IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

STRONG HOP STEINER DOMINATION IN GRAPHS

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Abstract

Let $G = (V, E)$ be a connected graph. A Steiner dominating set $S \subseteq V$ is said to be Strong hop steiner dominating set if every vertex $v \in V - S$ is strongly dominated by some $u \in S$ and for every vertex $v \in V - S$ there exists $u \in S$ such that $d(u, v) = 2$. The minimum cardinality of a strong hop steiner dominating set of G is its strong hop steiner domination number and is denoted by $\gamma_{sths}(G)$. In this paper, we determine the strong hopsteiner domination number of some special graphs. Some general properties satisfied by this concept are studied.

Keywords: Steiner dominating set, Hop dominating set, Strong dominating set, Strong hop steiner dominating set.

1.INTRODUCTION

A vertex in a graph G dominates itself and its neighbors. A set of vertices D in a graph G is a **dominating set** if each vertex of G is dominated by some vertex of D. The **domination number $\gamma(G)$** of G is the minimum cardinality of a dominating set of G.

The concept of Steiner number of a graph was introduced by G. Chatrand and P. Zhang [2]. For a nonempty set W of vertices in a connected graph G, the **Steiner distance** d(W) of W is the minimum size of a connected subgraph of G containing W. Necessarily, each such subgraph is a tree and is called a Steiner tree with respect to W or a Steiner W-tree. It is to be noted that d(W) = d(u ,v), when W = {u ,v}. If v is an end vertex of a Steiner W-tree, then $v \in W$.Also if W is connected, then any Steiner W-tree contains the elements of W only. The set of all vertices of G that lie on some Steiner W-tree is denoted by S(W). If S(W) = V, then W is called a **Steiner set** for G. A Steiner set of minimum cardinality is a minimum Steiner set or simply a s-set of G and this cardinality is the **Steiner number s(G)** of G.

N(v) = { u ∈ V(G) : uv ∈ E(G)} is called the neighborhood of the vertex v in G. A vertex v is an **extreme vertex** of a graph G if the subgraph induced by its neighbors is complete. If $e = uv$ is an edge of a graph G with $d(u) = 1$ and $d(v) > 1$, then we call e a pendant edge, u a leaf or end vertex and v a support vertex.Each extreme vertex of a graph G belongs to every Steiner set of G. In particular, each end-vertex of G belongs to every Steiner set of G.

The concept of Steiner domination number of a graph was introduced by J. John et al., [4]. Let G be a connected graph. A set of vertices W in G is called a **Steiner dominating set** if W is both a Steiner set and a dominating set. The minimum cardinality of a Steiner dominating set of G is its **Steiner domination number** and is denoted by $\boldsymbol{\gamma_s(G)}$**.** A Steiner dominating set of size $\gamma_s(G)$ is said to be a γ_s-set of G.

The concept of hop domination was introduced by Ayyaswamy and Natarajan [1]. A set S ⊆ V of a graph G = (V, E) is a hop dominating set of G iffor every vertex $v \in V - S$ there exists $u \in S$ such that $d(u, v) = 2$.

The concept of strong domination was introduced by Sampathkumar and Pushpa Latha [5]. For a graph G and $uv \in E(G)$, we say u strongly dominates v if $\deg(u) \geq \deg(v)$. A subset S of $V(G)$is a **strong dominating set (sd-set)** if every vertex $v \in V - S$ is strongly dominated by some $u \in S$. The strong domination number $\gamma_{st}(G)$ is the minimum cardinality of a sd-set.

An Fire cracker F(m,n) is a graph obtained by the series of interconnected m copies of n stars by linking one leaf from each.A Ladder graph L_n is a graph defined by $L_n = P_n \times K_2$ where P_n is path with n vertices and $\times$ denotes the Cartesian product and K_2 is a complete graph with two vertices.A Helm graph H_n is a graph obtained by attaching a single edge and node to each node of the outer circuit of wheel graph W_n.

2.STRONG HOP STEINER DOMINATION NUMBER OF A GRAPH

Definition 2.1 Let $G = (V, E)$ be a connected graph. A Steiner dominating set $S \subseteq V$ is said to be **Strong hop steiner dominating set** if every vertex$v \in V - S$ is strongly dominated by some $u \in S$ and for every vertex $v \in V - S$ there exists $u \in S$ such that $d(u, v) = 2$. The minimum cardinality of a strong hop steiner dominating set of G is its **strong hop steiner domination number** and is denoted by $\boldsymbol{\gamma_{sths}(G)}$**.**

Example 2.2: For the graph G in Figure 2.1, S= $\{v_1, v_5, v_8, v_9\}$ is a minimum strong hop steiner dominating set of G so that $\gamma_{sths}(G) = 4$

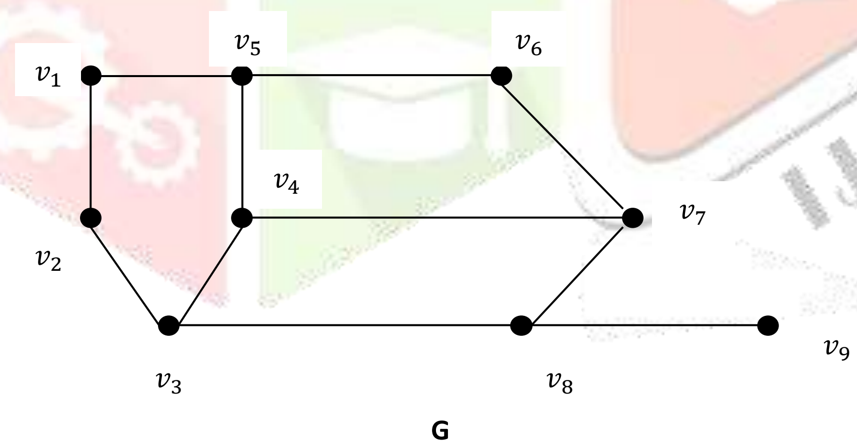


figure 2.1

Observation 2.3: Each extreme vertex of a connected graph G belongs to every strong hop steiner dominating set.

Observation 2.4: If G is a connected graph of order n, then

$2 \leq \max\{\gamma_s(G), \gamma_h(G), \gamma_{st}\} \leq \gamma_{sths}(G) \leq n$

3.STRONG HOP STEINER DOMINATION NUMBER OF SOME STANDARD GRAPHS.

Theorem 3.1: For the complete graph K_n $(n \leq 2)$, $\gamma_{sths}(K_n) = n$.

Proof:

Since every vertex of a complete graph K_n $(n \leq 2)$ is an extreme vertex, the vertex set of K_n is the unique strong hop steiner dominating set of K_n.

Thus the strong hop steiner domination number, $\gamma_{sths}(K_n) = n$.

Theorem 3.2: Let $n \geq 5$ & $k \geq 2$ be a positive integer, then for a path graph P_n,

$$\gamma_{sths}(P_n) = \begin{cases} k+1, & when\, n = 3k-1\, and\, 3k+1 \\ k+2, & when\, n = 3k \end{cases}$$

Proof:

Given $n \geq 5$ is a positive integer.

Let $V(P_n) = \{v_1, v_2, \dots, v_n\}$ so that it is of order n.

Here we consider two cases,

Case 1:

When $n = 3k-1\ and\ n = 3k+1$, where $k = 2,3, \dots$

then $V(P_{3k-1}) = \{v_1, v_2, \dots, v_{3k-1}\}$ and $V(P_{3k+1}) = \{v_1, v_2, \dots, v_{3k+1}\}$ and it is of order $3k-1\ and\ 3k+1$ respectively.

To get minimum strong hop steiner dominating set of P_{3k-1} and P_{3k+1}.

Consider the set $S_1 = \{v_1, v_4, v_7, v_{10}, v_{13}, v_{16}, v_{19}, \dots, v_{3k+1}, \dots, v_n\ / n = 3k-1\ and\ 3k+1\}$.

=> The set S_1 is a steiner dominating set and also for every vertex $v_i \in V - S_1$ there exists $v_j \in S_1$ such that $d(v_i, v_j) = 2$ and also that every vertex $v_i \in V - S_1$ is strongly dominated by some $v_j \in S_1$.

Thus S_1 is a minimum strong hop steiner dominating set.

$\therefore \gamma_{sths}(P_{3k-1}) = \gamma_{sths}(P_{3k+1}) = k+1$, where k=2, 3,…

Case 2:

When $n = 3k$, where $k = 2,3, \dots$

then $V(P_{3k}) = \{v_1, v_2, \dots, v_{3k}\}$ and it is of order $3k$.

To get minimum strong hop steiner dominating set of P_{3k}.

Here we consider two subcases,

Subcase 1:

When 3k is even,

Consider the set $S_2 = \left\{v_1, v_4, \dots, v_{\left\lceil\frac{3k}{2}\right\rceil}, v_{\left\lceil\frac{3k}{2}\right\rceil+1}, \dots, v_{3k}\right\}$.

=> The set S_2 is a steiner dominating set and also for every vertex $v_i \in V - S_2$ there exists $v_j \in S_2$ such that $d(v_i, v_j) = 2$ and also that every vertex $v_i \in V - S_2$ is strongly dominated by some $v_j \in S_2$.

Thus S_2 is a minimum strong hop steiner dominating set.

$\therefore \gamma_{sths}(P_{3k}) = k + 2,$ when 3k is even.

Subcase 2:

When 3k is odd,

Consider the set $S_3 = \left\{v_1, v_4, \dots, v_{\left\lceil\frac{3k}{2}\right\rceil - 1}, v_{\left\lceil\frac{3k}{2}\right\rceil}, v_{\left\lceil\frac{3k}{2}\right\rceil + 1}, \dots, v_{3k}\right\}$.

=> The set S_3 is a steiner dominating set and also for every vertex $v_i \in V - S_3$ there exists $v_j \in S_3$ such that $d(v_i, v_j) = 2$ and also that every vertex $v_i \in V - S_3$ is strongly dominated by some $v_j \in S_3$.

Thus S_3 is a minimum strong hop steiner dominating set.

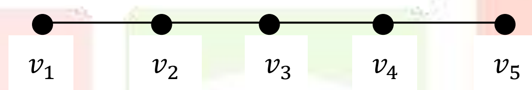
$\therefore \gamma_{sths}(P_{3k}) = k + 2,$ when 3k is odd.

Hence in both the subcase, $\gamma_{sths}(P_{3k}) = k + 2$ where k=2,3,…

$$\therefore \gamma_{sths}(P_n) = \begin{cases} k+1, & when\ n = 3k - 1\ and\ 3k + 1 \\ k+2, & when\ n = 3k \end{cases}$$

Illustration:

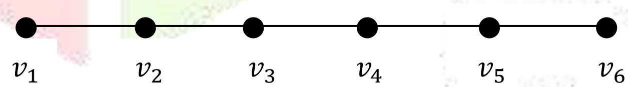
1) For the path graph P_5in the figure 3.1, $S_1 = \{v_1, v_4, v_5\}$ so that $\gamma_{sths}(P_5) = 3$.



$\boldsymbol{P_5}$

figure 3.1

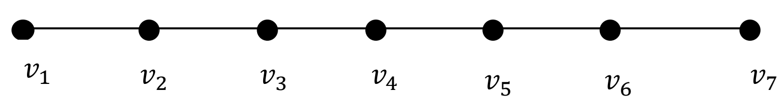
2) For the path graph P_6in the figure 3.2, $S_2 = \{v_1, v_3, v_4, v_6\}$ so that $\gamma_{sths}(P_6) = 4$.



$\boldsymbol{P_6}$

figure 3.2

3) For the path graph P_7 in the figure 3.3, $S_3 = \{v_1, v_4, v_7\}$ so that $\gamma_{sths}(P_7) = 3$.



$\boldsymbol{P_7}$

figure 3.3

Corollary 3.3: For the path graph P_n $(n \geq 5)$, the end vertices belongs to the strong hop steiner dominating set.

Observation 3.4: For any$n \geq 2$, $\gamma_{sths}(K_{1,n}) = n + 1$.

4. STRONG HOP STEINER DOMINATION NUMBER OF SOME SPECIAL GRAPHS.

Theorem 4.1: Let $m \geq 2, n \geq 4$ be a positive integer. For a fire cracker graph F(m,n),

$$\gamma_{sths}(F(m,n)) = m(n-1)$$

Proof:

Given: $m, n \geq 2$ be a positive integer

Let $V(F(m,n)) = \{v_1, v_2, \dots v_m\} \cup \{v_{ij}/i = 1,2, \dots m; j = 1,2, \dots n-1\}$ and it is of order mn.

Consider the set$S = \{v_1, v_2, \dots v_m\}$,

We observe that any vertex in $V - S$ is adjacent with atleast one vertex of S.

∴ S is the minimum dominating set.

But S is not a strong hop steiner dominating set.

Let $W = \{v_{ij}/i = 1,2, \dots m; j = 2, \dots n-1\}$ is the set of all pendant vertices and it is of order n-1.

Since every end vertex is an extreme vertex, W is the set of all extreme vertices.

To get a minimum strong hop steiner dominating set of $F(m,n)$,

Consider the set $S_1 = \{v_1, v_2, \dots v_m\} \cup W$

=>S_1 consists of all extreme verteices.

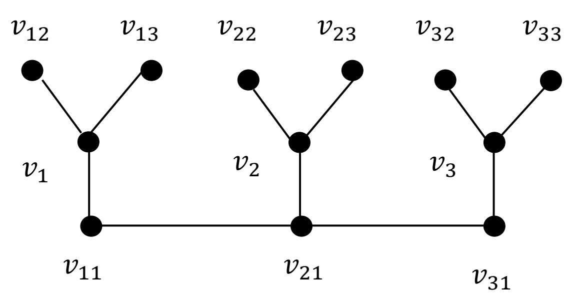
By observation 2.3, S_1 is the minimum hop steiner dominating set, and also that every vertex belongs to $V - S_1$ is strongly dominated by some vertex belongs to S_1.

∴ $\gamma_{sths}(F(m,n)) = |S_1| = m(n-1)$.

∴ $\gamma_{sths}(F(m,n)) = m(n-1)$.

Illustration:

For the fire cracker graph F(3,4), in the figure 4.1, $S_1 = \{v_1, v_2, v_3, v_{12}, v_{13}, v_{22}, v_{23}, v_{32}, v_{33}\}$ so that$\gamma_{sths}(F(3,4)) = 9$.



F(3,4)

Figure 4.1

Theorem 4.2: Let $n \geq 7$ be a positive integer. The Ladder graph L_n has same hop steiner domination number for the set of five consecutive ladder graph.

$$\gamma_{sths}(L_{5k-3}) = \gamma_{sths}(L_{5k-2}) = \gamma_{sths}(L_{5k-1}) = \gamma_{sths}(L_{5k}) = \gamma_{sths}(L_{5k+1}) = m + 5$$

where m is a (k-1)thodd numbers.

Proof:

Given $n \geq 7$ is a positive integer.

Let m be the (k-1)th odd number

Let $V(L_n) = \{v_{i,1}, v_{i,2}, \dots, v_{i,n} / \ i = 1,2; n = 1,2,3, \dots\}$ so that it is of order $2n$.

To get a minimum strong hop steiner dominating set of L_n,

Consider the set $S_1 = \{v_{i,1}, v_{i,6}, v_{i,11}, v_{i,16}, v_{i,21}, v_{i,26}, \dots, v_{i,n}/i = 1,2; k = 2,3, \dots\}$

=> The set S_1 is a steiner dominating set and also for every vertex $v_i \in V - S_1$ there exists $v_j \in S_1$ such that $d(v_i, v_j) = 2$ and every vertex $v_i \in V - S_1$ is strongly dominated by some $v_j \in S_1$

Thus S_1 is a minimum strong hop steiner dominating set.

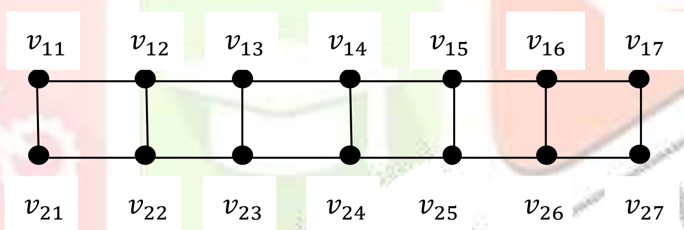
$$\therefore \gamma_{sths}(L_n) = |S_1| = m + 5.$$

$\therefore \gamma_{sths}(L_{5k-3}) = \gamma_{sths}(L_{5k-2}) = \gamma_{sths}(L_{5k-1}) = \gamma_{sths}(L_{5k}) = \gamma_{sths}(L_{5k+1}) = m + 5,$

where m is a (k-1)th odd numbers.

Illustration:

For the Ladder graph L_7, in the figure 4.2, $W_3 = \{v_{11}, v_{21}, v_{16}, v_{26}, v_{17}, v_{27}\}$ so that $\gamma_{trs}(L_7) = 6$.



L_7 ıre 4.2

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