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EQUITABLE POWER EDGE DOMINATION OF CERTAIN GRAPHS

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Abstract: We define G as a graph, where V denotes the collection of vertices and E represents the collection of edges. [7] If b is the adjacent observed vertex for every vertex a such that the difference between degrees a and b is less than or equal to 1 that is $|d(a) - d(b)| \leq 1$, then the equitable power dominating set S is a subset of V in G is power dominating set. The new concept of "Equitable Power Edge Domination" is defined as follows: If an edge f_1 that is not in S' is observed to be adjacent to an edge f_2 such that $|d(f_1) - d(f_2)| \leq 1$, the edge equitable dominating set $S' \subseteq E$ in G is said to be a "Equitable Power Edge Dominating set". Equitable power edge domination number of graph represented by $\gamma'_{eped}(G)$, represents the fewest elements of an equitable power edge domination set of graph.

Index Terms: Equitable Power Domination, Power Edge Domination, Equitable Power Edge Domination Number.

I. INTRODUCTION

The discussion revolves around graphs that are simple, finite, undirected, and connected. The degree of each edge in the graph is determined by the sum of the degrees of its endpoints minus two. [6]An Edge Dominating Set in the graph ensures that there is at least one edge from the set adjacent to every non-included edge. The minimum number of such sets is referred to as the Edge Dominating Number. In the context of power dominating sets [7], each vertex in the graph dominates itself as well as its neighboring vertices. If a vertex with multiple neighbors observes all but one, it will also observe the remaining unobserved neighbor. $\gamma_{pd}(G)$ represents the minimal number of a power-dominating set.

Equitable Dominating Sets [7] ensure that for each vertex not in the set, there exists a neighboring vertex with a degree difference of at most one. The Equitable Domination Number is defined as the smallest possible number within all equitable dominating sets.

In this paper, the concept of edge dominance [6] was merged with Equitable Power dominance [1], giving rise to Equitable Power Edge Domination (EPED) in graphs. In EPED, an equitable power edge dominating set, denoted as S' is a subset of the edges in a graph, is defined in such manner that for every edge f_1 not in S', there is an observed neighbouring edge f_2 with a degree difference of at most one such that, $|d(f_1) - d(f_2)| \leq 1$. The Equitable Power Edge Domination Number of graph, represented by $\gamma'_{eped}(G)$, is equivalent to the fewest elements among all equitable power edge dominating sets .

This cognition finds its inspiration in the electric power system sector, specifically in the optimal placement of phase measuring units (PMUs). Power dominance reduces the number of PMUs while determining their locations. The Equitable Power Edge Domination Number (EPEDN) explores relationships and analyzes various graph types.

II. EQUITABLE POWER EDGE DOMINANCE OF CERTAIN GRAPH SPECIAL CLASSES

RESULTS

1. Let P_n be a path, then $\gamma'_{eped}(P_n) = 1$ for $n \geq 2$.
2. Let C_n be a cycle, then $\gamma'_{eped}(C_n) = 1$ for $n \geq 3$.
3. Let $W_{1,n}$ be a wheel graph, then $\gamma'_{eped}(W_{1,n}) = 1$ for $n \geq 4$.
4. Let S_n be a star graph, then $\gamma'_{eped}(S_n) = 1$ for $n \geq 3$.

Theorem 2.1

Let L_n be the ladder graph, then $\gamma'_{eped}(L_n) = \left\lceil \frac{n-1}{3} \right\rceil$ for $n \geq 2$

Proof:

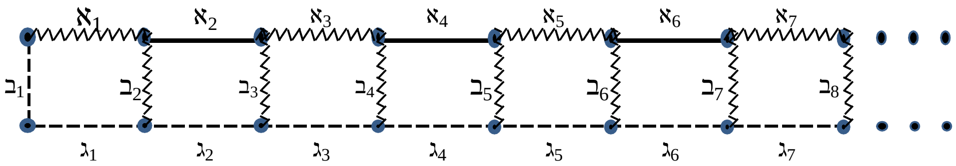

Fig.1.Equitable Power Edge Domination of Ladder graph L_n

Let, $G = L_n$, be ladder graph of $2n$ vertices and $3n-2$ edges. The theorem can be explained with an illustration. Let, $\aleph_2$ to be in the set S' for minimality. $\aleph_2$ dominates $\aleph_1, \aleph_3, \beth_3$ and $\beth_2$. Now, $\aleph_1$ observe $\beth_1$ and $\beth_1$ will inturn observes $\gimel_1$. It is simple to observe that $\beth_2$ and $\beth_3$ observe the only non-observed edges $\gimel_2$ and $\gimel_3$ respectively. There are more than one non observed edges for $\aleph_3$ and $\beth_4$. So, for the sake of minimality, $\aleph_4$ must be chosen to be in EPED set S'. $\beth_4, \beth_5, \aleph_5$ will be dominated by $\aleph_4$. Now for $\beth_4$ *and* $\beth_5$ the only non-observed edges are $\gimel_4$ and $\gimel_5$. Proceeding in the way one can obtain EPED set S'.

Hence, $\gamma'_{eped}(L_n) = \left\lceil \frac{n-1}{3} \right\rceil$ for $n \geq 2$.

Theorem 2.2

Consider K_{m_1,m_2} be complete bipartite graph, then $\gamma'_{eped}(K_{m_1,m_2}) = m_1 - 1, m_1 < m_2$

Proof:

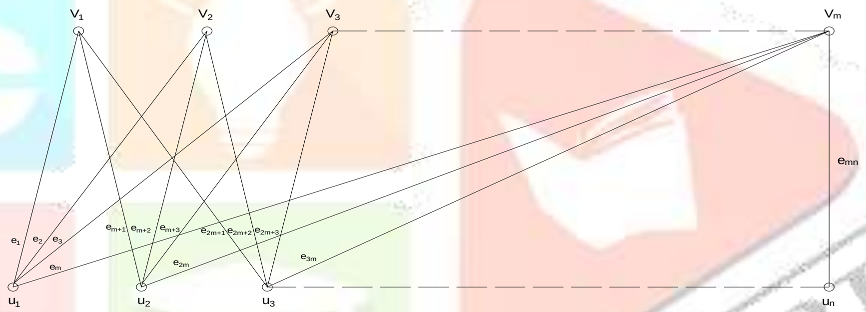

Fig.2 Equitable Power Edge Domination of Complete Bipartite graph $k_{m,n}$

Without loss of generality let e_{m+1} $(v_1u_2)$to be in the set EPED S'. e_{m+1} dominates the edges joining the vertices $v_1u_1, v_1u_3, v_1u_4, v_1u_5, v_1u_6 \ldots\ldots v_1u_n$ *and* $u_2v_2, u_2v_3, u_2v_4 \ldots\ldots u_2v_m$. It dominates $m_1 + m_2 - 2$ edges. It is observed that every dominated edge there are more non observed edges. So, one has to choose the edge joining the vertices v_2u_3 say $e_i, i > m_1$. It dominates $m_1 + m_2 - 2$ edges. Similarly, every dominated edge there are more than one non observed edges. Proceeding in this way, one can obtain the EPED set

$S' = \{v_1u_2, v_2u_3, v_3u_4 \ldots\ldots v_{m-1}u_{n-1}\}$. Therefore, $\gamma'_{eped}(K_{m_1,m_2}) = m_1 - 1, m_1 < m_2$

Theorem 2.3

Let C_n^+be a crown graph. Then $\gamma'_{eped}(C_n^+) = n + 1$.

Proof:

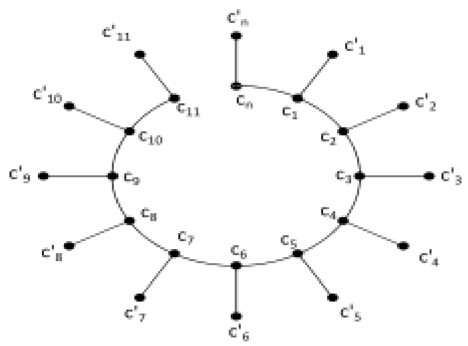

Fig.3 Equitable Power Edge Domination of Crown graph C_n^+

Consider C_n^+ be a crown graph with $\{c_1, c_2, c_3 \ldots\ldots c_n, c'_1, c'_2 \ldots\ldots c'_n\}$ edge set. From the Fig.3 it can be seen that $d(c'_i) = 2$ for $1 \leq i \leq n$ and $d(c_i) = 4$ for $1 \leq i \leq n$. None of the c'_i equitably power dominates any of c_i as the degree difference between c'_i to c_i be 2. To acquire the equitable power edge dominating set S', all c'_i for $1 \leq i \leq n$ must be chosen, as the pendant edges will not dominate any of their adjacent edges. It is sufficient to select at least one edge among the remaining edges in the rim that equitably powers all of the other non-observed edges, like in the case of the wheel graph. So from the edges c'_i, $1 \leq i \leq n$ in the rim, choose one edge say c_1 to be in S'. So $S' = \{c_1, c'_1, c'_2 \ldots\ldots c'_n\}$. Thus $\gamma'_{eped}(C_n^+) = n + 1$.

III. COMPARISON OF EQUITABLE EDGE POWER DOMINATION NUMBER WITH EQUITABLE POWER DOMINATION NUMBER.

The following observations are obtained using the above theorem

1. $\gamma'_{eped}(C_n^+) = \gamma_{epd}(C_n^+)$
2. $\gamma'_{eped}(P_n) = \gamma_{epd}(P_n), n \geq 2$
3. $\gamma'_{eped}(C_n) = \gamma_{epd}(C_n)$ for $n \geq 3$
4. $\gamma'_{eped}(S_n) = \gamma_{epd}(S_n)$ for $n \geq 3$
5. $\gamma'_{eped}(P_n \times P_2) \leq \gamma_{epd}(P_n \times P_2)$
6. $\gamma'_{eped}(K_{m_1,m_2}) = m_1 - 1, if\ m_1 < m_2 \leq \gamma_{epd}(K_{m_1,m_2}) = m_1 + m_2, if |m_1 - m_2| \geq 2$
7. $\gamma'_{eped}(W_{1,n}) \leq \gamma_{epd}(W_{1,n})$ for $(n \geq 5)$

IV. CONCLUSION

In this study, the Equitable Power Edge Domination Number (EPEDN) has been computed for various graphs, path, cycle, ladder, crown, wheel, bipartite graph, and star. Additionally, a comprehensive examination of special graphs and a comparative analysis between EPD and EPED have been undertaken. The findings reveal that EPED exhibits a lower minimal cardinality compared to EPD. Reducing the quantity of phase measurement units (PMUs) is of utmost importance in cost reduction for electric companies. With further reductions in bounds, it is anticipated that EPED will prove increasingly advantageous in the context of PMUs.

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