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Relatively Prime Edge Detour Domination Number Of Graphs

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Abstract: In this paper, we introduce the new concept relatively prime edge detour domination number of a graph and obtain the relatively prime edge detour domination number for some well known graphs.

Key words: Domination, Edge detour domination, Relatively prime detour domination, Relatively prime edge detour domination.

I.Introduction

The concept of domination was introduced by Ore and Berge [5]. Let G be a finite, undirected connected graph with neither loops nor multiple edges. A subset D of V(G) is a dominating set of G if every vertex in $V - D$ is adjacent to at least one vertex in D. The minimum cardinality among all dominating sets of G is called the domination number $\gamma(G)$ of G. For basic definitions and terminologies, we refer Harary [1].

For vertices u and v in a connected graph G, the detour distance $D(u, v)$ is the length of longest $u - v$ path in G. A $u - v$ path of length $D(u, v)$ is called a $u - v$ detour. A subset S of V is called a detour set if every vertex in G lies on a detour joining a pair of vertices of S. The detour number $dn(G)$ of G is the minimum order of a detour set and any detour set of order $dn(G)$ is called a detour basis of G. These concepts were studied by chartrand [2].

A subset S of V is called an edge detour set if every edge in G lies on a detour joining a pair of vertices of S. The edge detour number $dn_1(G)$ of G is the minimum order of its edge detour set and any edge detour set of order dn_1 is an edge detour basis. A graph G is called an edge detour graph if it has an edge detour set. Edge detour graphs were introduced and studied by Santhakumaran and Athisayanathan [6].

A set $S \subseteq V$ is said be relatively prime detour dominating set of a graph G if it is a detour set and a dominating set with at least two elements and for every pair of vertices u and v such that $(\deg(u), \deg(v)) = 1$. The minimum cardinality of a relatively prime detour dominating set is called the relatively prime detour domination number of a graph G and is denoted by $\gamma_{rpdn}(G)$. This concept were introduced by C. Jayasekaran and L.G. Binoja [3].

The helm graph H_n is a graph obtained from wheel W_n by attaching a pendent edge to each rim vertex. It contains three types of vertices, the vertex of degree n called apex, n pendant vertices and n vertices of degree four.

The complete bipartite graph $K_{1,p}$ is called a Star. The vertex of degree $p - 1$ in $K_{1,P-1}$ is called its Center.

II.Relatively Prime Edge Detour Domination Number of Graphs

Definition 2.1: A set $S \subseteq V$ is said to be relatively prime edge detour dominating set of a graph G if it is a edge detour set and a dominating set with at least two elements and for every pair of vertices u and v such that $(\deg(u), \deg(v)) = 1$. The minimum cardinality of relatively prime edge detour dominating set is called the relatively prime edge detour domination number of a graph G and is denoted by $\gamma_{rped}(G)$. If the relatively prime edge detour dominating sets does not exist then the relatively prime edge detour domination number is zero.

Example 2.2: $S = \{v_1, v_3\}$ is a edge detour dominating set. Also, $(\deg(v_1), \deg(v_3)) = 1$. Therefore, S is a relatively prime edge detour dominating set. Hence, $\gamma_{rped}(G) = 2$.

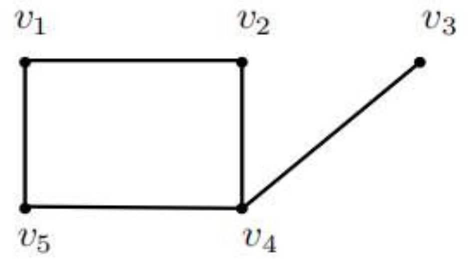

Figure 2.1

Theorem 2.3:

Every end vertex of a graph G belong to every relatively prime edge detour dominating set of G.

Proof:

Every relatively prime edge detour dominating set is a dominating set of G. Therefore, every end vertex of a graph G belongs to every relatively prime edge detour dominating set of G.

Theorem 2.4:

If G is a Path P_n of order $n \geq 2$, then $\gamma_{rped}(P_n) = \begin{cases} 2 & if\ 2 \leq n \leq 4 \\ 3 & if\ 5 \leq n \leq 7 \\ 0 & otherwise \end{cases}$

Proof:

Let $v_1, v_2, \ldots, v_n$ be a path P_n.

Case 1: $2 \leq n \leq 4$

Let S be an edge detour dominating set of P_n. By theorem 2.3, $\{v_1, v_n\} \subseteq S$. Clearly $\{v_1, v_n\}$ is a minimum edge detour dominating set of P_n and $(\deg(v_1), \deg(v_n)) = 1$. Therefore, $S = \{v_1, v_n\}$ is a minimum relatively prime edge detour dominating set. Hence, $\gamma_{rped}(P_n) = 2$.

Case 2: $5 \leq n \leq 7$

Sub case i): If n = 5 and 6. In this case, $S = \{v_1, v_3, v_n\}$ is a minimum edge detour dominating set. Also, $(\deg(v_1), \deg(v_3)) = (\deg(v_3), \deg(v_n)) = (\deg(v_1), \deg(v_n)) = 1$. Therefore, S is a minimum relatively prime edge detour dominating set. Hence, $\gamma_{rped}(P_n) = 3$.

Sub case ii): If n = 7. $S = \{v_1, v_4, v_7\}$ is a minimum edge detour dominating set. Also, $(\deg(v_1), \deg(v_7)) = (\deg(v_1), \deg(v_4)) = (\deg(v_4), \deg(v_7)) = 1$. Therefore, S is a minimum relatively prime edge detour dominating set. Hence, $\gamma_{rped}(P_7) = 3$.

Case 3: $n \geq 3$

Since the dominating set of P_n containing at least any two internal vertices v_i, v_j ; $3 \leq i \neq j \leq n-2$ and $(\deg(v_i), \deg(v_j)) = 2$. Therefore, relatively prime edge detour dominating set does not exist. Hence, $\gamma_{rped}(P_n) = 0$.

Theorem 2.5:

If G is a Star $K_{1,n-1} (n \geq 3)$, then $\gamma_{rped}(G) = n - 1$.

Proof:

Let G be the Star $K_{1,n-1}$ with $V(K_{1,n-1}) = \{v, v_i : 1 \leq i \leq n-1\}$ and $E(K_{1,n-1}) = \{vv_i : 1 \leq i \leq n-1\}$.

Let S be the edge detour dominating set of G. By theorem 2.3, $\{v_1, v_2, \dots, v_{n-1}\} \subseteq S$. Also, $\{v_1, v_2, \dots, v_{n-1}\}$ is a minimum edge detour dominating set of G and $(\deg(v_i), \deg(v_j)) = 1$ for $i \leq i \neq j \leq n-1$. Therefore, $S = \{v_1, v_2, \dots, v_{n-1}\}$ is a minimum relatively prime edge detour dominating set. Hence, $\gamma_{rped}(G) = n-1$.

Theorem 2.6:

If G is a Bistar $B_{m,n}$, then $\gamma_{rped}(G) = m+n$.

Proof:

Let $G = B_{m,n}$ with $V(G) = \{v, v_i, u, u_j : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(G) = \{uv, uu_j, vv_i : 1 \leq i \leq m, 1 \leq j \leq n\}$. Therefore, $|V(G)| = m+n$. Let S be a edge detour dominating set of G. By theorem 2.3, $\{v_i, u_j : 1 \leq i \leq m, 1 \leq j \leq n\} \subseteq S$. Clearly, $\{v_1, v_2, \dots, v_m, u_1, u_2, \dots, u_n\}$ itself is a minimum edge detour dominating set of G. Also, $(\deg(v_i), \deg(v_j)) = (\deg(u_x), \deg(u_y)) = (\deg(v_i), \deg(u_x)) = 1$; $1 \leq i \neq j \leq m, 1 \leq x \neq y \leq n$. Therefore, $S = \{v_1, v_2, \dots, v_m, u_1, u_2, \dots, u_n\}$ is a relatively prime edge detour dominating set. Hence, $\gamma_{rped}(G) = m+n$.

Theorem 2.7:

If G is a Complete graph $K_p (p \geq 2)$, then $\gamma_{rped}(G) = \begin{cases} 2 & if\ p = 2 \\ 0 & otherwise \end{cases}$.

Proof:

Case 1: If $p = 2$.

Then, $\{v_1, v_2\}$ is the relatively prime edge detour dominating set and $\gamma_{rped}(G) = 2$.

Case 2: If $p > 2$.

Every three element subset of $V(K_p)$ is an edge detour dominating set. Let $S = \{v_i, v_j, v_k\}$. Since, any two vertices in K_p is adjacent and $\deg(v_i) = p-1$ for all i. Also, $(\deg(v_i), \deg(v_j)) = (p-1, p-1) = p-1 = (\deg(v_i), \deg(v_k)) = (\deg(v_j), \deg(v_k))$. Therefore, $\{v_i, v_j, v_k\}$ is not a relatively prime edge detour dominating set. Hence, $\gamma_{rped}(G) = 0$.

Theorem 2.8: If G is the complete bipartite graph $K_{m,n}$ then,

$$\gamma_{rped}(G) = \begin{cases} 2 & if\ m = n = 1\ and\ (m,n) = 1\ where\ m,n \geq 2 \\ n & if\ m = 1\ ;\ n \geq 2\quad (or)\quad m\ \ if\ n = 1\ ;\ m \geq 2 \\ 0 & if\ (m,n) \neq 1\ and\ m,n \geq 2 \end{cases}$$

Proof:

Let $G = K_{m,n}$ with bi-partition $V_1 = \{a_1, a_2, \dots, a_m\}$ and $V_2 = \{b_1, b_2, \dots, b_n\}$ and $|V(G)| = m+n$.

Case 1: If $m = n = 1$, then $G \cong K_2$ and $\gamma_{rped}(G) = 2$.

Case 2: If $m = 1$ and $n \geq 2$ or $n = 1$ and $m \geq 2$, then $K_{m,n} = K_{1,n}$ or $K_{m,1}$. v_2 or v_1 is a minimum edge detour dominating set of G.

Hence, $\gamma_{rped}(G) = \begin{cases} n & if\ \ n \geq 2,\ m = 1 \\ m & if\ \ m \geq 2,\ n = 1 \end{cases}$

Case 3: $m = n \geq 2$. A minimum edge detour dominating set $S = \{u_i, u_{i+1}, v_j\}$; $(\deg(u_i), \deg(v_j)) = (\deg(u_{i+1}), \deg(v_j)) = (m, n)$ and $(\deg(u_i), \deg(u_{i+1})) = (m, m) \neq 1$. Therefore, S is not a relatively prime edge detour dominating set. Hence, $\gamma_{rped}(G) = 0$.

Theorem 2.9:

If G is a Helm graph H_n, then $\gamma_{rped}(G) = n$.

Proof:

Let $v_1, v_2, \dots, v_{n-1}, v_1$ be the cycle C_n. Add a vertex v which is adjacent to v_i ; $1 \leq i \leq n-1$. The resultant graph is the Wheel W_p. For, $1 \leq i \leq n-1$ add u_i which is adjacent to v_i. The resultant graph is the Helm graph H_n. Also, $|V(G)| = 2n - 1$; $\deg(v) = n - 1$; $\deg(v_i) = 4$; $\deg(u_i) = 1$ for each $i = 1, 2, \ldots, n - 1$.

Let $S = \{v, u_1, u_2, \ldots, u_{n-1}\}$. Then, S is a minimum edge detour dominating set of H_n. Also, $(\deg(v), \deg(u_i)) = (n - 1, 1) = 1$; $1 \leq i \leq n - 1$ and $(\deg(u_i), \deg(u_j)) = 1$; $1 \leq i \neq j \leq n - 1$. Therefore, S is a relatively prime edge detour dominating set. Hence, $\gamma_{rped}(G) = n$.

Theorem 2.10:

For $n \geq 2$, $C_n \odot K_1$, then $\gamma_{rped}(C_n \odot K_1) = n$.

Proof:

Let $v_1, v_2, \ldots, v_n, v_1$ be the cycle. For $1 \leq i \leq n$, add vertex u_i which is adjacent to v_i which is the graph $C_n \odot K_1$. Therefore, $|V(C_n \odot K_1)| = 2n$. Also, $\deg(u_i) = 1$ and $\deg(v_i) = 3$.

Let $S = \{u_1, u_2, \ldots, u_n\}$. Then, S is a minimum edge detour dominating set of $C_n \odot K_1$. Also, $(\deg(u_i), \deg(u_j)) = 1$; $1 \leq i, j \leq n$. Therefore, S is a minimum relatively prime edge detour dominating set. Hence, $\gamma_{rped}(C_n \odot K_1) = n$.

Theorem 2.11:

For $n \geq 2$, $\gamma_{rped}(P_n \odot K_1) = n$.

Proof:

Let $v_1, v_2, \ldots, v_n$ be the path P_n. Add a vertex u_i which is adjacent to v_i for $1 \leq i \leq n$. The resultant graph $P_n \odot K_1$, $|V(P_n \odot K_1)| = 2n$. Also, $\deg(v_1) = \deg(v_n) = \deg(u_i) = 2$; $1 \leq i \leq n$ and $\deg(v_j) = 3$; $2 \leq j \leq n - 1$.

Let S be a edge dominating set of $P_n \odot K_1$. By theorem 2.3, the end vertices $\{u_1, u_2, \ldots, u_n\}$ itself is a minimum relatively prime edge detour dominating set. Hence, $\gamma_{rped}(P_n \odot K_1) = n$.

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