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Role Of Intuitionistic Fuzzy Rings Through Intuitionistic Fuzzy Spaces

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Abstract:

The role played by Intuitionistic Fuzzy Rings through Intuitionistic Fuzzy Spaces is established by the interplay between these two concepts. Through a comprehensive analysis of the induction and correspondence principle, we found a relationship between Intuitionistic Fuzzy Rings and ordinary rings. This study not only provides a bridge between intuitionistic fuzzy sets and ring theory but also offers real-world challenges which contributes a deeper understanding of the connections between Intuitionistic Fuzzy Rings, Intuitionistic Fuzzy Spaces, and conventional ring structures.

Keywords: Fuzzy rings and ideals, intuitionistic fuzzy space, fuzzy binary operations, Intuitionistic Fuzzy Rings.

1. INTRODUCTION

In 1982, W. J. Liu introduced the groundbreaking concept of fuzzy rings and fuzzy ideals [5]. This pivotal work marked a significant advancement in the field. Subsequently, in 1985, Ren delved explored fuzzy ideals and fuzzy quotient rings, further expanding the landscape of this field [6]. Liu and Ren's exploration of fuzzy rings and fuzzy ideals was an extension of Rosenfield's fuzzy group theory.

The foundational role of fuzzy spaces as a universal set in classical algebra, K. A. Dib, in a seminal work [3], presented a formulation for fuzzy rings and fuzzy ideals. This formulation introduced a novel perspective that enriched the theoretical framework.

In this paper, we have generalized Dib's pioneering notion for fuzzy rings through the concept of a fuzzy space and intuitionistic fuzzy sets. This concept will serve as a universal set in the classical context, thus creating a comprehensive framework for intuitionistic fuzzy rings.

2. PRELIMINARIES

In this section we will recall some of the fundamental concepts and definitions required in the sequel.

Let $L = I \times I$, where $I = [0,1]$. Define a partial order on L, in terms of the partial order on I, as follows: For every $(r_1, r_2), (s_1, s_2) \in L$

1. $(r_1, r_2) \leq (s_1, s_2)$ if and only if $r_1 < s_1, r_2 < s_2$ (or $r_1 = s_1$ *and* $r_2 = s_2$) whenever $s_1 \neq 0 \neq s_2$

2. $(0,0) = (s_1, s_2)$ whenever $s_1 = 0 = s_2$

Thus the cartesian product $L = I \times I$ is a distributive, not complemented lattice. The operation of infimum and supremum in L are given respectively by

$(r_1, r_2) \wedge (s_1, s_2) = (r_1 \wedge s_1, r_2 \wedge s_2)$ and $(r_1, r_2) \vee (s_1, s_2) = (r_1 \vee s_1, r_2 \vee s_2)$.

DEFINITION.1 : (Atanassov [1]): Let X be a nonempty fixed set. An intuitionistic fuzzy set A is an object having the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\},$$

where the functions $\mu_A(x): X \to I$ and $\gamma_A(x): X \to I$ denote the degree of membership and the degree of non-membership respectively of each element $x \in X$ to the set A, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in X$.

REMARK 1: The intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, in X will be denoted by

$A = \{\langle x, A(x), A(x)\rangle : x \in X\}$

The support of the intuitionistic fuzzy set $A = \{\langle x, A(x), A(x)\rangle : x \in X\}$ in X is the subset A◦ of X defined by: $A\circ = \{x \in X : A(x) \neq 0 \; and \; A(x) \neq 1\}$.

DEFINITION 2.2 : Let X be a nonempty set. An intuitionistic fuzzy space (simply IFS) denoted by (X, I, I) is the set of all ordered triples (x, I, I) where $(x, I, I) = (x, r, s) : r, s \in I$ with $r + s \in I$ *and* $x \in X$. the ordered triplet (x, I, I) is called an intuitionistic fuzzy element of the intuitionistic fuzzy space (X, I, I) and the condition $r, s \in I$ with $r + s \in I$ will be referred to as the "intuitionistic condition".

DEFINITION 2.3 : Let U◦ be a given subset of X. An intuitionistic fuzzy sub-space U of the IFS (X, I, I) is the collection of all ordered triples (x, u_x, u_x), where x ∈ U◦ and u_x, u_x are subsets of I such that ux contains at least one element beside the zero element and u_x contains at least one element beside the unit. If $x \in U\circ$, then $u_x = 0$ and $u_x = 1$. The ordered triple (x, u_x, u_x) will be called an intuitionistic fuzzy element of the intuitionistic fuzzy subspace U. The empty fuzzy subspace denoted by φ is defined to be

$\varphi = \{(x, I, I) : x \in \varphi\}$.

DEFINITION 2.4: An intuitionistic fuzzy function between two intuitionistic fuzzy spaces X and Y is an intuitionistic fuzzy relation F from the X to Y satisfying the following conditions:

1. For every $x \in X$ with $r, s \in I$, there exists a unique element $y \in Y$ with $w, z \in I$; such that $((x, y), (r, w), (s, z)) \in A$ for some $A \in F$.

2. If $((x, y), (r_1, w_1), (s_1, z_1)) \in A \in F$, and $((x, y*), (r_2, w_2), (s_2, z_2)) \in B \in F$ then $y = y$.

3. If $((x, y), (r_1, w_1), (s_1, z_1)) \in A \in F$, and $((x, y), (r_2, w_2), (s_2, z_2)) \in B \in F$ then $(r_1 > r_2)$ implies $(w_1 > w_2)$ and $(s_1 > s_2)$ implies $(z_1 < z_2)$.

4. If $((x, y), (r, w), (s, z)) \in A \in F$, then $r = 0$ implies $w = 0, s = 1$ implies $z = 0$, and $r = 1$ implies $w = 1, s = 0$ implies $z = 1$.

Thus conditions (1) and (2) imply that there exists a unique (ordinary) function from $X \; to \; Y$, namely

$F : X \to Y$ and that for every $x \in X$ there exists unique (ordinary) functions from $I \; to \; I$, namely f_x ,

$\dot{f}_x : I \to I$. On the x other hand conditions (3) and (4) are respectively equivalent to the following conditions:

(i) f_x is nondecreasing on I and $\dot{f}_x$ is increasing on I.

(ii) $f_x(0) = 0 = \dot{f}_x(1) \; and \; f_x(1) = 1 = \dot{f}_x(0)$.

That is, an intuitionistic fuzzy function between two intuitionistic fuzzy spaces X and Y is a function F from $X \; to \; Y$ characterized by the ordered triple

$F(x), \{f_x\}, x \in X, \{\dot{f}_x\}, x \in X,$

where $F(x)$ is a function from $X \; to \; Y$ and $\{f_x\}, x \in X, \{\dot{f}_x\}, x \in X$ are family of functions from $I \; to \; I$ satisfying the conditions (i) and (ii) such that the image of any intuitionistic fuzzy subset A of the IFS X under F is the intuitionistic fuzzy subset $F(A)$ of the IFS Y defined by

$$F_A(y) = \begin{cases} \left\{ \bigvee_{x \in F^{-1}(y)} f_x(\mu_A(x)) \quad \bigwedge_{x \in F^{-1}(y)} \dot{f}_x(\gamma_A(x)) \right. & if \; F^{-1}(y) \neq \varphi \\ (0,1) & if \; F^{-1}(y) = \varphi \end{cases}$$

We will call the functions $f_x, \dot{f}_x$ the comembership functions and the conon- membership functions respectively. The intuitionistic fuzzy function F will be denoted by $F = (F, f_x, \dot{f}_x)$.

DEFINITION 2.5 : An intuitionistic fuzzy binary operation F on an IFS (X, I, I) is an intuitionistic fuzzy function $F : X \times X \to X$ with comembership functions f_{xy} and cononmembership functions f_{xy} satisfying:

1. $f_{xy}(r, s) = 0$ iff $r = 0$ and $s = 0$, and $f_{xy}(w, z) = 1$ iff $w = 1$ and $z = 1$.

2. f_{xy}, f_{xy} are onto. That is, $f_{xy}(I \times I) = I$ and $f_{xy}(I \times I) = I$.

Thus for any two intuitionistic fuzzy elements $(x, I, I), (y, I, I)$ of the IFS X

and any intuitionistic fuzzy binary operation $F = (F, f_{xy}, f_{xy})$ defined on an IFS X. The action of the intuitionistic fuzzy binary operation F over the IFS X is given by

$(x, I, I)F(y, I, I) = F((x, I, I), (y, I, I)) = (F(x, y), f_{xy})$

$((I \times I), f_{xy}(I \times I)) = (F(x, y), I, I)$.

3. INTUITIONISTIC FUZZY RING

We will define intuitionistic fuzzy ring by adding two intuitionistic fuzzy binary operations to a given fuzzy space with similar conditions to the ordinary case.

DEFINITION 3.1: An intuitionistic fuzzy ring, denoted by $(R, I, I), F+, F*$, is an IFS (R, I, I) together with two intuitionistic fuzzy binary operations, namely $F+, F*$ satisfying the following conditions

(1) $(R, I), F+$ is an abelian intuitionistic fuzzy group,

(2) $((R, I), F*)$ is an intuitionistic fuzzy semi-group,

(3) $F*$ is distributive over $F+$. That is,

$$(x, I, I)F*(y, I, I)F+(z, I, I) = (x, I, I)F*(y, I, I)F+(x, I, I)F*(z, I, I),$$

$$(x, I, I)F+(y, I, I)F*(z, I, I) = (x, I, I)F*(z, I, I)F+(y, I, I)F*(z, I, I).$$

DEFINITION 3.2: $(R, I, I), F+, F*$ be an intuitionistic fuzzy ring.

fuzzy unity and denoted by $(1, I, I)$ if $(1, I, I)F*(b, I, I) = (b, I, I)F*(1, I, I) = (b, I, I)$ for all $(b, I, I), (R, I, I), F+, F*$. An intuitionistic fuzzy ring having a unity is called an intuitionistic fuzzy ring with unity.

(2) An intuitionistic fuzzy element $(a, I, I) \in (R, I, I), F+, F*$ is called a unit.

REMARK 2: For the intuitionistic fuzzy ring $(R, I, I), F+, F*$ we will call $F+$ and $F*$ the additive and multiplicative intuitionistic fuzzy binary operation respectively. The intuitionistic fuzzy identity will be denoted by $(0, I, I)$ and for any intuitionistic fuzzy element (a, I, I) $(R, I, I), F+, F*$ the additive and multiplicative intuitionistic fuzzy inverse will be denoted by (a, I, I) and (a^{-1}, I, I) respectively. That is;

$$(a, I, I)F + (a, I, I) = (a, I, I)F + (a, I, I) = (0, I, I),$$

$$(a, I, I)F * (a^{-1}, I, I) = (a^{-1}, I, I)F * (a, I, I) = (1, I, I).$$

The next theorem gives a correspondence relation between intuitionistic fuzzy rings and both ordinary and fuzzy rings.

where $F+= F+, f+, f+$ and $F*= F*, f*, f*$ two fuzzy rings $(R, I), F+, F*$ and

$(R, I), F+, F*$ where $F+= (F+, f+)$, $F*= (F*, f*)$ and $F+= (F^{+}, 1^{-}, f^{+})$, $F*= (F^{*}, 1^{-}, f^{*})$ which are isomorphic to the intuitionistic fuzzy ring.

DEFINITION 3.3 : Let S be an intuitionistic fuzzy subspace of the intuitionistic fuzzy space (R, I, I). The ordered pair $(S;\ F+, F*)$ is an intuitionistic fuzzy subring of the intuitionistic fuzzy ring $((R, I, I), F+, F*)$ if and only if $(S;\ F+, F*)$ defines an intuitionistic fuzzy ring under the intuitionistic fuzzy binary operations $F+, F*$.

THEOREM 1: Let $S = (x, s_x, s_x : x)\ S \circ$ be an intuitionistic fuzzy subspace of the intuitionistic fuzzy space (R, I, I). Then $(S, F+, F*)$ is an intuitionistic fuzzy subring of the IFR $((R, I, I), F+, F*)$ if and only if :

(1) $(S \circ, F+, F*)$ is an (ordinary) subring of the ring $(R, F+, F*)$,

(2) for all $x, y \in S \circ$ we have $s_x f_{xy} s_y = s_{xFy}$ *and* $\dot{s}_x \dot{f}_{xy} \dot{s}_y = \dot{s}_{xFy}$

PROOF:

Suppose (1) and (2) are satisfied, then:

The intuitionistic fuzzy subspace S is closed under the intuitionistic fuzzy binary operations $F+$ and $F*$: Let $(x, s_x, \dot{s}_x), (y, s_y, \dot{s}_y)$ be in S then

$$(x, s_x, \dot{s}_x)F + (y, s_y, \dot{s}_y) = F + ((x, s_x, \dot{s}_x), (y, s_y, \dot{s}_y))$$

$$= (F + (x, y), f_{xy} + (s_x, s_y), \dot{f}_{xy}(\dot{s}_x, \dot{s}_y))$$

$$= ((xF + y), s_{xFy}, \dot{s}_{xFy}) \in S$$

Similarly, for the intuitionistic fuzzy binary operation $F*$

$$(x, s_x, \dot{s}_x)F * (y, s_y, \dot{s}_y) = F * ((x, s_x, \dot{s}_x), (y, s_y, \dot{s}_y))$$

$$= (F * (x, y), f_{xy} * (s_x, s_y), \dot{f}_{xy}(\dot{s}_x, \dot{s}_y))$$

$$= ((xF * y), sxF * y, sxF * y) \in S.$$

From (1) and since $(S\ \circ, F\ +, F\ *)$ is an (ordinary) subring of the ring $(R, F\ +, F\ *)$ it is easy to check that $(S\ \circ, F\ +, F\ *)$ is an intuitionistic fuzzy subring of the IFR $((R, I, I), F\ +, F\ *)$

Conversely if $(S\ \circ, F\ +, F\ *)$ is an intuitionistic fuzzy subring of $((R, I, I), F\ +, F\ *)$ then (1) holds by the associativity theorem. Also the following hold

$$s_f\ +\ s_{xy}$$

$$= f\ +\ (s_{xy}\ \times\ s_y\ )\ =\ s_{xF+y}$$

$$s_x + f_{xy} + s_y\ =\ +f_{xy}(s_x\ \times\ s_y)\ =\ s_{xF+y}$$

$$s\ f * xy\, s_x f *$$

$$s\ \ = f\ *\ xy$$

$$s_y\ =\ f\ *\ (s_x\ \times\ s_y)\ =\ s_x F\ * y\, (s_x\ \times\ s_y)\ =\ s_x F\ * y$$

DEFINITION 3.4: (1) An intuitionistic fuzzy zero-divisor is an intuitionistic fuzzy element $(a, I, I)\ =\ (0, I, I)$ of a commutative intuitionistic fuzzy ring.

$((R, I, I), F+, F\ *)$ such that there is an intuitionistic fuzzy element (b, I, I) $((R, I, I), F+, F\ *)$ where

$(b, I, I)\ =\ (0, I, I)$ with $(a, I, I)F * (b, I, I)\ =\ (0, I, I)$.

(2) An intuitionistic fuzzy integral domain is a commutative intuitionistic fuzzy ring with unity and no intuitionistic fuzzy zero divisor.

THEOREM 2: Let $(a, I, I), (b, I, I)$ and (c, I, I) belong to an intuitionistic fuzzy integral domain $((R, I, I), F+, F\ *)$. If (a, I, I) $(0, I, I)$ and $(a, I, I)F * (b, I, I)\ =\ (a, I, I)F * (c, I, I)$,
then $(b, I, I)\ =\ (c, I, I)$.

PROOF:

From $(a, I, I)\ F * (b, I, I) =\ (a, I, I)\ F * (c, I, I)$

we have: $(a, I, I)\ F *\ (b, I, I)\ F + (c, I, I)\ \ =\ (0, I, I)$.

Since $(a, I, I)\ =\ (0, I, I)$ we must have:

$(b, I, I)F\ + (c, I, I)$. That is $(b, I, I)\ =\ (c, I, I)$.

A necessary and sufficient condition for intuitionistic fuzzy rings in terms of the cancellation property is given the next corollary which is a direct result from the above theorem.

COROLLARY 1 : A commutative intuitionistic fuzzy ring with unity is an intuitionistic fuzzy integral domain I and only if the cancellation property holds.

DEFINITION 3.5: An intuitionistic fuzzy field is a commutative intuitionistic fuzzy ring $((R, I, I), F+, F*)$ with unity in which every intuitionistic fuzzy element $(a, I, I) = (0, I, I)$ in $((R, I, I), F+, F*)$ is a unit.

The next theorem is a straightforward result to ideals obtained by intuitionistic fuzzy elements in an intuitionistic fuzzy ring.

THEOREM 3 : Let $((R, I, I), F+, F*)$ be a commutative intuitionistic fuzzy ring having an intuitionistic fuzzy identity $(0, I, I)$ and let (p, I, I) be an arbitrary intuitionistic fuzzy element in $((R, I, I), F+, F*)$ then

(1) $P = (p, I, I)F*(r, I, I) : (r, I, I)((R, I, I), F+, F*)$ is an intuitionistic fuzzy ideal in $((R, I, I), F+, F*)$.

(2) $T = (r, I, I) : (r, I, I)F*(t, I, I) = (0, I, I) : (r, I, I), (t, I, I)((R, I, I), F+, F*)$

is an intuitionistic fuzzy ideal in $((R, I, I), F+, F*)$.

CONCLUSION

Our study in this paper takes a novel approach by introducing the concept of an intuitionistic fuzzy ring, grounded in the framework of an intuitionistic fuzzy space. This innovative inclusion of the intuitionistic fuzzy space as a universal set serves as a pivotal correction mechanism through the intuitionistic fuzzy subgroups and rings within a well-defined structure. By synthesizing the concepts of intuitionistic fuzzy space and intuitionistic fuzzy rings, we offer a more refined and coherent framework for the exploration of this field, allowing for a deeper analysis of intuitionistic fuzzy ring structures.

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