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The Total Chromatic Number in Various Graphs

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Abstract: In this paper, we develop that the total coloring conjecture. A total coloring of a graph G is marked of colors to both the points and lines of G, such that no two adjacent or incident points and lines of G are marked the same colors. Throughout this paper we prove that the Tcc of an Arrow graph,square grid graph, p-th power graph of a path, shell graph and shell subdivided graph.

***Index Terms* - :** Total coloring conjecture, Arrow graph, square grid graph, p-th power graph of a path, shell graph, shell subdivided graph

INTRODUCTION:

In this paper, we have finite, Simple and cycle graphs. Let$G = (V(G), E(G))$, be a graph with point set $V(G)$ and the line set $E(G)$ respectively. In (Behzad 1965) has introduced the concept of total chromatic number. One of the most interested coloring of graph theory is total coloring. This coloring was introduced by (Rosenfeld et al, 1967). (Behzad in 1971) came out new ideology the total chromatic number of the complete graph. They conjectured that the total chromatic number $\chi_{tc}(G)$ for any graph G satisfies the condition.

$\Delta(G) + 1 \leq \chi_{tc}(G) \leq \Delta(G) + 2$. Where $\Delta(G)$ is the maximum degree of G and this is called as total coloring conjecture(Tcc). If it is total colors with $\Delta(G) + 1$ colors it is called type-I. If it is total colors with $\Delta(G) + 2$ colors it is called a type-II

Referring to the arrow graph definition was explained by (Kaneriaa et al) in this year 2015.Square grid graph definition was explained by (Acharya et al 1981). Power graph definition was explained by (Brandstädt et al 1996), (Kheddouci et al 2000). Shell graph definition and shell subdivided graph definition was explained by (Ezhilarasi Hilda et al 2018).

In this research paper, we obtain the Tcc of an Arrow graph, Square grid graph, p-th Power graph of a path, Shell graph and Shell subdivided graph.

Definition: 1 Total Chromatic Number

A total coloring of G is a function$f: S \to C$. Where $S = V(\mathrm{G}) \cup E(G)$ and C is a set of colors which satisfies the given conditions.

(i) Two adjacent points accept the different colors.

(ii) Two adjacent lines accept the different colors.

(iii) Line and its end points accept the different colors.

Definition: 2 Arrow Graph

An arrow graph A_n^t with breath t and length n is obtained by joining a vertex X with superior vertices of Pm × Pn by new edges from one end.

Definition: 3 Grid Square graph.

A two dimensional grid graph $m \times n$ is called a rectangular Grid graph

If m=n then $n \times n$ is called a Square Grid graph.

Definition: 4 Power graph

The Power graph of a path is considered as respectively. Subclasses of distance graph. The p-th Power graph of G is a graph obtained from Gp by adding an edge between every pair of vertices Distance p or less with for p

Definition: 5 Shell graph

A shell graph as a cycle Cn with (n-3) chords a common end point is called the apex. The shell graph is join of a complete graph k_1 andP_m, with m vertices.

Definition: 6 subdivided shell graphs

A subdivided shell graph is obtained from the shell graph G=P_mvK $_1$ by subdividing the edges in the path P_m of the shell graph.

THEOREM: 1

The total chromatic number of an arrow graph of order n. Then $\chi_{tc}(A_n^2) = 5$ for $n \geq 3$

Proof:

Let G =A_n^2 be an arrow graph obtained by joining a vertex X with superior vertices of $P_2 \times P_n$ by 2 new edges.

The number of points and lines of an arrow graph A_n^2 is 2n+1 and 3n.

The point set =$\{\mathrm{X}\} \cup \{u_i: 1 \leq i \leq n\} \cup \{v_i: 1 \leq \mathrm{i} \leq \mathrm{n}\}$

The line set = $\{\mathrm{X}u_1\} \cup \{\ \mathrm{X}v_1)\} \cup \{u_i u_{i+1}\ , 1 \leq i \leq n-1\ \} \cup \{v_i v_{i+1}, 1 \leq i \leq n-1\ \} \cup$

$\{f(u_i v_i, 1 \leq \mathrm{i} \leq n\}$

Maximum degree =$\Delta(A_n^2) = 3$

Minimum degree= $\delta(A_n^2) = 2$

First to the 2n+1 point in this graph should be colored.

u_i And v_i are two adjacent points, so needed two colors. X point joined with u_i and v_i points; therefore apart from 1 and 2, the X point should be given a different color.

If n is odd

$f(u_1), f(u_3), f(u_5), , , , ,f(u_n)$=1 for i= 1 to $\frac{n+1}{2}$ if i is odd

$f(u_2), f(u_4), f(u_6),, , , , ,f(u_{n-1})$=2 for i= 1 to $\frac{n-1}{2}$ if i is even

$f(v_1), f(v_3), f(v_5),, , , , ,f(v_n)$=2 for i= 1 to $\frac{n+1}{2}$ if i is odd

$f(v_2), f(v_4), f(v_6),, , , , ,f(v_{n-1})$=1 for i= 1 to $\frac{n-1}{2}$ if i is even

If n is even

$f(u_1), f(u_3), f(u_5),, , , , ,f(u_n)$=1 for i= 1 to $\frac{n}{2}$ if i is odd

$f(u_2), f(u_4), f(u_6),, , , , ,f(u_{n-1})$=2 for i= 1 to $\frac{n}{2}$ if i is even

$f(v_1), f(v_3), f(v_5),, , , , ,f(v_n)$=2 for i= 1 to $\frac{n}{2}$ if i is odd

$f(v_2), f(v_4), f(v_6),, , , , ,f(v_{n-1})$=1 for i= 1 to $\frac{n}{2}$ if i is even

f(X)=5

Next the lines should be colored. Already three colors are assigned to the 2n+1 points.

$f(Xu_1)$=3, $f(Xv_1)$=4

$f(u_i v_i) = 5$ for i = 1 to n

If n is odd

$f(u_1u_2), f(u_3u_4), f(u_5u_6),,,,f(u_{n-2}u_{n-1}) = 4$ for i= 1 to $\frac{n-1}{2}$ If i is odd

$f(u_2u_3), f(u_4u_5), f(u_6u_7),,,,,,f(u_{n-1}u_n) = 3$ for i= 1 to $\frac{n-1}{2}$ If i is even

$f(v_1v_2), f(v_3v_4), f(v_5v_6),,,,f(v_{n-2}v_{n-1}) = 3$ for i= 1 to $\frac{n-1}{2}$ If i is odd

$f(v_2v_3), f(v_4v_5), f(v_6v_7),,,,,,f(v_{n-1}v_n) = 4$ for i= 1 to $\frac{n-1}{2}$ If i is even

If n is even

$f(u_1u_2), f(u_3u_4), f(u_5u_6),,,,f(u_{n-2}u_{n-1}) = 4$ for i= 1 to $\frac{n}{2}$ If i is odd

$f(u_2u_3), f(u_4u_5), f(u_6u_7),,,,,,f(u_{n-1}u_n) = 3$ for i= 1 to $\frac{n-2}{2}$ If i is even

$f(v_1v_2), f(v_3v_4), f(v_5v_6),,,,f(v_{n-2}v_{n-1}) = 3$ for i= 1 to $\frac{n}{2}$ If i is odd

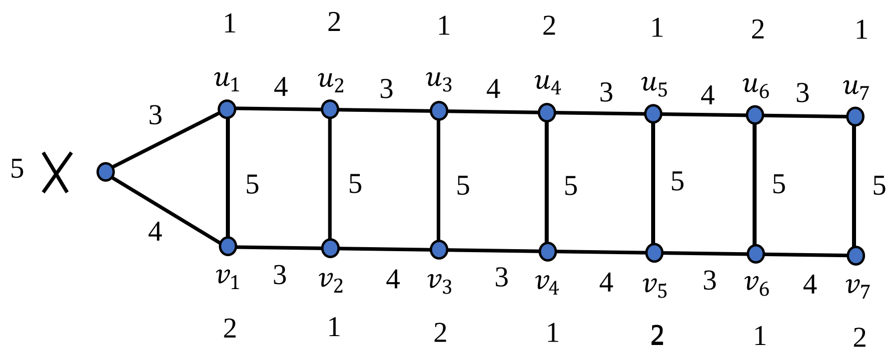
$f(v_2v_3), f(v_4v_5), f(v_6v_7),,,,,,f(v_{n-1}v_n) = 4$ for i= 1 to $\frac{n-1}{2}$. If i is even

In this graph, a maximum number of points have a maximum degree. So it belongs to category type II

Thus, the complete coloring of this graph requires only five colors.

Hence $\chi_{tc}(A_n^2) = 5$ for $n \geq 3$

Example: 1



$\chi_{tc}(A_7^2) = 5$

THEOREM: 2

The total chromatic number of an arrow graph of order n. Then $\chi_{tc}(A_n^3) = 6$ for $n \geq 3$.

Proof: Let G =A_n^3 be an arrow graph obtained by joining a point X with superior points of $P_3 \times P_n$ by 3 new lines.

The number of points and lines of an arrow graph A_n^3 is 3n+1 and 5n.

The point set =$\{X\} \cup \{u_i : 1 \leq i \leq n\} \cup \{v_i : 1 \leq i \leq n\} \cup \{w_i : 1 \leq i \leq n\}$

The line set=$\{Xu_1\} \cup \{Xv_1\} \cup \{Xw_1\} \cup \{u_i u_{i+1}, \quad 1 \leq i \leq n-1\} \cup$
$\{vv_{i+1}, 1 \leq i \leq n-1\} \cup \{w_i w_{i+1}, \quad 1 \leq i \leq n-1\} \cup \{u_i v_i, \quad 1 \leq i \leq n\} \cup$
$\{v_i w_i, \quad 1 \leq i \leq n\}$

Maximum degree =$\Delta(A_n^3) = 4$

Minimum degree= $\delta(A_n^3) = 2$

Now we identify the Tcc and C= {1, 2, 3, 4, 5, 6}

First to the 3n+1 point in this graph should be colored.

u_i and v_i are two adjacent points, v_i and w_i are two adjacent points but u_i and w_i are not adjacent points, so needed only two colors. X point joined with u_i , v_i and w_i are adjacent points; therefore, apart from 1 and 2, X should be given a different colour.

If n is odd

$f(u_1), f(u_3), f(u_5),, , , , ,f(u_n)$=1 for i= 1 to $\frac{n+1}{2}$ if i is odd

$f(u_2), f(u_4), f(u_6),, , , , ,f(u_{n-1})$=2 for i= 1 to $\frac{n-1}{2}$ if i is even

$f(v_1), f(v_3), f(v_5),, , , , ,f(v_n)$=2 for i= 1 to $\frac{n+1}{2}$ If i is odd

$f(v_2), f(v_4), f(v_6),, , , , ,f(v_{n-1})$=1 for i= 1 to $\frac{n-1}{2}$ If i is even

$f(w_1), f(w_3), f(w_5),, , , , ,f(w_n)$=1 for i= 1 to $\frac{n+1}{2}$ If i is odd

$f(w_2), f(w_4), f(w_6),, , , , ,f(w_{n-1})$=2 for i= 1 to $\frac{n-1}{2}$ If i is even

If n is even

$f(u_1), f(u_3), f(u_5),, , , , ,f(u_n)$=1 for i= 1 to $\frac{n}{2}$ If i is odd

$f(u_2), f(u_4), f(u_6),, , , , ,f(u_{n-1})$=2 for i= 1 to $\frac{n}{2}$ If i is even

$f(v_1), f(v_3), f(v_5),, , , , ,f(v_n)$=2 for i= 1 to $\frac{n}{2}$ If i is odd

$f(v_2), f(v_4), f(v_6),,,,,,f(v_{n-1})$=1 for i= 1 to $\frac{n}{2}$ If i is even

$f(w_1), f(w_3), f(w_5),,,,,,f(w_n)$=1 for i= 1 to $\frac{n}{2}$ If i is odd

$f(w_2), f(w_4), f(w_6),,,,,,f(w_{n-1})$=2 for i= 1 to $\frac{n}{2}$ If i is even

f(X)=6

Next the lines should be colored. Already three colors are assigned to the points.

$f(Xu_1) = 2$ $f(X) = 6$ $f(Xv_1) = 3$ $f(Xw_1) = 4$
$f(u_i v_i) = 5$ for $1 \leq i \leq$n
$f(v_i w_i) = 6$ for $1 \leq i \leq$n

If n is odd

$f(u_1u_2), f(u_3u_4), f(u_5u_6),,,,f(u_{n-2}u_{n-1}) = 4$ for i= 1 to $\frac{n-1}{2}$ If i is odd

$f(u_2u_3), f(u_4u_5), f(u_6u_7),,,,,,f(u_{n-1}u_n) = 3$ for i= 1 to $\frac{n-1}{2}$ If i is even

$f(v_1v_2), f(v_3v_4), f(v_5v_6),,,,f(v_{n-2}v_{n-1}) = 3$ for i= 1 to $\frac{n-1}{2}$ If i is odd

$f(v_2v_3), f(v_4v_5), f(v_6v_7),,,,,,f(v_{n-1}v_n) = 4$ for i= 1 to $\frac{n-1}{2}$ If i is even

$f(w_1w_2), f(w_3w_4), f(w_5w_6),,,,f(w_{n-2}w_{n-1}) = 4$ for i= 1 to $\frac{n-1}{2}$ If i is odd

$f(w_2w_3), f(w_4w_5), f(w_6w_7),,,,,,f(w_{n-1}w_n) = 3$ for i= 1 to $\frac{n-1}{2}$ If i is even

If n is even

$f(u_1u_2), f(u_3u_4), f(u_5u_6),,,,f(u_{n-2}u_{n-1}) = 4$ for i= 1 to $\frac{n}{2}$ If i is odd

$f(u_2u_3), f(u_4u_5), f(u_6u_7),,,,,,f(u_{n-1}u_n) = 3$ for i= 1 to $\frac{n-2}{2}$ If i is even

$f(v_1v_2), f(v_3v_4), f(v_5v_6),,,,f(v_{n-2}v_{n-1}) = 3$ for i= 1 to $\frac{n}{2}$ If i is odd

$f(v_2v_3), f(v_4v_5), f(v_6v_7),,,,,,f(v_{n-1}v_n) = 4$ for i= 1 to $\frac{n-1}{2}$ If i is even

$f(w_1w_2), f(w_3w_4), f(w_5w_6),,,,f(w_{n-2}w_{n-1}) = 4$ for i= 1 to $\frac{n}{2}$ If i is odd

$f(w_2w_3), f(w_4w_5), f(w_6w_7),,,,,,f(w_{n-1}w_n) = 3$ for i= 1 to $\frac{n-2}{2}$ If i is even

In this graph, a greater number of points have a maximum degree. So it belongs to category type II

Thus, the complete coloring of this graph required only six colors.

Hence the total chromatic numberof $\chi tc(A_n^3) = 6$ for $n \geq 3$.

Note:1 Maximum degree =$\Delta(A_n^4) = 4$

Minimum degree= $\delta(A_n^4) = 2$

The maximum degree is 4, when the value of t is 4

In this graph, a maximum number of points have a maximum degree. So it belongs to Category type-II

Thus, the complete coloring of this graph required only six colors.

Hence the total chromatic numberof $\chi tc(A_n^4) = 6$ for $n \geq 3$.

Note:2 Maximum degree $=\Delta(A_n^5) = 5$

Minimum degree= $\delta(A_n^5) = 2$

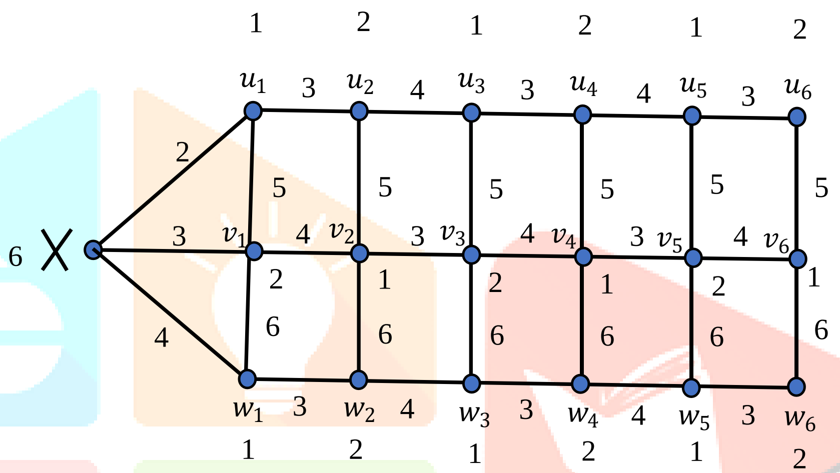
The maximum degree is 5, when the value of t is 5

It is category type-I, as only one point has a maximum degree and remaining points have a minimum degree. Thus, the complete coloring of this graph required six colors.

Hence the total chromatic numberof $\chi tc(A_n^5) = 6$ for $n \geq 3$

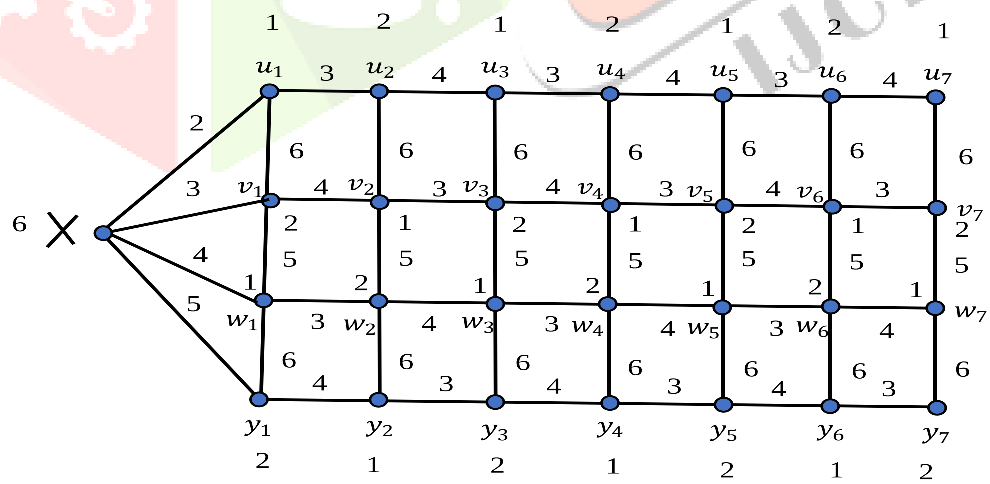
Note:3 If the t value is increasing then the maximum degree value increases. Therefore the value of the total chromatic number is changed.

Example: 2



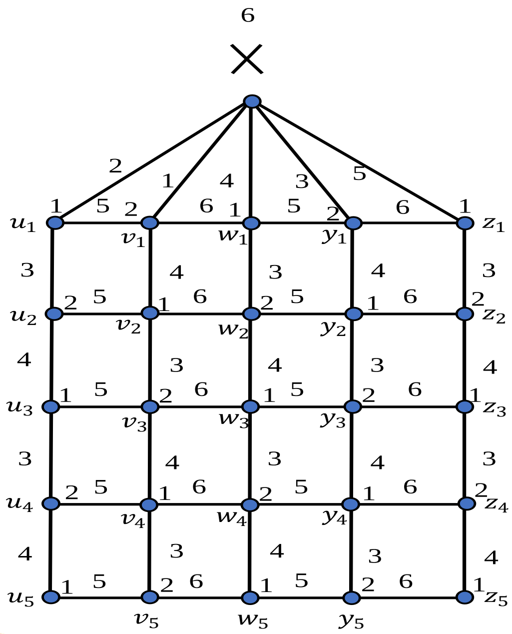
$\chi_{tc}(A_6^3) = 6$

Example: 3



$\chi_{tc}(A_7^4) = 6$

Example: 4



$\chi_{tc}(A_5^5)$=6

THEOREM:3

The total chromatic number of the square Grid graph is given by $\chi_{tc}[G(n,n)] = 6$ for $n \geq 3$.

Proof:

The number of points and lines of the Square Grid graph $G(n,n)$ is n^2 and $2n^2 - 2n$ respectively.

Now we define the total coloring f such that $f: S \rightarrow C$ as follows.

Maximum degree $=\Delta(G(n,n)) = 4$

Minimum degree= $\delta(G(n,n)) = 2$

Case: I:

If n is odd

If n is an odd number then the number of squares will be even.

First assign the total coloring for the points as follows.

$$f(v_i) = \begin{cases} 1 & \text{if} \quad i \equiv 1(\text{mod}2) \\ 2 & \text{if} \quad i \equiv 0(\text{mod}2) \end{cases} \quad 1 \leq i \leq n^2$$

Next, assign the total coloring for the lines as follows.

There are two types of lines. horizontal line and vertical line.

$$\text{Horizontal line} = f(v_i v_{i+1}) = \begin{cases} 3 & \text{if} \quad i \equiv 1(\text{mod}2) \\ 4 & \text{if} \quad i \equiv 0(\text{mod}2) \end{cases} \quad \text{for } 1 \leq i \leq n(n-1)$$

$$\text{Vertical line} = f(u_i u_{i+1}) = \begin{cases} 5 & \text{if} \quad i \text{ is odd} \\ 6 & \text{if} \quad i \text{ is even} \end{cases} \quad \text{for } 1 \leq i \leq n(n-1)$$

Case II:

If n is even

If n is an even number then the number of squares will be an odd number First assign the total coloring for the points as follows. Number of the square is being an odd number, so we are giving color to the point as 1,2,1,2 ... So the last point color 2. After when we came to the next row which has been ended that color 2, We have to start 2,1,2,1. ,,,,

Similarly, we have to give color n^2 points.

Next, assign the total coloring for the lines as follows.

Horizontal lines

$$f(v_i v_{i+1}) = \begin{cases} 3 & \text{if} \quad i \equiv 1(\text{mod}2) \\ 4 & \text{if} \quad i \equiv 0(\text{mod}2) \end{cases} \quad \text{for } 1 \le i \le n(n-1)$$

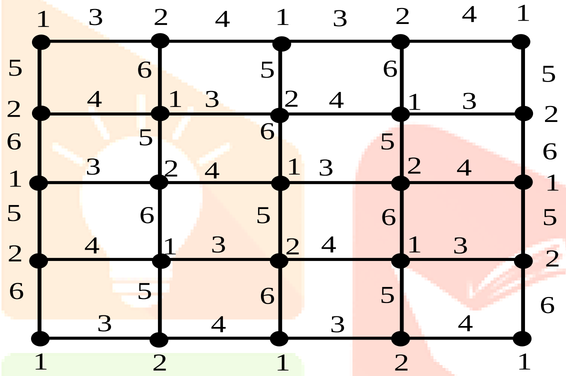
Vertical lines

$$f(u_i u_{i+1}) = \begin{cases} 5 & \text{if} \quad i \text{ is odd} \\ 6 & \text{if} \quad i \text{ is even} \end{cases} \quad \text{for } 1 \le i \le n(n-1)$$

So the opposite sides of the horizontal lines are the same colors. Because these lines are unrelated to each other.In this graph, a maximum number of points have a maximum degree. So it belongs to Category type II Thus, the complete coloring of this graph requires only six colors.

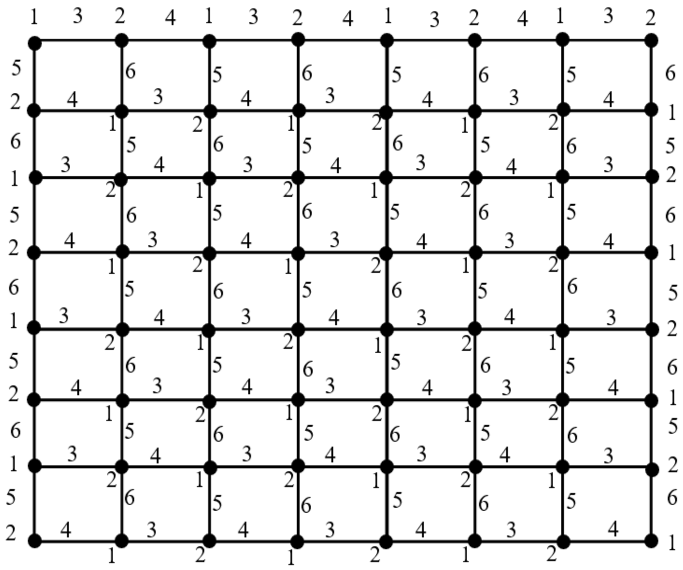
Therefore $\chi_{tc}[G(n,n)] = 6$ for $n \ge 3$

Example: 5



$\chi_{tc}[G(5,5)] = 6$ if n is odd

Example:6



$\chi_{tc}[G(8,8)] = 6$ If n is even

THEOREM: 4

Let P_n be a Path on points $x_1, x_2, \cdots x_n$. The total chromatic number of p-th power graph of a path is

$$\chi_{tc}(P_n^p) = \begin{cases} 2p+1 & \text{if } p = 2 \text{ and } n \ge 5 \\ 2p+2 & \text{if } p = 3 \text{ and } n \ge 7 \\ 2p+2 & \text{if } p = 4 \text{ and } n \ge 8 \\ 2p+2 & \text{if } p = 5 \text{ and } n \ge 9 \end{cases}$$

Proof:

Let P_n^p be a path on n points

Case I:

Let P_n^2 be a path 2nd power graph of path with n points and $2n-3$ lines respectively. The point set and the line set (P_n^2) as given as follows

$$V(P_n^2) = \{v_1, v_2 \dots v_n\},$$

$$E(P_n^2) = \{v_i v_{i+1} : 1 \le i \le n-1\} \cup$$

$$\{v_i v_{i+2} : 1 \le i \le n-2\}$$

$$f(v_i) = \begin{cases} 1 & \text{if } i \equiv 1(\text{mod}3) \\ 2 & \text{if } i \equiv 2(\text{mod}3) \\ 3 & \text{if } i \equiv 0(\text{mod}3) \end{cases} \quad \text{for } 1 \le i \le n$$

$$f(v_i v_{i+1}) = \begin{cases} 3 & \text{if } i \equiv 1(\text{mod}3) \\ 1 & \text{if } i \equiv 2(\text{mod}3) \\ 2 & \text{if } i \equiv 0(\text{mod}3) \end{cases} \quad \text{for } 1 \le i \le n-1$$

If n is even

$$f(v_{2i-1} v_{2i+1}) = \begin{cases} 4 & \text{If } i \text{ is odd} \\ 5 & \text{If } i \text{ is even} \end{cases} \quad \text{for } 1 \le i \le \frac{n-2}{2}$$

$$f(v_{2i} v_{2i+2}) = \begin{cases} 4 & \text{If } i \text{ is odd} \\ 5 & \text{If } i \text{ is even} \end{cases} \quad \text{for } 1 \le i \le \frac{n-2}{2}$$

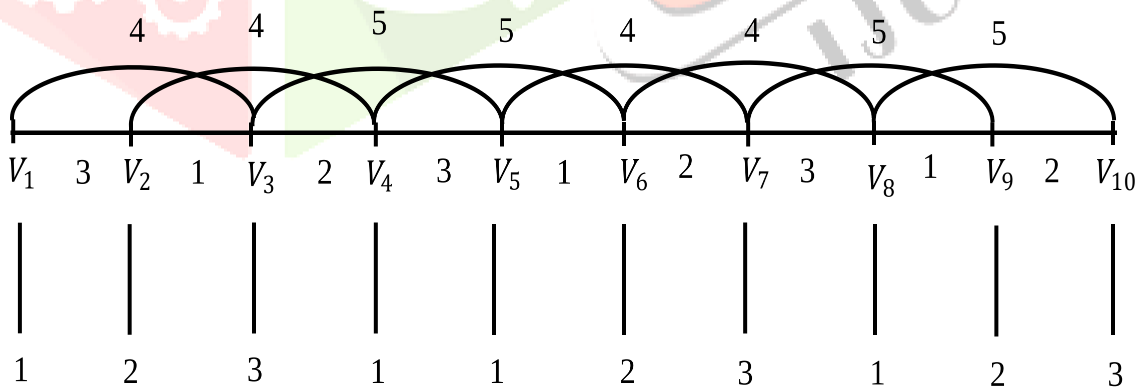
If n is odd

$$f(v_{2i-1} v_{2i+1}) = \begin{cases} 4 & \text{If } i \text{ is odd} \\ 5 & \text{If } i \text{ is even} \end{cases} \quad \text{for } 1 \le i \le \frac{n-1}{2}$$

$$f(v_{2i} v_{2i+2}) = \begin{cases} 4 & \text{if } i \text{ is odd} \\ 5 & \text{if } i \text{ is even} \end{cases} \quad \text{for } 1 \le i \le \frac{n-3}{2}$$

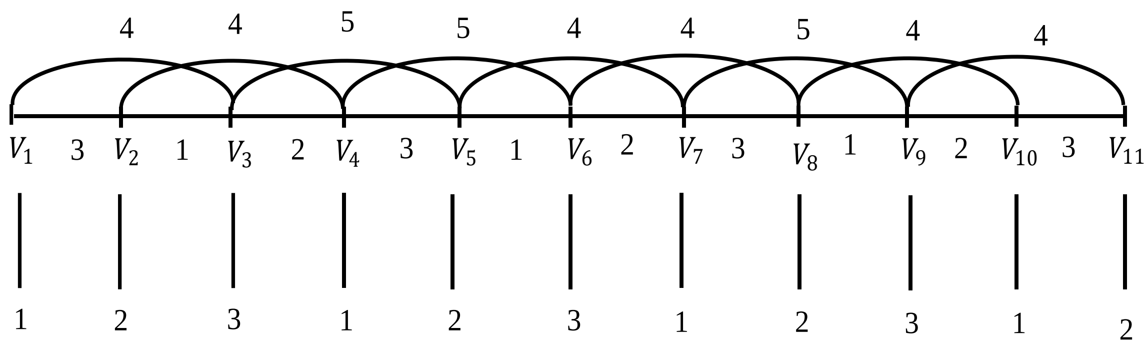
Hence the total chromatic number of $\chi tc(P_n^2) = 2p + 1 \; for \; n \ge 5$.

Example: 7



$\chi_{tc}(P_{10}^2)$=5

Example: 8



$\chi_{tc}(P_{11}^2)$=5

***Case II*:**

Let P_n^3 be a path 3rd power graph of path with n points and $3n-6$ lines respectively.

The point set and the line set (P_n^3) as given as follows

$$V(P_n^3) = \{v_1, v_2, v_3 \dots v_n\}$$
$$E(P_n^3) = \{v_i v_{i+1} : 1 \le i \le n-1\} \cup \{v_i v_{i+3} : 1 \le i \le n-3\}$$
$$\{v_i v_{i+2} : 1 \le i \le n-2\}$$

Now we define the total coloring f such that $f : S \to C$ as follows,

$$f(v_i) = \begin{cases} 1 & \text{f} \quad i \equiv 1(\text{mod}4) \\ 2 & \text{if} \quad i \equiv 2(\text{mod}4) \\ 3 & \text{f} \quad i \equiv 3(\text{mod}4) \\ 4 & \text{if} \quad i \equiv 0(\text{mod}4) \end{cases} \quad \text{for } 1 \le i \le n$$

$$f(v_i v_{i+1}) = \begin{cases} 4 & \text{f} \quad i \equiv 1(\text{mod}4) \\ 1 & \text{if} \quad i \equiv 2(\text{mod}4) \\ 2 & \text{if} \quad i \equiv 3(\text{mod}4) \\ 3 & \text{if} \quad i \equiv 0(\text{mod}4) \end{cases} \text{for } 1 \le i \le n-1$$

$f(v_i v_{i+3}) = f(v_4 v_7), f(v_5 v_8), f(v_6 v_9), f(v_{10} v_{13}), f(v_{11} v_{14}),$,

$f(v_{n-4} v_{n-1})$ =$\{6\}$ for i= 4,5,6,10,11,12,,18,19,20,,, , ,

$f(v_i v_{i+3}) = f(v_1 v_4), f(v_2 v_5), f(v_3 v_6), f(v_7 v_{10}), f(v_8 v_{11}), f(v_9 v_{12}),$, ,

$f(v_{n-3} v_n)$ ={5} for i= 1,2 3,7,8,9,,13,14,15,17,21,22,23, ,, ,

If n is even

$$f(v_{2i-1} v_{2i+1}) = \begin{cases} 7 & \text{If } i \text{ is odd} \\ 8 & \text{If } i \text{ is even} \end{cases} \quad \text{for } 1 \le i \le \frac{n-2}{2}$$

$$f(v_{2i} v_{2i+2}) = \begin{cases} 7 & \text{If } i \text{ is odd} \\ 5 & \text{If } i \text{ is even} \end{cases} \quad \text{for } 1 \le i \le \frac{n-2}{2}$$

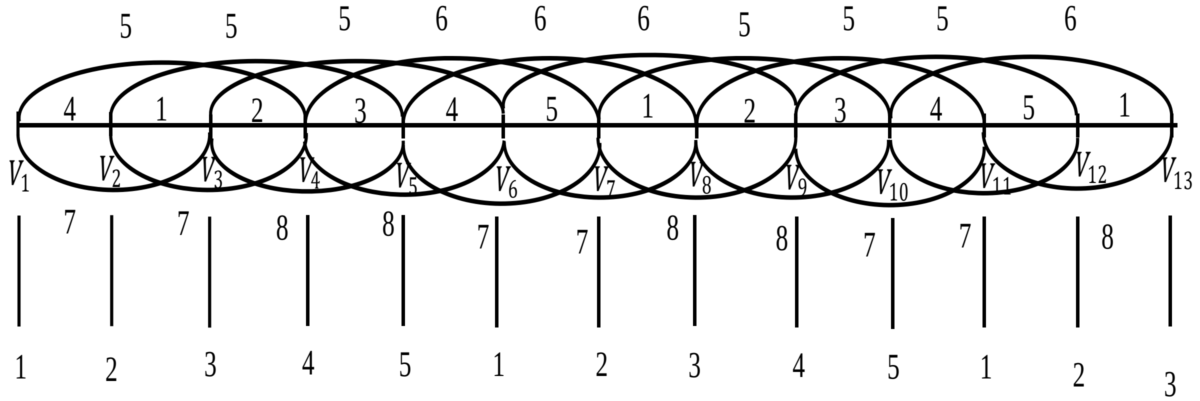
If n is odd

$$f(v_{2i-1} v_{2i+1}) = \begin{cases} 7 & \text{If } i \text{ is odd} \\ 8 & \text{If } i \text{ is even} \end{cases} \quad \text{for } 1 \le i \le \frac{n-1}{2}$$

$$f(v_{2i} v_{2i+2}) = \begin{cases} 7 & \text{if } i \text{ is odd} \\ 8 & \text{if } i \text{ is even} \end{cases} \quad \text{for } 1 \le i \le \frac{n-3}{2}$$

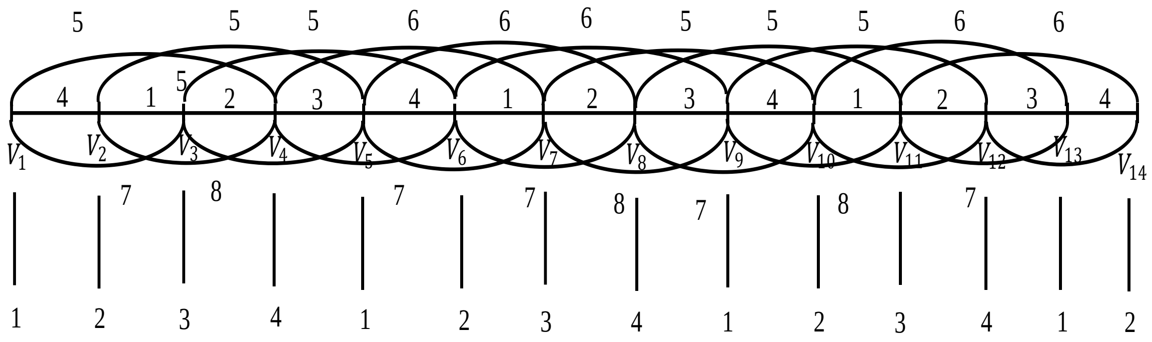
Hence the total chromatic number of $\chi tc(P_n^3) = 2p + 2 \; for \; n \ge 5$.

Example: 9



$\chi_{tc}(P_{13}^3)$=6

Example: 10



$\chi_{tc}(P_{14}^3)$=6

Case III:

In other similar graphs P_n^4, P_n^5 ..., as the p value changes, its maximum degree also changes. So add two to the maximum degree value. It is clear that the above procedure of total coloring, The p[th] power graph of a path is properly totally colored with $2p + 2$ Colors.Hence the total chromatic number of the pth power graph of a path graph is

$\chi_{tc}G(P_n^2) = 2P + 1$ for $n \geq 5$

$\chi_{tc}G(P_n^3) = 2P + 2$ for $n \geq 7$

$\chi_{tc}G(p_n^4) = 2p + 2$ for $n \geq 9$.

THEOREM: 5

The total chromatic number of shell graph is $\chi tc[P_n \vee K_1] = n$ +1 for $n \geq 4$.

Proof:

Let G $= P_n \vee K_1$be a shellgraph. With $n + 1$ points and $2n - 1$ lines respectively. The point set and the line set $C(n; n - 3)$ is given as follows.

Denote apex of G as V

$V(P_n \vee K_1) = \{v_1, v_2, v_3 \ldots, v_n\} \cup \{V\}$

$E((P_n \vee K_1) = \{Vv_n: 1 \leq i \leq n\} \cup \{v_i v_{i+1}: 1 \leq i \leq n - 1\}$

$S = V[(P_n \vee K_1)] \cup E[(P_n \vee K_1)]$

Maximum degree=$\Delta(P_n \vee K_1) = n$

Minimum degree= $\delta(P_n \vee K_1) = 2$

Now we define the total coloring f such that $f: S \to C$ as follows,$C = \{1,2,3 \ldots n + 1\}$

Assign the total coloring for the points and lines as follows

$$f(V) = 1$$

$$f(v_i) = i+1 \quad \text{for } 1 \leq i \leq n.$$

$$f(Vv_i) = i + 2 \quad \text{for } 1 \leq i \leq n - 1$$

$$f(v_i v_{i+1}) = i \quad \text{for } 1 \leq i \leq n - 1$$

$$f(Vv_n) = 2.$$

It is category type I, as only one point has a maximum degree and the remaining n points have a minimum degree. Thus, the complete coloring of this graph requires n+1 color.

Therefore $\chi tc(P_n \vee K_1) = n$+1

THEOREM: 6

The total chromatic number of subdividing shell graph n is $n+2$ colors. for $n \geq 3$

Proof:

Let G be a subdivided shell graph with $2n$ points and $3n-2$ lines respectively.

Denote apex of G as V.

The number of path points in each subdivided shell is denoted as $n(n \geq 3)$.

The points in the path of the first subdivided shell as $v_1, v_2 v_3 \ldots v_n$.

The points in the path of the second subdivided shell are denoted as $u_1, u_2, u_3, \ldots\ldots\ldots u_{n-1}$

The $= \{v_i : 1 \leq i \leq n\} \cup \{V\} \cup \{u_j : 1 \leq j \leq n-1\}$

The line set $= \{V v_n : 1 \leq i \leq n\} \cup \{v_i u_i : 1 \leq i \leq n-1\} \cup \{v_i u_j : 1 \leq i \leq j \leq n-1\}$

$$f(V) = n+1$$

$$f(v_i) = n+2 \quad 1 \leq i \leq n$$

$$f(u_j) = j \quad 1 \leq j \leq n-1$$

$$f(v_1 v_i) = i+1 \quad 2 \leq i \leq n-1$$

$$f(v_i u_j) = \begin{cases} 1 & \text{for } 2 \leq i \leq n-2,\ 1 \leq j \leq n-1 \\ 2 & \text{for } 3 \leq i \leq n-2,\ 1 \leq j \leq n-1 \end{cases}$$

Only one point has degree n, (n+1) points have degree 3 and the remaining n-2 points have

Degree 2.

Thus, the complete coloring of this graph requires n+2 colors.

Hence the total chromatic number of subdividing shell graph n is $n+2$ colors. for $n \geq 3$

CONCLUSION:

In this paper, we have explored the total chromatic number of various graphs.

1. $\chi_{tc}(A_n^2) = 5$. for $n \geq 3$.

2. $\chi_{tc}(A_n^3) = 6$. for $n \geq 3$.

3. $\chi_{tc}(A_n^4) = 6$. for $n \geq 3$.

4. $\chi_{tc}(A_n^5) = 6$. for $n \geq 3$.

5. $\chi_{tc}[G(n,n)] = 6$ for $n \geq 3$.

6. Let P_n be a Path on points $x_1, x_2, \cdots x_n$ path is $\chi_{tc}(P_n^p) = \begin{cases} 2p+1 & \text{if } p = 2 \text{ and } n \geq 5 \\ 2p+2 & \text{if } p = 3 \text{ and } n \geq 7 \\ 2p+2 & \text{if } p = 4 \text{ and } n \geq 8 \\ 2p+2 & \text{if } p = 5 \text{ and } n \geq 9 \end{cases}$

7. $\chi_{tc}[(\text{Pn v K1})] = n+1$ for $n \geq 4$.

8. The total chromatic number of subdividing shell graph n is $n+2$ colors. for $n \geq 4$.

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