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# On Fuzzy And Intuitionistic Fuzzy Topological BP-Algebras

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***Abstract:*** We discuss subalgebras and BP-ideal properties in BP-algebras whose topology is generated by a fuzzy set family. We present and explore some of the features of fuzzy topological subalgebras and ideals in BP-algebras in this work. The characteristics of the inverse and homomorphic images of the fuzzy topological ideals of BP-algebras are also covered. Additionally, we study some of the results of intuitionistic fuzzy topological structures in BP-algebras and present the idea of such structures in this paper.

***Index Terms***– (Fuzzy) continuous maps, (Fuzzy topological) ideals, C5-disconnectness, Compactness, BP-algebras, Intuitionistic fuzzy (topological) BP-algebras, Intuitionistic fuzzy continuous.

## I. Introduction

Zadeh [13] first proposed the idea of fuzzy sets in 1965.The notion of fuzzy sets was expanded upon by Atanassov [1] in 1986 to create intuitionistic fuzzy sets. Inspired by Rosenfeld's work [15], other mathematicians fuzzified a number of algebras. The concept of BCK and BCI algebras was first developed by Imai and Iseki [7] in 1966. Sun Shin Ahn and Han [11] first proposed the idea of BP-algebras in 2012. O.G. Xi [12] introduced fuzzy sets to BCK algebras in 1991. A study of intuitionistic fuzzy ideals of BCK algebras was conducted in 2000 by Jun and Kim [8]. Several ideas from general topology can be naturally extended to what could be called fuzzy topological spaces using the notion of a fuzzy set, which was first presented in. Foster [17] developed the components of a theory of fuzzy topological groups by fusing the structure of a fuzzy group introduced by A. Rosenfeld [10] with that of a fuzzy topological space. In this study, we define fuzzy topological subalgebras of BP-algebras and we apply to fuzzy topological subalgebras some of the conclusions obtained by Foster on homomorphic and inverse images. While intuitionistic fuzzy sets, or IFSs, provide both degrees of membership and non-membership, fuzzy sets only provide the degree of membership of an element in a particular set. Since both degrees are part of the interval [0,1], their sum cannot be more than1.The concept of intuitionistic fuzzy topological spaces was first presented by D. Coker [22]. We provide an intuitionistic fuzzy topological of BP-algebras based on this concept, and we derive some of its results.

## II. Preliminaries

We review several fundamental terminology in this part that are relevant to our work.

**Definition 2.1:** A nonempty set W with a constant 0 and a binary operation * that satisfies the following criteria is called a BCK-Algebra (W, *, 0):

(BCK 1) $[(a * b) * (a * c)] * (c * b) = 0$

(BCK 2) $(a * (a * b)) * b = 0$

(BCK 3) $a * a = 0$

(BCK 4) $0 * a = 0$

(BCK 5) $a * b = 0$ & $b * a = 0$ implies $a = b$, for any a, b, $c \in W$.

**Definition 2.2.** A non-empty set W with a constant 0 and a binary operation * that satisfies the following requirements is called a BP-algebra (W, *, 0).

i. $a * a = 0$

ii. $a * (a * b) = b$

iii. $(a * c) * (b * c) = a * b$ for any a, b, c ∈ W.

**Note:** Consider the BP - Algebra. Then, $a \leq b$ defines the partial order relation "$\leq$" iff $a * b = 0$.

**Definition 2.3.** Let any arbitrary set W be used. Let I = [0, 1] represent the unit interval. A fuzzy set on W is referred to as any function $\mu : W \to [0,1]$. Let $I^W$ be the collection of all fuzzy sets defined on W.

**Definition 2.4.** If the following condition is satisfied for every a, b∈ W, then a fuzzy set $\mu$ of a BP-algebra (W, *, 0) is referred to as a fuzzy BP-Subalgebra of W:

$\mu(a * b) \geq \min\{\mu(a), \mu(b)\}$.

**Definition 2.5.** A BP-Homomorphism is defined as a mapping f:(W,*,0)→(X, *′,0′) of BP-Algebras such that for any w, x∈ W, f (w ∗ x) = f(w) ∗′ f(x).

**Definition 2.6.** In a non-empty set W, an Intuitionistic Fuzzy Set A (IFSA) is defined as an object of the type $A = \{< w,\ \mu_A(w),\ \nu_A(w) > / \ w \in W\}$ or A= $(\mu_A, \nu_A)$ where $\mu_A : W \to [0,1]$ is the degree of membership and $\nu_A$ : W → [0,1] is the degree of non - membership of the element w ∈ W satisfying $0 \leq \ \mu_A(w) + \nu_A(w) \ \leq 1$.

**Definition 2.7.** Let A = $\{< w,\ \mu_A(w),\ \nu_A(w) > /w \in W\}$ and B = $\{< w,\ \mu_B(w),\ \nu_B(w) >/w \in W\}$ be two Intuitionistic Fuzzy
Sets of the set W, then define

(i) $\overline{A} = \{< w, \nu_A(w), \mu_A(w) > / \ w \in W\}$

(ii) $A \subseteq B$ *iff* $\mu_A(w) \leq \mu_B(w)$ *and* $\nu_A(w) \geq \nu_B(w)\ \forall w \in W$

(iii) $A = B$ *iff* $\mu_A(w) = \mu_B(w)$ *and* $\nu_A(w) = \nu_B(w)\ \forall w \in W$

(iv) $A \cup B = \{< w, \max(\mu_A(w), \mu_B(w)), \min(\nu_A(w), \nu_B(w)) / \ w \in W\}$

(v) $A \cap B = \{< w, \min(\mu_A(w), \mu_B(w)), \max(\nu_A(w), \nu_B(w)) / \ w \in W\}$

**Definition 2.8.** An Intuitionistic Fuzzy set A = $\{< w,\ \mu_A(w),\ \nu_A(w) > /w \in W\}$ in a BP-Algebra W is said to be an Intuitionistic Fuzzy BP-Subalgebra of W if

(i) $\mu_A(w * x) \ \geq \min \{\ \mu_A(w),\ \mu_A(x)\}$
(ii) $\nu_A(w * x) \ \leq \max \{\ \nu_A(w),\ \ \nu_A(x)\}\ \forall\ w,\ x \ \in$ W.

**Definition 2.9.** An Intuitionistic Fuzzy Set A of a BP-Algebra W is said to be an Intuitionistic Fuzzy BP-ideal of W, if

1. $\mu_A(0) \ \geq \mu_A(w)$ and $\nu_A(0) \leq \nu_A(w)$
2. $\mu_A(w) \ \geq \min \ \{\mu_A(w * x), \mu_A(x)\}$
3. $\nu_A(w) \ \leq \min \ \{\nu_A(w * x), \nu_A(x)\}$ for all w, x∈ W.

**Definition 2.10.** Let f : W →X be a BP-Homomorphism of BP-Algebras. For any Intuitionistic Fuzzy Set A = $(\mu_A, \nu_A)$ in X, we define a new intuitionistic Fuzzy Set $A^f = (\mu_A^f, \nu_A^f)$ in W by $\mu_A^f(W) = \mu_A(f(W))$, $\nu_A^f(W) = \nu_A(f(W))$ for all w, x ∈ W.

**Definition 2.11.** Let f be a mapping from a set W to a set X.
(a) If Q = $\{< x,\ \mu_Q(x),\ \nu_Q(x) >/x \in X\}$ is an IFS in X, then the preimage of Q under f, denoted by $f^{-1}(Q)$, is the IFS in W defined by

$$f^{-1}(Q) = \{< w, f^{-1}\mu_Q(w), f^{-1}\nu_Q(x) >: w \in W\}$$

(b) If P = $\{< w,\ \mu_Q(w),\ \nu_Q(w) >/w \in W\}$ is an IFS in W, then the image of P under f, denoted by $f(P)$, is the IFS in X defined by

$$f(P) = \{< x, f_{\cup}(\mu_P)(x), f_{\cap}(\nu_Q)(x) >: x \in X\},$$

Where

$$f_{\cup}(\mu_P)(x) = \begin{cases} \vee\, w \in f^{-1}(x)\mu_P(w), & if\ f^{-1}(x) \neq \phi, \\ 0, & otherwise \end{cases}$$

$$f_{\cap}(\nu_P)(w) = \begin{cases} \wedge\, w \in f^{-1}(x)\nu_P(w), & if\ f^{-1}(x) \neq \phi, \\ 0, & otherwise \quad for \quad each \quad x \in X \end{cases}$$

**Definition 2.12.** An intuitionistic fuzzy topology (IFT) on a non empty set W is a family $\tau$ of IFSs in W satisfies the following conditions:
(i) 0, 1 $\in \tau$ ,
(ii) If $H_1, H_2 \in \tau$, then $H_1 \cap H_2 \in \tau$,
(iii) If $H_j \in \tau$ for all j $\in J$, then $\cup_{i \in I} H_i \in \tau$.
Any IFS in $\tau$ is referred to as an intuitionistic fuzzy open set (IFOS), and the pair (W, $\tau$) is referred to as an intuitionistic fuzzy topological space (IFTS).

**Definition 2.13.** Given two fuzzy IFTSs (W, $\tau_1$) and (X, $\tau_2$), let f : (W, $\tau_1$) $\rightarrow$(X, $\tau_2$) be a function. Then f is said to be fuzzy continuous if and only if the preimage of each IFS in $\tau_2$ is an IFS in $\tau_1$.

**Definition 2.14.** Let (W, $\tau_1$) and (X, $\tau_2$) be two fuzzy IFTSs and let f : (W, $\tau_1$) $\rightarrow$(X, $\tau_2$) be a function. Then f is said to be fuzzy open if and only if the image of each IFS in $\tau_1$ is an IFS in $\tau_2$.

## III. Fuzzy Topological BP-subalgebras

**Definition 3.1.** A fuzzy topology τ on a BP-algebra, $\mathfrak{B}\wp$ is said to be an indiscrete fuzzy topology if its only elements are empty fuzzy set ($\emptyset_{\mathfrak{B}\wp}$) and whole fuzzy set ($1_{\mathfrak{B}\wp}$). A fuzzy topology τ on a BP-algebra $\mathfrak{B}\wp$ is said to be a discrete fuzzy topology if it contains all fuzzy subsets of $\mathfrak{B}\wp$.

**Definition 3.2.** A fuzzy set S in a BP-algebra $\mathfrak{B}\wp$ with membership function $\mu_S$ is called a fuzzy subalgebra of $\mathfrak{B}\wp$ if
$\mu_S(x * y) \geq \min\{\mu_S(x), \mu_S(y)\}$ for all x, y $\in \mathfrak{B}\wp$.

**Definition 3.3.** Consider the BP-algebra $\mathfrak{B}\wp$ = (W, *, 0), where W = {0, x, $x^2$, $x^3$} be a set with the following Cayley table:

| * | 0 | x | $x^2$ | $x^3$ |
|---|---|---|---|---|
| 0 | 0 | x | $x^2$ | $x^3$ |
| x | x | 0 | $x^3$ | $x^2$ |
| $x^2$ | $x^2$ | $x^3$ | 0 | x |
| $x^3$ | $x^3$ | $x^2$ | x | 0 |

(i) Let we define a fuzzy set,

$$S = < w, \frac{0}{0.4}, \frac{x}{0.3}, \frac{x^2}{0.2}, \frac{x^3}{0.1} >$$

$$T = < w, \frac{0}{0.3}, \frac{x}{0.2}, \frac{x^2}{0.2}, \frac{x^3}{0.1} >$$

Then the family {$\emptyset_{\mathfrak{B}\wp}$, $1_{\mathfrak{B}\wp}$, S, T} of fuzzy sets in $\mathfrak{B}\wp$ is a fuzzy topology on $\mathfrak{B}\wp$ because the empty fuzzy set $\emptyset_{\mathfrak{B}\wp}$ and the whole fuzzy set $1_{\mathfrak{B}\wp}$ are in τ , and the intersection of any two members and arbitrary union of members of τ is a member of τ.

(ii) Define a fuzzy set S in $\mathfrak{B}\wp$ with membership function $\mu_S$ is defined by $\mu_S(0) = 0.5$ and $\mu_S(w) = 0.01$ for all w $\neq$ 0 in $\mathfrak{B}\wp$. So it is clear that S is called the fuzzy subalgebra of $\mathfrak{B}\wp$.

**Definition 3.4.** Let $(T_1, \tau_{T_1})$ and $(T_2, \tau'_{T_2})$ is a fuzzy subspaces of fuzzy topological spaces $(\mathfrak{B}\wp_1, \tau)$ and $(\mathfrak{B}\wp_2, \tau')$ respectively, and let f be a mapping from $(\mathfrak{B}\wp_1, \tau)$ to $(\mathfrak{B}\wp_2, \tau')$. Then f is a mapping of $(\mathfrak{B}\wp_1, \tau_{T_1})$ into $(\mathfrak{B}\wp_2, \tau'_{T_2})$ if $f(\mathfrak{B}\wp_1) \subset \mathfrak{B}\wp_2$ .
Further, f is relatively fuzzy continuous if for each open fuzzy set $U_{T_2}$ in $\tau'_{T_2}$, the intersection $f^{-1}(U_{T_2}) \cap T_1$ is in $\tau_{T_1}$.
Moreover, f is relatively fuzzy open if for each open fuzzy set $V_{T_1}$ in $\tau_{T_1}$, the image $f(V_{T_1})$ is in $\tau'$.

**Theorem 3.5.** Let $\mathfrak{B}\wp_1$ and $\mathfrak{B}\wp_2$ be BP-algebras and let $(S, \tau_S)$ and $(T, \tau_T)$ be fuzzy subspaces of $(\mathfrak{B}\wp_1, \tau)$ and $(\mathfrak{B}\wp_2, \tau')$ respectively. Let f be a fuzzy continuous mapping of $\mathfrak{B}\wp_1$ into $\mathfrak{B}\wp_2$ such that $f(S_1) \subset S_2$. Then f is relatively fuzzy continuous mapping of $S_1$ into $S_2$.

**Proof.** Let $U_{S_2}$ be a fuzzy set in $\tau'_{S_2}$, then there exists $U \in \tau'$ such that $U_{S_2} = U \cap S_2$. Since f is fuzzy continuous, it follows that $f^{-1}(U)$ is a fuzzy set in $\tau$.
Hence $f^{-1}(U_{S_2}) \cap S_1 = f^{-1}(U \cap S_2) \cap S_1 = f^{-1}(U) \cap f^{-1}(S_2) \cap S_1 = f^{-1}(U) \cap S_1$ is a fuzzy set in $\tau_{S_1}$. This completes the proof.

**Definition 3.6.** Let $\tau_1$ and $\tau_2$ be fuzzy topologies on BP-algebras $\mathfrak{B}\wp_1$ and $\mathfrak{B}\wp_2$ respectively and S be a fuzzy set with membership function. A function $f : (\mathfrak{B}\wp_1, \tau_1) \to (\mathfrak{B}\wp_2, \tau_2)$ is said to be a fuzzy continuous map from $(\mathfrak{B}\wp_1, \tau_1)$ to $(\mathfrak{B}\wp_2, \tau_2)$ if it satisfies following conditions:

(i) For every $S \in \tau_2$ , $f^{-1}(S) \in \tau_1$,
(ii) For every fuzzy subalgebras S (of $\mathfrak{B}\wp_2$) in $\tau_2$ , $f^{-1}(S)$ is a fuzzy subalgebra (of $\mathfrak{B}\wp_1$) in $\tau_1$.

**Definition 3.7.** Let $(\mathfrak{B}\wp_1, \tau_1)$ and $(\mathfrak{B}\wp_2, \tau_2)$ be any two fuzzy topological spaces. A function $f : (\mathfrak{B}\wp_1, \tau_1) \to (\mathfrak{B}\wp_2, \tau_2)$ is said to be a fuzzy homomorphism if it satisfies the following conditions:
- f is bijective,
- both f and $f^{-1}$ are fuzzy continuous maps.

**Definition 3.8.** Let $\tau$ be a fuzzy topology on a BP-algebra $\mathfrak{B}\wp$. A FTS $(\mathfrak{B}\wp, \tau)$ is said to be a fuzzy $C_5$ - disconnected if there exists a fuzzy open and fuzzy closed set S with membership $\mu_S$ such that $\mu_S \neq 1_{\mathfrak{B}\wp}$ and $\mu_S \neq \emptyset_{\mathfrak{B}\wp}$. A FTS $(\mathfrak{B}\wp, \tau)$ is said to be a fuzzy $C_5$ - connected if it is not a fuzzy $C_5$ - disconnected.

**Theorem 3.9.** Let $\tau_1$ and $\tau_2$ be the fuzzy topologies on BP(H)-algebras $\mathfrak{B}\wp_1$ and $\mathfrak{B}\wp_2$ respectively. Let $f : (\mathfrak{B}\wp_1, \tau_1) \to (\mathfrak{B}\wp_2, \tau_2)$ be a fuzzy continuous surjective mapping. If $(\mathfrak{B}\wp_1, \tau_1)$ is a fuzzy $C_5$ - connected, then $(\mathfrak{B}\wp_2, \tau_2)$ is a fuzzy $C_5$ -connected.

**Proof.** Assume that $(\mathfrak{B}\wp_2, \tau_2)$ is a fuzzy $C_5$ -disconnected. Then there exist a fuzzy open and closed set S with membership function $\mu_S$ such that $\mu_S \neq 1_{\mathfrak{B}\wp_2}$ and $\mu_S \neq \emptyset_{\mathfrak{B}\wp_2}$. Since f is a fuzzy continuous mapping, $f^{-1}(\mu_S)$ is both OFS and CFS.

Thus $f^{-1}(\mu_S) = 1_{\mathfrak{B}\wp_1}$ or $f^{-1}(\mu_S) = \emptyset_{\mathfrak{B}\wp_1}$ which is impossible.[since $\mu_S = f(f^{-1}(\mu_S)) = f(1_{\mathfrak{B}\wp_1}) = 1_{\mathfrak{B}\wp_2}$, and $\mu_S = f(f^{-1}(\mu_S)) = f(\emptyset_{\mathfrak{B}\wp_1}) = \emptyset_{\mathfrak{B}\wp_2}$] This is contradiction to our assumption. Hence $(\mathfrak{B}\wp_2, \tau_2)$ is also a fuzzy $C_5$ - connected.

**Definition 3.10.** Let $\tau$ be a fuzzy topology on a BP-algebra $\mathfrak{B}\wp$ and S be a fuzzy set in $\mathfrak{B}\wp$ with membership function $\mu_S$ . If a class $\{< x, \mu_{S_i}\} >: i \in I\}$ of OFS in $\mathfrak{B}\wp$ satisfies the condition $S \subseteq \cup \{< x, \mu_{S_i} >: i \in I\}$ , then it is called a fuzzy open cover of S.

A finite subclass of the fuzzy open cover $\{< x, \mu_{S_i} >: i \in I\}$ of S, which is also a fuzzy open cover of S, is called a finite subcover of $\{< x, \mu_{S_i} >: i \in I\}$. A FS $S = < x, \mu_S >$ in a FTS $(X, \tau)$ is called a fuzzy compact, if every fuzzy open cover of S has a finite subcover.

**Theorem 3.11.** Let $\tau_1$ and $\tau_2$ be the fuzzy topologies on BP-algebras $\mathfrak{B}\wp_1$ and $\mathfrak{B}\wp_2$ respectively. Let f : $\mathfrak{B}\wp_1 \to \mathfrak{B}\wp_2$ be a fuzzy continuous mapping. If S is a fuzzy compact in ($\mathfrak{B}\wp_1$,$\tau_1$), then f(S) is a fuzzy compact in ($\mathfrak{B}\wp_2$,$\tau_2$).

**Proof**. Let A = $\{\mu_{S_i}$: i ∈ I}, where $\mu_{S_i} = < x, \mu_{S_i} >$ be a fuzzy open cover of f(S). Then B = $\{f^{-1}(\mu_{S_i})$: i ∈ I} is a fuzzy open cover of S. Since S is a fuzzy compact, there exists an finite subcover $\mu_{S_i}$(i = 1, 2, …, n) of S such that S ⊆ $\bigcup_{i=1}^{n} f^{-1}(\mu_{S_i})$. Thus

$$f(S) \subseteq (f(\bigcup_{i=1}^{n} f^{-1}(\mu_{S_i}))$$

$$f(S) \subseteq \bigcup_{i=1}^{n} f(f^{-1}(\mu_{S_i}))$$

$$f(S) \subseteq \bigcup_{i=1}^{n}(\mu_{S_i})$$

Therefore, f (S) is a fuzzy compact in ($\mathfrak{B}\wp_2$,$\tau_2$).

## IV. FUZZY TOPOLOGICAL BP-IDEALS

**Definition 4.1.** A fuzzy set B in a BP-algebra $\mathfrak{B}\wp$ with membership function $\mu_B$ is called a fuzzy ideals of $\mathfrak{B}\wp$ if it satisfies:

(i) $\mu_B(0) \geq \mu_B(y)$

(ii) $\mu_B(y) \geq \min\{\mu_B(y * z), \mu_B(z)\}$ for all y, z ∈ $\mathfrak{B}\wp$.

**Proposition 4.2.** Let β be a homomorphism of a BP-algebra P into a BP-algebra Q and B a fuzzy BP-ideal of Q with membership function $\mu_B$. Then the inverse image $\beta^{-1}(B)$ of B is a fuzzy BP-ideal of P.

**Proof:** Since β is a homomorphism of (P, *, 0) into (Q, *, $\theta$), then β (0) = $\theta$ and, by the assumption, $\mu_B(\beta(0)) = \mu_B(\theta) \geq \mu_B(q)$ for every q ∈ $Q$. In particular, $\mu_B(\beta(0)) \geq \mu_B(\alpha(p))$ for every p ∈ P. Thus $\mu_{\beta^{-1}(B)}(0) \geq \mu_{\beta^{-1}(B)}(p)$, hence the proof (i).

Let y, z ∈ P. Then, since by assumption on $\mu_B$ , we have

$$\mu_{\beta^{-1}(B)}(y) = \mu_B(\beta(y)) \geq \min\{\mu_B(\beta(y) * \beta(z)), \mu_B(z)\}$$

All β (y), β (z) ∈ Q, this gives

$$\mu_{\beta^{-1}(B)}(y) = \mu_B(\beta(y)) \geq \min\{\mu_B(\beta(y) * \beta(z)), \mu_B(\beta(z))\}$$

$$= \min\{\mu_B(\beta(y * z)), \mu_B(\beta(z))\}$$

$$\mu_{\beta^{-1}(B)}(y) \geq \min\{\mu_{\beta^{-1}(B)}(y * z), \mu_{\beta^{-1}(B)}(z))\}$$

Hence the proof (ii) since by putting y = 0.

**Corollary 4.3**. Given BP-algebras P, Q and a homomorphism $\beta$ of P onto Q , let τ be a fuzzy topology on Q and V be the fuzzy topology on Q such that $\beta$ (τ) = V. Let E be a fuzzy topological BP-ideal (BP-ideal) of P. If the membership function $\mu_E$ of E is $\beta$-invariant, then $\beta$(E) is a fuzzy topological BP-ideal (BP-ideal) of Q.

## V. INTUITIONISTIC FUZZY TOPOLOGICAL BP-ALGEBRAS

**Definition 5.1.** An IFT $\tau$ is said to be an indiscrete intuitionistic fuzzy topology if its only elements are (0) and (1). An IFT $\tau$ is said to be a discrete intuitionistic fuzzy topology if it contains all intuitionistic fuzzy subsets of W.

**Example 5.2.** Let BP-algebra W = {0, p, q, r} with the following table:

| * | 0 | p | q | r |
|---|---|---|---|---|
| 0 | 0 | p | q | r |
| p | p | 0 | r | q |
| q | q | r | 0 | p |
| r | r | q | p | 0 |

Suppose we define IFS

$$L = < w, \left(\frac{0}{0.4}, \frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.5}\right), \left(\frac{0}{0.2}, \frac{0}{0.4}, \frac{0}{0.1}, \frac{0}{0.3}\right) >$$
$$M = < w, \left(\frac{0}{0.1}, \frac{p}{0.2}, \frac{q}{0.3}, \frac{r}{0.2}\right), \left(\frac{0}{0.3}, \frac{0}{0.3}, \frac{0}{0.2}, \frac{0}{0.1}\right) >$$
$$N = < w, \left(\frac{0}{0.2}, \frac{p}{0.1}, \frac{q}{0.3}, \frac{r}{0.4}\right), \left(\frac{0}{0.3}, \frac{0}{0.2}, \frac{0}{0.1}, \frac{0}{0.4}\right) >$$

Then the family τ = {0, 1, L, M, N} of IFSs in W is an IFTs on W.

**Example 5.3.** Consider a BP- algebra W = {0, 1, 2, 3}.Suppose we define IFS as follows (n ∈ ℕ):

$$B_n = < w, \left(\frac{0}{\frac{n}{n+1}}, \frac{1}{\frac{n}{n+2}}, \frac{2}{\frac{n+1}{n+2}}, \frac{3}{\frac{n+1}{n+3}}\right), \left(\frac{0}{\frac{1}{n+4}}, \frac{1}{\frac{1}{n+3}}, \frac{2}{\frac{1}{n+2}}, \frac{3}{\frac{1}{n+1}}\right) >$$

Then the family {0, 1} ∪ $\{B_n$: n ∈ ℕ} of IFSs in W is an IFTs on W.

**Definition 5.4.** Let W be a BP-algebra. An IFS B = $< w, \mu_B, \nu_B >$ in W is called an intuitionistic fuzzy subalgebra of W if it satisfies:

(i) $\mu_B(p * q) \geq \min\{\mu_B(p) * \mu_B(q)\}$

(ii) $\nu_B(p * q) \leq \max\ \{\nu_B(p) * \nu_B(q)\} \forall\ p, q \in W.$

**Example 5.5.** Consider the BP-algebra W = {0, l, m, n} with the following table:

| * | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 |
| 1 | 1 | 0 | 3 | 2 |
| 2 | 2 | 3 | 0 | 1 |
| 3 | 3 | 2 | 1 | 0 |

Define,

$$\mu_B(p) = \begin{cases} 0.7 & if \quad p \neq 2 \\ 0.2 & if \quad p = 2 \end{cases}$$

And

$$\nu_B(p) = \begin{cases} 0.1 & if \quad p \neq 2 \\ 0.6 & if \quad p = 2 \end{cases}$$

Then the IFS $B = \{< p, \mu_B(p), \nu_B(p) > / p \in W\}$ of W is an IFBP-Subalgebra of W.

**Definition 5.6.** Let B be an IFS in an IFTS (W, $\tau$). Then the induced intuitionistic fuzzy topology on B is the family of IFSs in B which are the intersection with B of IFOSs in W. The induced intuitionistic fuzzy topology is denoted by $\tau_B$, and the pair (B, $\tau_B$) is called an intuitionistic fuzzy subspace of (W, $\tau$).

**Definition 5.7.** Let (E, $\tau_E$) and (F, $\tau_F$) be an intuitionistic fuzzy subspaces of intuitionistic fuzzy topological spaces (W, $\tau_1$) and (X, $\tau_2$) respectively, and let f be a mapping from (W, $\tau_1$) to (X, $\tau_2$). Then f is a mapping of (E, $\tau_E$) into (F, $\tau_F$) if f(E) ⊆ F. Furthermore, f is relatively intuitionistic fuzzy continuous if for each open fuzzy set Q in $\tau_F$, the intersection $f^{-1}(Q) \cap E$ is in $\tau_E$. Conversely, f is relatively intuitionistic fuzzy open if for each open fuzzy set P in $\tau_E$, the image $f^{-1}(P)$ is in $\tau_2$.

**Theorem 5.8.** Let (E, $\tau_E$) and (F, $\tau_F$) be an intuitionistic fuzzy subspaces of (W, $\tau_1$) and (X, $\tau_2$) respectively. Let f be an intuitionistic fuzzy continuous mapping of (W, $\tau_1$) into (X, $\tau_2$) such that f(E) ⊆ F. Then f is relatively an intuitionistic fuzzy continuous mapping of (E, $\tau_E$) into (F, $\tau_F$).

**Proof.** Let $Q_F$ be an IFS in $\tau_F$, then there exists Q $\epsilon$ $\tau$ such that $Q_F = Q \cap F$. Since f is an intuitionistic fuzzy continuous, it follows that $f^{-1}(Q)$ is an IFS in $\tau$.

Hence $f^{-1}(Q_F) \cap E = f^{-1}(Q \cap F) \cap E = f^{-1}(Q) \cap f^{-1}(E) \cap E = f^{-1}(Q) \cap E$ is an intuitionistic fuzzy set in $\tau_E$.

Hence the proof.

**Definition 5.9**. Let $\tau_1$ and $\tau_2$ be the intuitionistic fuzzy topologies on BP-algebras W and X respectively. A function f : (W, $\tau_1$) → (X, $\tau_2$) is said to be an intuitionistic fuzzy continuous map from (W, $\tau_1$) to (X, $\tau_2$) if it satisfies following conditions:

(i) For every B $\epsilon$ $\tau_2, f^{-1}(B) \epsilon \tau_1$,

(ii) For every intuitionistic fuzzy subalgebras B (of X ) in $\tau_2$, $f^{-1}(B)$ is an intuitionistic fuzzy subalgebra (of W) in $\tau_1$.

**Theorem 5.10**. If $\tau_1$is an intuitionistic fuzzy topology on a BP-algebra W and $\tau_2$ is an indiscrete intuitionistic fuzzy topology on a BP-algebras Y, then every function f : (W, $\tau_1$) → (X, $\tau_2$) is an intuitionistic fuzzy continuous map.

**Proof.** Since $\tau_2$ is an indiscrete intuitionistic fuzzy topology, $\tau_2$ = {0, 1}.

Let f : W → X be any function. Let 0 $\epsilon$ $\tau_2$, then for any w $\epsilon$ $W$,

$f^{-1}$ (0)(w) = 0(f(w)) = 0 [as f(w) $\epsilon$ X] = 0(w).

Thus ($f^{-1}$ (0)) = 0 $\tau_1$.

Let 1 $\epsilon$ $\tau_2$ and w $\epsilon$ W, then we have

($f^{-1}$ (1))(w) = 1(f(w)) = 1 = 1(w) [ by definition of whole intuitionistic fuzzy set].

Thus ($f^{-1}$(1))) = 1 $\epsilon$ $\tau_1$.

Hence f is an intuitionistic fuzzy continuous map from W to X .

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