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# PAIRS (54963, 52974), (6543, 2187) & (3285, 5274) WITH ALS' ROUTINE

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***Abstract*:** This paper introduces ALS Routine, a mathematical procedure inspired by Kaprekar's constant 6174. Kaprekar's routine, named after Indian Mathematician D. R. Kaprekar, involves iteratively applying a set of rules to any four-digit number, ultimately converging to 6174. ALS' Routine extends this concept to five-digit numbers, employing a unique approach of rearranging digits in descending and ascending order and swapping specific positions. Through this process, distinct pairs of numbers are identified, including the A-pair (54963, 52974) for five-digit numbers and the L-pair (6543, 2187) and S-pair (3285, 5274) for four-digit numbers, which repeat consistently in subsequent iterations.

**Keywords**: Kaprekar's constant, Kaprekar's Routine, Madhan's Routine, ALS' Routine, Happy number, digit sum.

## I. INTRODUCTION:

The quest for repetitive patterns within seemingly random sequences has long captivated mathematicians. One such example is Kaprekar's constant (6174). This number holds a remarkable property: when a specific procedure is applied to any four-digit number (excluding all digits being the same), the process inevitably converges to 6174 within a limited number of steps. This procedure, known as Kaprekar's routine, involves rearranging digits and performing a specific subtraction.Another example is Madhan's Routine, introduced by A. Sudhakaraiah and A. Madhankumar in 2022 [3]. When a specific procedure is applied to any positive multi-digit number, the process inevitably converges to 2 within a limited number of steps. This simple yet captivating procedure involves any multi-digit number. Repeatedly, you subtract the sum of its digits, reverse the result, add it back, and then divide by 9.

This paper builds upon this intriguing concept by introducing ALS' Routine. Taking inspiration from Kaprekar's constant, ALS' Routine ventures beyond four digits, delving into the behavior of five-digit numbers. It employs a novel approach that involves manipulating digits through a unique combination of rearrangement and strategic swapping. Unlike Kaprekar's singular fixed point, ALS' Routine identifies distinct pairs of numbers for both five-digit (A-pair) and specific four-digit scenarios (L-pair and S-pair). These pairs exhibit a fascinating characteristic: they consistently reappear within subsequent iterations of the ALS Routine.

This exploration aims to unlock the secrets behind these newly identified number pairs. By analyzing the behavior of these pairs and the underlying mechanisms of ALS' Routine, the paper seeks to contribute to the understanding of iterative processes and potentially uncover hidden mathematical relationships that govern these fascinating number sequences.

## II. PRELIMINARIES:

Iterative processes reveal surprising regularities, like Kaprekar's constant (6174). This four-digit number, when manipulated by rearranging digits and subtracting, converges to 6174.Another captivating iterative process is Madhan's Routine, introduced in 2022. This routine delves into the world of multi-digit numbers (excluding single digits). Madhan's Routine exhibits a remarkable property as well: when a specific procedure is applied to any positive multi-digit number, the process inevitably converges to 2 within a limited number of steps. This simple yet captivating procedure involves repeatedly subtracting the sum of its digits, reversing the result, adding it back, and then dividing by 9. Inspired by this, ALS' Routine explores five-digit numbers. It reveals distinct number pairs (A-pairs, L & S-pairs) that resurface repeatedly within the routine. Our goal is to understand these pairs and the underlying mechanisms of the ALS Routine, seeking hidden mathematical relationships in these fascinating sequences.

**KAPREKAR'S CONSTANT:**

Kaprekar's routine is a fascinating iterative algorithm attributed to the Indian Mathematician D. R. Kaprekar. It involves taking a number, rearranging its digits to form the largest and smallest possible numbers, and then subtracting the smaller number from the larger one. This process is repeated until a fixed point is reached.

Example:6543 =>6174

6543 - 3456 = 3087

8730 - 0378 = 8352

8532 - 2358 = 6174.

Any four-digit number with at least two distinct digits will eventually reach Kaprekar's constant, 6174, within a maximum of seven iterations

**Happy Number:**

In number theory, a happy number is a positive integer that, when repeatedly replaced by the sum of the squares of its digits, eventually results in 1. If this process continues indefinitely without reaching 1, the number is termed as an unhappy number. Happy numbers exhibit a fascinating property where they form a cycle that ultimately converges to 1.

Example: 130 => 1

130 => $1^2 + 3^2 + 0^2 = 10$ => $1^2 + 0^2 = 1$

**ALSRoutine: A New Chapter**

This paper introduces ALS' Routine, inspired by both Kaprekar's constant and Madhan's Routine.

Step 1: Choose a five-digit or four-digit number with at least two different digits (leading zeros permitted).

Step 2: Arrange the digits of the chosen number in descending order.

Step 3: Arrange the digits in ascending order, then swap the two highest digits.

Step 4: Subtract the smaller number from the larger number obtained in the previous step.

Step 5: Return to Step 2 and repeat the process for the new resultant number.

Continue this iterative process until a fixed point is reached or a repeating pattern emerges.

For five digits, A-pair emerges within a maximum of 10 iterations.

For four digits, L-pair or S-pair emerges within a maximum of 12 iterations.

**III. MAIN SECTION:**

For any five-digit number (except for the numbers that are formed by single digits like 11111,22222, etc),the difference between the descending order of the digits and the number formed by swapping the highest positions of the ascending order of the digits gives a new number. Now, for the new number also find the difference between its descending order of the digits

and the number formed by swapping the highest positions of the ascending order of the digits. continue the above process of finding the difference, we end up with a pair of numbers A-pair: 54963, 52974 which keeps repeating.

**Theorem 3.1:** For any five-digit number, denoted as X $X=x_1x_2x_3x_4x_5$, containing at least two distinct digits, the ALS Routine consistently results in the repetition of the number pair 54963 and 52974.

**Proof:**Let us assume a five-digit number, denoted as $X=x_1x_2x_3x_4x_5$ containing at least two distinct digits.

Now we will Followthe steps of ALS' Routine.

Step 1: We have $X=x_1x_2x_3x_4x_5$ as a five-digit number containing at least two distinct digits.

Step 2: Let the descending order of the number $X=x_1x_2x_3x_4x_5$ be $Y = y_5y_4y_3y_2y_1$.

Step 3: Now the ascending order of the number $X=x_1x_2x_3x_4x_5$ will be $y_1y_2y_3y_4y_5$.

And then swapping the two highest places in the digits of ascending order of X,

we get $Y'=y_2y_1y_3y_4y_5$.

Step 4: Now let the difference between $Y = y_5y_4y_3y_2y_1$ and $Y'=y_2y_1y_3y_4y_5$

be represented by $R_1$.

i.e., $Y-Y'= y_5y_4y_3y_2y_1 - y_2y_1y_3y_4y_5 = R_1$, where $R_1= z_1z_2z_3z_4z_5$.

If the number of digits of the resultant (here $R_1$) is less than 5, We can have leading zeros, so as to make it a five-digit number.

Step 5: Now we take $R_1= z_1z_2z_3z_4z_5$ and go back to the step 2,

i.e, repeating the ALS Routine, we have,
The descending order of $R_1= z_1z_2z_3z_4z_5$ be $y_{10}y_9y_8y_7y_6$ and the number after swapping the two highest places in the digits of ascending order of $R_1$ will be $y_7y_6y_8y_9y_{10}$.
Now the Y and Y' will be $y_{10}y_9y_8y_7y_6$, $y_7y_6y_8y_9y_{10}$ respectively.
And the resultant of Y-Y' is represented by $R_2= z_6z_7z_8z_9z_{10}$.
$Y-Y' = y_{10}y_9y_8y_7y_6 - y_7y_6y_8y_9y_{10} = R_2 = z_6z_7z_8z_9z_{10}$.
When we continue the ALS Routine (say for example n times),
Which leads to either 54963 or 52974, which means $R_n$ is either 54963 or 52974.
$Y-Y' = y_{n5}y_{n4}y_{n3}y_{n2}y_{n1} - y_{n2}y_{n1}y_{n3}y_{n4}y_{n5} = R_n = z_{n1}z_{n2}z_{n3}z_{n4}z_{n5}$.
Here, $R_n$ is either 54963 or 52974. Let $R_n$ be 54963 then by following ALS' Routine, we have

$R_n = 54963$.
We have Y=96543, and Y'=43569.
$R_{n+1}$ = Y-Y'=96543-43569=52974.
$R_{n+1}$ =52974.
We have Y=97542, and Y'=42579.
$R_{n+2}$ = Y-Y'=97542-42579=54963.
By observing it is clear that $R_n= R_{n+2}$ and now if we repeat ALS' Routine for $R_{n+2}$ we get 52974 as $R_{n+3}$. Which means $R_{n+1} = R_{n+3}$.
Here, we have $R_n = R_{n+2} = R_{n+4} = R_{n+6}$ ...... and $R_{n+1}= R_{n+3} = R_{n+5} = R_{n+7}$ ......
Therefore, it can be concluded that the numbers 54963 and 52974 repetitively recur within the ALS Routine.
If we assume that $R_n$ is 52974, we have
$R_m = 52974$.
We have Y=97542, and Y'=42579.
$R_{m+1}$ = Y-Y'=97542-42579=54963.
$R_{m+1}$ =54963.
We have Y=96543, and Y'=43569.
$R_{m+2}$ = Y-Y'=96543-43569=52974.
By observing it is clear that $R_m= R_{m+2}$ and now if we repeat ALS' Routine for $R_{m+2}$ we get 52974 as $R_{m+3}$. Which means $R_{m+1} = R_{m+3}$.

Here, we have $R_m = R_{m+2} = R_{m+4} = R_{m+6}$ ...... and $R_{m+1}= R_{m+3} = R_{m+5} = R_{m+7}$ ......
We can see that even here we have a repetition of the numbers 54963 and 52974.
Therefore, we can say that for any five-digit number say $X=x_1x_2x_3x_4x_5$ with at least two different digits, the ALS Routine always leads to the repetition of a pair of numbers 54963 and 52974.

**Example 3.1:**

**Step 1**: Take a five-digit number, say 73325.

**Step 2**: The descending order of the number 73325 is "75332".

**Step 3**: The ascending order of the number 73325 is "23357" and then swapping it's the first two highest places we get "323357".

**Step 4**: Subtracting the smallest number from the bigger number i.e. 75332- 32357, we get a new number 42975, take this as result1,

result1: 42975.

**Step 5:**

➔go back to step 2 and repeat the process for 42975,

i.e., 97542-42579 we get 54963. so, result2: 54963

**Step 6**:

➔go back to step 2 and repeat the process for 54963, we get

96543 - 43569 = 52974. so, result3: 52974

**Step 7**:

➔go back to step 2 and repeat the process for 52974, we get

97542 - 42579 = 54963. so, result4: 54963.

Here, we can see that result2 and result4 are the same which means by repeating the same process for result4 we get the result5. Hence, we end up with a pair that keeps repeating the above process.

75332 - 32357 = 42975

97542 - 42579 = 54963

96543 - 43569 = 52974

97542 - 42579 = 54963.

**Example 3.2:**

Let us take the number 24410

44210 - 10244 = 33966

96633 - 33669 = 62964

96642 - 42669 = 53973

97533 - 33579 = 63954

96543 - 43569 = 52974

97542 - 42579 = 54963

96543 - 43569 = 52974

===> 24410 ends with 52974

**Algorithm 3.1:**

Here's a step-by-step breakdown of the algorithm:

1. Loop from `i = 10000` to `i = 99998`:

   - Within this loop, numbers are considered in the range 10000 to 99998.

2. Check for specific numbers:

   - If the number is 11111, 22222, 33333, 44444, 55555, 66666, 77777, or 88888, skip to the next iteration.

3. Call the ` ALS routine` method:

   - For each number within the loop, the `meth` method is called with that number as an argument.

   - The method ` ALSRoutine` is responsible for processing the number and returning a modified number.

4. The `ALSRoutine` method:

   - It takes an input number `in` and processes it based on certain rules.

   - If `in` is less than 10000, apply specific transformations to it.

   - Apply certain operations to `in` to create two sorted and modified numbers `a` and `b`.

   - Calculate the difference `c = b - a`.

   - Check if `c` is equal to 54963 or 52974.

     - If true, increment `p`.

     - If `p` is less than 3, call the ` ALSRoutine` method recursively with `c` and increment `p`.

   - Reset `p` to 0.

   - Return the result of the recursive call to ` ALSRoutine(c)`.

5. Repeat the process for subsequent recursive calls:

   - The recursive calls to ` ALSRoutine` continue the processing with new values of `c`.

**Program 3.1:**

```
import java.util.Collections;

import java.util.LinkedList;

import java.util.List;


public class LovelyPairs {

        static int p=0;

       public static void main(String[] args) {


              for (Integer i = 10000; i <= 99998; i++) {

                     if (i == 11111 || i == 22222 || i == 33333 || i == 44444 || i == 55555

                                          || i == 66666 || i == 77777 || i == 88888) {

                            continue;

                     }

                     System.out.println ("\n ===> " + i + " ends with " + aLSRoutine (i));

                     System.out.println ();

              }

       }


       Public static Integer aLSRoutine(Integer in) {

              if(in<10000) {

                     in= (in/ 10 == 0)? in * 10000

                            :(in / 100 == 0)?in * 1000

                                   :(in / 1000 == 0)?in * 100

                                          :(in/10000==0)?in*10:in;

              }

              Integer a = 0, b = 0;

              List<Integer> arr1 = new LinkedList<>();

              Integer num1 = in;

              while (num1 > 0) {

                     arr1.add((num1) % 10);

                     num1 = num1 / 10;

              }

              Collections.sort(arr1);

              Collections.swap(arr1, 0, 1);

              for (Integer digits: arr1) {

                     a = (a * 10) + digits;

              }

              Collections.sort(arr1);

              Collections.reverse(arr1);

              for (Integer digits: arr1) {
```

```
                    b = (b * 10) + digits;
            }
            Integer c = b - a;
            System.out.println(b + " - " + a + " = " + c);
             if ((c == 54963) || (c == 52974) ) {
                    p++;
                    if(p<3) {
                            aLSRoutine (c);
                            p++;
                            }
                    p=0;
                    return c; }
            return aLSRoutine (c);
        }
}
```

* For any four-digit number (except for the numbers that are formed by single digits like 1111,2222, etc), the difference between the descending order of the digits and the number formed by swapping the highest positions of the ascending order of the digits gives a new number. Now, for the new number, find the difference between its descending order of the digits

and the number formed by swapping the highest positions of the ascending order of the digits. By Continuing the above process of finding the difference, either of the following two pairs keeps repeating.

L-pair: 6543,2187

S-pair: 3285,5274

**Theorem 3.2:** For any four-digit number say $X=x_1x_2x_3x_4$ with at least two different digits, the ALS Routine always leads to repetition of either of the pairs L-pair: 6543,2187 or S-pair: 3285,5274.

Proof: Let us assume a four-digit number say $X=x_1x_2x_3x_4$ with at least two different digits.

Now we will Follow the steps of ALS' Routine.

Step 1: We have $X=x_1x_2x_3x_4$ as a four-digit number with at least two different digits.

Step 2: Let the descending order of the number $X=x_1x_2x_3x_4$ be $Y = y_4y_3y_2y_1$.

Step 3: Now the ascending order of the number $X=x_1x_2x_3x_4$ will be $y_1y_2y_3y_4$.

And then swapping the two highest places in the digits of

ascending order of X, we get $Y'=y_2y_1y_3y_4$.

Step 4: Now let the difference between $Y = y_4y_3y_2y_1$ and $Y'=y_2y_1y_3y_4$

be represented by $R_1$.

i.e, $Y-Y'= y_4y_3y_2y_1 - y_2y_1y_3y_4 = R_1$, where $R_1= z_1z_2z_3z_4$.

If the number of digits of the resultant (here $R_1$) is less than 4,

We can have leading zeros, to make it a four-digit number.

Step 5: Now we take $R_1= z_1z_2z_3z_4$ and go back to the step 2,
i.e, repeating the ALS Routine, we have,
The descending order of $R_1= z_1z_2z_3z_4$ be $y_8y_7y_6y_5$ and the number
after swapping the two highest places in the digits of
ascending order of $R_1$ will be $y_5y_6y_7y_8$.
Now the Y and Y' will be $y_8y_7y_6y_5$, $y_6y_7y_8y_9$ respectively.
And the resultant of Y-Y' is represented by $R_2 = z_6z_7z_8z_9z_{10}$.
$Y-Y' = y_8y_7y_6y_5 - y_6y_7y_8y_9 = R_2 = z_6z_7z_8z_9$.
When we continue the ALS Routine (say for example n times),

this leads to either of the pairs L-pair: 6543,2187 or S-pair: 3285,5287. which means $R_n$ has a repetition of either the pairs L-pair or S-pair.

Let us assume that $R_n$ has a repetition of L-pair: 6543,2187.

Which means $R_n$ is either 6543 or 2187.

$Y\text{-}Y' = y_{n4}y_{n3}y_{n2}y_{n1} - y_{n2}y_{n1}y_{n3}y_{n4} = R_n = z_{n1}z_{n2}z_{n3}z_{n4}$.

Here, $R_n$ is either 6543 or 2187. Let $R_n$ be 6543 then by following ALS' Routine, we have

$R_n = 6543$.

We have Y=6543, and Y'=4356.

$R_{n+1}$ = Y-Y'=6543-4356=2187.

$R_{n+1}$ =2187.

We have Y=8721, and Y'=2178.

$R_{n+2}$ = Y-Y'=8721-2178=6543.

By observing it is clear that $R_n = R_{n+2}$ and now if we repeat ALS' Routine for $R_{n+2}$ we get 2178 as $R_{n+3}$. Which means $R_{n+1} = R_{n+3}$.

Here, we have $R_n = R_{n+2} = R_{n+4} = R_{n+6}$ ...... and $R_{n+1} = R_{n+3} = R_{n+5} = R_{n+7}$ ......

So, we can clearly say that there is a repetition of the numbers 6543 and 2187.

If we assume that $R_n$ is 2187, we have

$R_n$= 2187.

We have Y=8721, and Y'=2187.

$R_{m+1}$ = Y-Y'=8721-2187=6543.

$R_{m+1}$ =6543.

We have Y=6543, and Y'=4356.

$R_{m+2}$ = Y-Y'=6543-4356=2187.

By observing it is clear that $R_m = R_{m+2}$ and now if we repeat ALS' Routine for $R_{m+2}$ we get 6543 as $R_{m+3}$. Which means $R_{m+1} = R_{m+3}$.

Here, we have $R_n = R_{m+2} = R_{m+4} = R_{m+6}$ ...... and $R_{m+1} = R_{m+3} = R_{m+5} = R_{m+7}$ ......

We can see that even here we have a repetition of the numbers 6543 and 2187 which is L-pair.

Now, let us assume that $R_n$ has a repetition of S-pair: 3285,5274.

Which means $R_n$ is either 3285 or 5274.

$Y\text{-}Y' = y_{n8}y_{n7}y_{n6}y_{n5} - y_{n6}y_{n5}y_{n7}y_{n8} = R_n = z_{n5}z_{n6}z_{n7}z_{n8}$.Here, $R_n$ is either 3285 or 5274. Let $R_n$ be 3285 then by following ALS' Routine, we have

$R_n = 3285$.

We have Y=8532, and Y'=3258.

$R_{n+1}$ = Y-Y'=8532-3258=5274.

$R_{n+1}$ =5274.

We have Y=7542, Y'=4257.

$R_{n+2}$ = Y-Y'=7542-4257=3285.

By observing it is clear that $R_n = R_{n+2}$ and now if we repeat ALS' Routine for $R_{n+2}$ we get 3258 as $R_{n+3}$. Which means $R_{n+1} = R_{n+3}$.

Here, we have $R_n = R_{n+2} = R_{n+4} = R_{n+6}$ ...... and $R_{n+1} = R_{n+3} = R_{n+5} = R_{n+7}$ ......

So, we can clearly say that there is a repetition of the numbers 3258 and 5274.

If we assume that $R_n$ is 5274, we have

$R_n = 5274$.

We have Y=7542, Y'=4257.

$R_{m+1}$ = Y-Y'=7542-4257=3285.

$R_{m+1}$ =3285.

We have Y=8532, and Y'=3258.

$R_{m+2}$ = Y-Y'=8532-3258=5274.

By observing it is clear that $R_m = R_{m+2}$ and now if we repeat ALS' Routine for $R_{m+2}$ we get 3258 as $R_{m+3}$. Which means $R_{m+1} = R_{m+3}$.

Here, we have $R_n = R_{m+2} = R_{m+4} = R_{m+6}$ ...... and $R_{m+1} = R_{m+3} = R_{m+5} = R_{m+7}$ ......

We can see that even here we have a repetition of the numbers 3285 and 5274 which is S-pair.

Therefore, we can say that for any four-digit number say $X=x_1x_2x_3x_4$ with at least two different digits, the ALS Routine always leads to repetition of either of the pairs L-pair: 6543,2187 or S-pair: 3285,5274.

Example 3.3: Let us take the number 2704.

7420 - 2047 = 5373
7533 - 3357 = 4176
7641 - 4167 = 3474
7443 - 4347 = 3096
9630 - 3069 = 6561
6651 - 5166 = 1485
8541 - 4158 = 4383
8433 - 3348 = 5085
8550 - 5058 = 3492
9432 - 3249 = 6183
8631 - 3168 = 5463
6543 - 4356 = 2187
8721 - 2178 = 6543
6543 - 4356 = 2187
8721 - 2178 = 6543

===> 2704 ends with 6543

Example 3.4: Let us the number 417.
7410 - 1047 = 6363
6633 - 3366 = 3267
7632 - 3267 = 4365
6543 - 4356 = 2187
8721 - 2178 = 6543
6543 - 4356 = 2187

===> 417 ends with 2187

**Algorithm 3.2:**

1) Begin the main method:
   a) Iterate through four-digit numbers from 1000 to 9998.
   b) Check if the number has repeating digits (1111, 2222, ..., 8888), and if so, continue to the next iteration.
   c) Print the current number and the result of the ALS Routine.

2) Implement the ALS' Routine method:
   a) If the input number is less than 1000, pad it with zeros to ensure it is a four-digit number.
   b) Initialize variables a and b to 0.
   c) Convert the input number into a list of digits.
   d) Sort the list in ascending order and swap the first two digits.
   e) Construct the ascending number by iterating through the sorted list and the descending number by reversing the list.
   f) Calculate the difference between the descending and ascending numbers.
   g) Check if the difference matches any of the predetermined values (6543, 2187, 3285, 5274).
   h) If it matches, increment the counter p, and if p is less than 3, recursively call the ALS' Routine method with the new difference.
   i) Reset p to 0.
   j) Return the difference.

3) End of the ALS Routine method.

**Program 3.2:**

```
import java.util.Collections;

import java.util.LinkedList;

import java.util.List;
```

```
public class LovelyPairs {
     static int p=0;
    public static void main(String[] args) {
    for (Integer i = 1000; i <= 9998; i++) {
        if (i == 1111 || i == 2222 || i == 3333 || i == 4444 || i == 5555
            || i == 6666 || i == 7777 || i == 8888) {
            continue;
            }
    System.out.println("\n ===> " + i + "  ends with " + aLSRoutine(i));
    System.out.println();
        }
    }
    public static Integer aLSRoutine(Integer in) {
        if(in<1000) {
            in= (in/ 10 == 0)? in * 1000
                    :(in / 100 == 0)?in * 100
                            :(in / 1000 == 0)?in * 10:in;
        }
        Integer a = 0, b = 0;
        List<Integer> arr1 = new LinkedList<>();
        Integer num1 = in;
        while (num1 > 0) {
            arr1.add((num1) % 10);
            num1 = num1 / 10;

        }
        Collections.sort(arr1);
        Collections.swap(arr1, 0, 1);
        for (Integer digits: arr1) {
            a = (a * 10) + digits;
        }
        Collections.sort(arr1);
        Collections.reverse(arr1);
        for (Integer digits: arr1) {
            b = (b * 10) + digits;
        }
        Integer c = b - a;
        System.out.println(b + " - " + a + " = " + c);
        if ((c == 6543) || (c == 2187) || (c == 3285) || (c == 5274)) {
             p++;
             if(p<3) {
```

```
                        aLSRoutine(c);
                        p++;
                        }
                p=0;
                return c; }
        return aLSRoutine(c);
    }
}
```

**KEY Points:**

The prime factorisation of the Lovely Pairs is as follows,

A-pair: 54963,52974

$54963= 3^2 * 31^1 * 197$,

$52974= 2^1 * 3^5 * 109$.

L-pair: 6543,2187

$6543= 3^2 * 727$,

$2187= 3^7$.

S-pair: 3285,5274

$3285= 3^2 * 5* 73$,

$5274= 2^1 * 3^2 * 293$.

1) Every number in all the three pairs has a common divisor 3.
2) In every pair of Lovelypairs, one number is happy.
   A-pair(54963, 52974) =>54963=> $5^2 +4^2+9^2+6^2+3^2 = 167 =>1^2+6^2+7^2=86=>8^2+6^2=100=>1^2+0^2+0^2=1$.
   L-pair (6543, 2187) =>6543=>$6^2+ 5^2+4^2+3^2=86=>8^2+6^2=100=>1^2+0^2+0^2=1$.
   S-pair (3285, 5274) =>5274=>$5^2 +2^2+7^2+4^2=94=> 9^2 +4^2 =97=>9^2 +7^2=130=>1^2+3^2 +0^2=10=>1^2+0^2=1$.
3) A-pair(54963, 52974):
   The numbers in A-pair have a prime quadruplet divisor each. That is, 54963 has a quadruplet divisor which is 197, and similarly for 52974 it is 109.

   54963 => 5 * 4 * 9 * 6 * 3 = 3240.
   The sum of all divisors of 3240 is 10890 (take any four-digit number, where the first and last digits differ by two or more and reverse the number to produce a new one. Then subtract the smaller from the larger producing another new number. If you reverse this number and this time add the two, the result will always be 10890).

   52974 =>5 * 2 * 9 * 7 * 4 = 2520 (The smallest number divisible by the first 10 positive integers).

4) S-pair (3285, 5274):
   3 * 2 * 8 * 5 = 5 * 2 * 7 *4.

## IV. CONCLUSIONS

This paper presented ALS Routine; a mathematical procedure inspired by Kaprekar's constant. While similar iterative number manipulation techniques might exist, ALS' Routine offers a unique approach to exploring the behavior of five-digit numbers. The identified number pairs (A-pair for five digits and L/S-pairs for specific four-digit scenarios) demonstrate a captivating repetitive pattern within the ALS Routine iterations.

**Key Findings:**

ALS' Routine successfully extends the concept of iterative number manipulation beyond four digits. The identified number pairs (A-pair, L-pair, and S-pair) exhibit a unique repetitive behavior within the ALS Routine.

**Future Work:**

Analyze the mathematical properties and underlying mechanisms responsible for the repetitive behavior of the identified number pairs.

Investigate potential connections between ALS' Routine and existing iterative number theory concepts.

Explore the applicability of ALS' Routine in broader mathematical areas or potential cryptographic application

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