac{d\theta}{dX}\right|_{X=1}}{N_c{}^2(1 - \theta_a) + N_r\left[1 + B(1 - \theta_s)\right]\left(1 - \theta_s{}^4\right)} \quad (31)$$

6. Results and discussion

The validation of results is achieved by considering both the sink temperature and surface emissivity parameter, along with the heat generation number. By setting the equation to zero, it aligns with previous work found in the literature [21]. The present work examines five terms, comparing accuracy results with previous studies as shown in **Tables 1 and 2**. In fin design applications, selecting appropriate values is crucial. Fins are classified as practical when the conductive-convective fin parameter is below 0.5."

Table1. Comparison of HPM solutions with Adomian Decomposition Method (ADM), Galerkin Method (GM) and Boundary Value Problem (NM)

$N_c = 1, N_r = 0.2, A = 0.2, Q = 0, \theta_s = 0, \theta_a = 0, B = 0$				
X	HPM (Present)	ADM [6]	GM [21]	$NM(BVP)$ [21]
0	0.666858541	0.666858541	0.667013018	0.667013597

Table2. Effect of number of terms in HPM on the temperature distribution of rectangular fin

$N_c = 1, N_r = 0.2, A = 0.2, Q = 0, \theta_s = 0, \theta_a = 0, B = 0$

Number of terms (m)	Distance, X										
	0.0000	0.1000	0.2000	0.300	0.400	0.5000	0.6000	0.7000	0.8000	0.9000	1.0000
2	0.6608	0.6638	0.6730	0.6884	0.7102	0.7389	0.7746	0.8180	0.8696	0.9300	0.9999
3	0.6687	0.6719	0.6812	0.6969	0.7194	0.7479	0.7835	0.8260	0.8760	0.9338	0.9999
Absolute error difference	0.0079	0.0081	0.0082	0.0085	0.0092	0.0090	0.0089	0.0080	00064	0.0038	0.0000

With m=2 and m=3

3	0.6687	0.6718	0.6812	0.6970	0.7192	0.7479	0.7835	0.8260	0.8760	0.9338	0.9999
4	0.6668	0.6700	0.6794	0.6951	0.7173	0.7462	0.7819	0.8284	0.8751	0.9334	0.9999
Absolute error difference	0.0019	0.0018	0.0018	0.0019	0.0019	0.0017	0.0016	0.0012	00009	0.0004	0.0000

With m=3 and m=4

The validation of results involves considering both the sink temperature and the surface emissivity parameter, along with the heat generation number. By setting the equation to zero, it is aligned with the findings from previous studies in the literature [21]. The current study examines five terms, comparing accuracy results with previous research found in the literature, as presented in Tables 1 and 2. In fin design applications, the selection of appropriate values is critical. Fins are classified as practical based on the conductive-convective fin parameter, which must be below 0.5. The surface emissivity of fin materials can vary with temperature, either increasing (B > 0) or decreasing (B < 0). In practice, values of B typically range from -0.5 to +0.5 [8]. To analyze the impact of surface emissivity on temperature distribution and fin efficiency, this study considers the extreme and constant values of emissivity: B = -0.5, 0, and +0.5. Additionally, it is important to maintain a nonzero environmental temperature, as it is unrealistic for a fin to keep one end at a high temperature while dissipating heat to absolute zero. The effect of environmental temperature on thermal performance has also been noted as significant. Relevant ranges for environmental temperature and heat generation numbers are available in the literature [6].

Figure 2 illustrates the variation of tip temperature with respect to the heat generation number for three different values of the surface emissivity parameter, i.e. $B=-0.5,\ 0,\ +0.5$ while all other parameters are maintained at zero except $N_r = 0.9$ for this one. The lines representing surface emissivity increase linearly with the internal heat generation number $Q=0$ to $Q=0.4$.The presence of an internal heat source influences the surface temperature of the fin materials. The spacing between the emissivity lines is greater when the conductive-convective parameter (Nc) is kept at low values. This occurs because, at lower Nc values, convective heat loss is higher, resulting in a steeper temperature gradient.

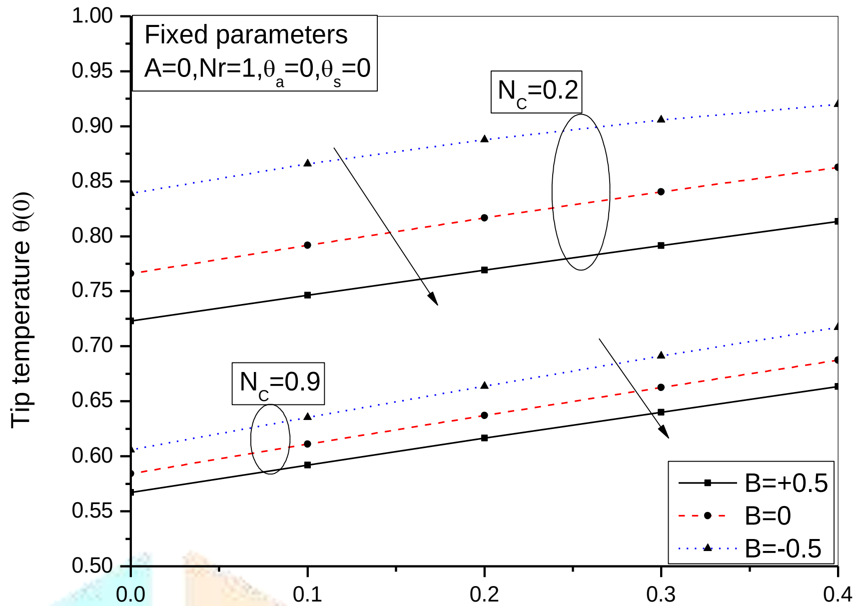


Figure 2. The tip temperatures function of heat generation number.

Figures 3 and 4 illustrate how the temperature of the fin materials varies with different thermal emissivity values. The thermal conductivity of the fin materials $A = -0.5$ and is kept constant, along with the sinks temperature, while heat generation is held at a fixed value. Both figures clearly show that the surface emissivity of the fin materials is significantly influenced when the values are kept below 0.5. Additionally, Figure 4 demonstrates that the surface of the fin materials is more impacted at higher values of the heat generation number.

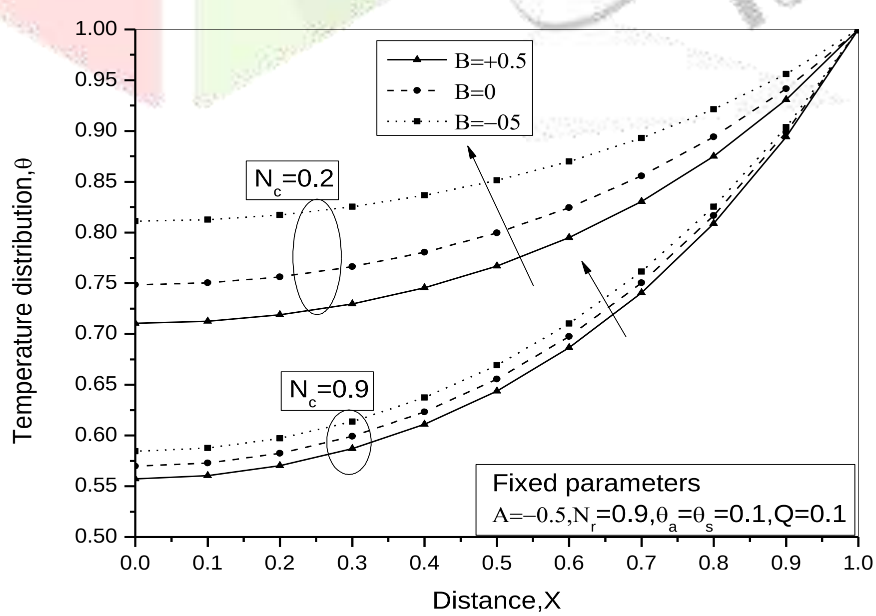


Figure 3. . The effect of surface emissivity on the temperature distribution for Q=0.1.

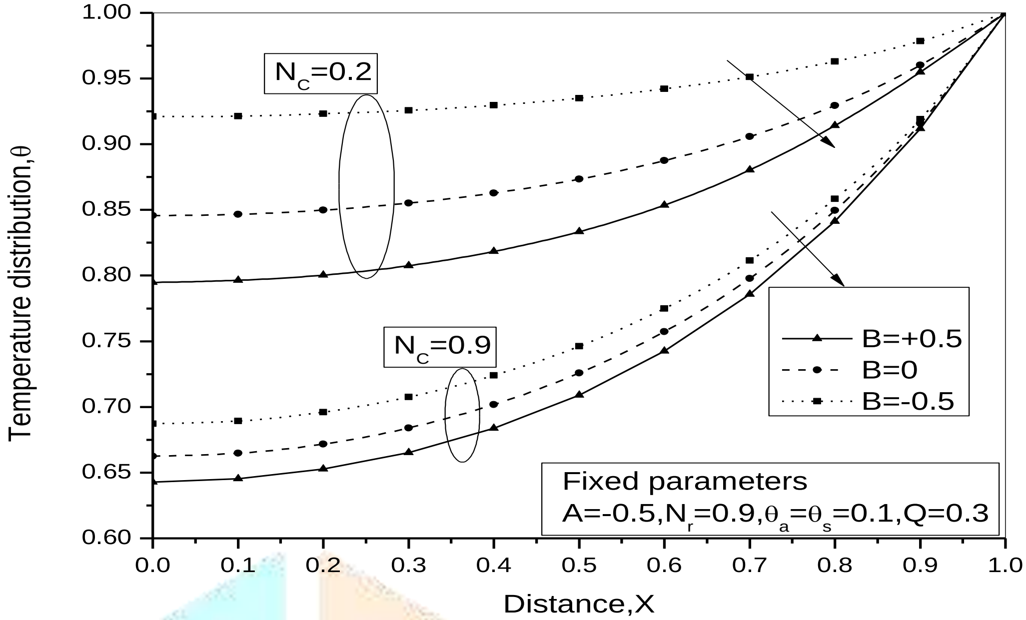


Figure 4. . The effect of surface emissivity on the temperature distribution for Q=0.3

Figure 5 illustrates the variation of fin efficiency of the materials in relation to the heat generation number. It is evident from the figure that the efficiency of the fin materials is quite low without internal heat generation. As the internal heat generation number QQQ increases, the fin efficiency also rises. The efficiency corresponding to lower values of the conduction-convection parameter NcN_cNc shows a steeper increase compared to higher values. This may be due to the fact that, at lower conduction-convection parameter values, the combined effects of internal heat generation and surface heat loss enhance the fin efficiency

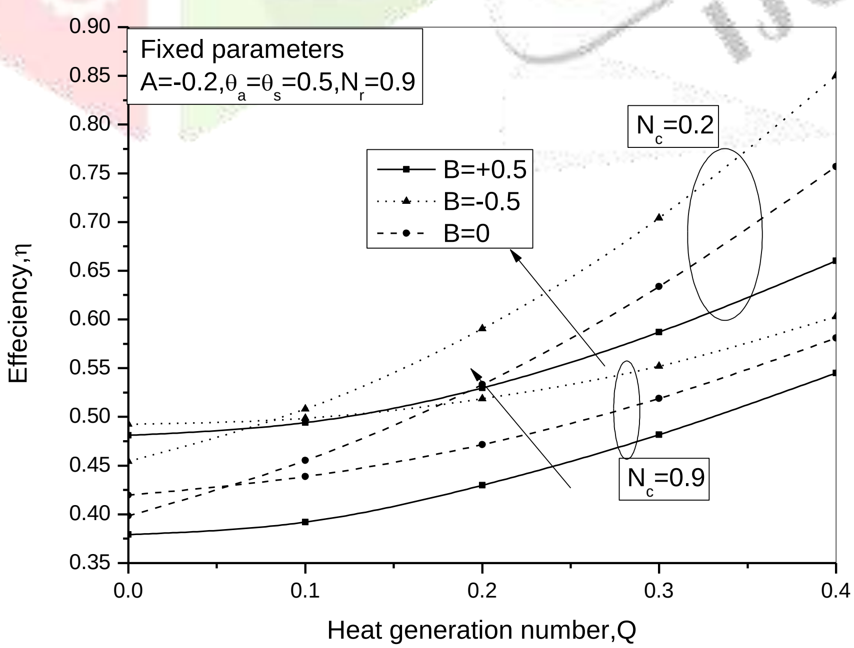


Figure 5. The effect of fin efficiency η on the heat generation number, Q

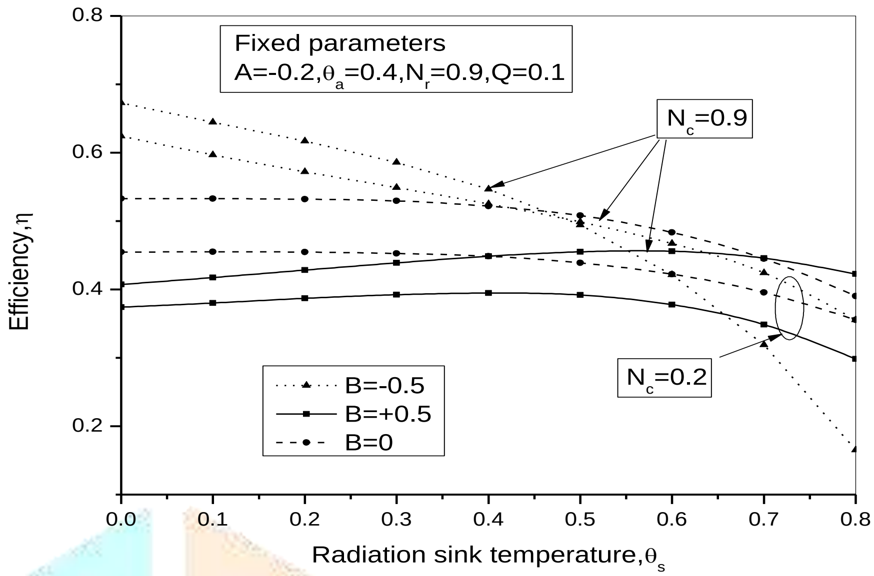


Figure 6. The variation of fin efficiency η with respect to the radiation sinks temperature θ_s .

Figure 6 shows the variation of fin efficiency with respect to the radiation sink temperature. The efficiency of the surface emissivity curves corresponding to negative values of surface emissivity is higher. This observation aligns with findings from studies on space radiative fins [9]. The efficiency peaks when the fin is maintained at a zero sink temperature and decreases as the radiation sink temperature exceeds 4.5.

7. Conclusion

In this study, the Homotopy Perturbation Method (HPM) is employed to solve the nonlinear energy equation for a straight rectangular fin with variable thermal conductivity and surface emissivity, along with a constant heat generation source. The analysis focuses on two key thermo-geometric fin parameters, N_c=0.2 and N_c=0.9. Results are presented for temperature distribution, efficiency, and optimum dimensionless parameters, demonstrating their effectiveness and convenience. Additionally, it is found that this method yields more favorable results compared to other approaches reported in the literature.

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