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## SMART AQUARIUM CONTROL USING IOT

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**ABSTRACT** The advancement of the Things connected to internet has made notable strides across various sectors such as town, houses, and agricultural fields, encompassing fish tank maintaining systems as well. Key variables in such systems include monitoring ammonia levels and water temperature. While prior research has proposed various systems to bolster fish tank monitoring and control, they often entail the drawback of necessitating active user access to cloud information. This study presents a resilient fish tank maintaining systems employing the things which has connectivity with internet, which had , aimed at addressing remote user engagement issues. A precise day to day monitoring and doing some operations is imperative in averting potential hazards, such as water temperature spikes. Our experimentation involved connecting an fish tank system to a cloud and implementing a controller application. Through meticulous testing, we evaluated data transmission late signals from a device which is used to measure the physical signals s to the cloud, processing late signals, actuator response times, user interaction late signals, and the time taken to reach critical points in the aquarium. Robustness assessment was conducted by gauging the likelihood of timely user information delivery during critical periods made some good comparison with time to get those hard points. Additionally, It was devised to complement experimental findings. Both analytically and experimentally, our results demonstrate the efficacy of the proposed system in meeting the demands of fish tank maintaining and achieving some power/authority within an thing that interact with internet framework.

**INDEX TERMS** Connectivity, Understanding, Likelihood, Strength, Fishery, Prediction

### I. INTRODUCTION

The IoT has witnessed substantial growth in recent years, largely driven by advancements in device to device connecting medium over the last 10 years. Hence these medium were designed to interconnect various devices, a device which is used to measure the physical signals s, and actuators. Concurrently, research has focused on middleware facilitating communication between hardware and IoT applications. An exemplar application of such developments is a fish maintenance system designed for remote monitoring and control, as explored in this study.

Ensuring optimal fish maintenance necessitates a reliable system that enables owners to monitor their fish's

well-being even while on the move. Continuous monitoring is essential as users cannot always oversee the condition of their ornamental fish. This monitoring ensures that the fish are housed in an environment with appropriate food and environmental conditions. Users seek assurance that both monitoring and control processes are functioning effectively. Factors such as the quality of fish feed and water conditions are crucial considerations. Maintaining an appropriate water temperature is particularly vital, as drastic fluctuations can lead to critical and precarious conditions for the fish. Under specific circumstances, fish cannot survive in environments with temperatures beyond certain limits. A system capable of proactively anticipating such conditions within acceptable variables is crucial. Both manual and IoT-based fish tank caretaking systems have been hugely adopted.

However, manual methods for fish tank management are labor-intensive and susceptible to human errors, such as neglect due to forgetfulness or busyness. Relying on human sensitivity to detect changes in water temperature is unreliable for maintaining optimal water conditions. An automated system is essential for accurately monitoring fish tank conditions, ensuring the well-being and longevity of pet fish.

In addition to manual methods, various studies have explored the function of IoT medium for fish tank monitoring and control. Previous research has proposed IoT-based fish tank control systems, but these systems have been passive in nature. In such systems, users are required to actively monitor the aquarium's condition, which may not always be feasible. This passive approach becomes impractical, especially when users are unaware of potential issues arising from busy schedules, leading to oversight of the aquarium's condition. Users find themselves monitoring the fish tank without prior knowledge of its status. While accessing fish tank information during optimal conditions may seem unnecessary, late signals in obtaining information could lead to catastrophic events during critical periods. There is a pressing need for an IoT system that not only provides information but also actively alerts users to the aquarium's condition, particularly during critical situations.

The study develops an internet-based fish tank management system encompassing both a giving forcing mechanism and handy remote accessibility for fish tank maintenance. The forward forcing system actively sends data to the cloud system, which subsequently relays it to users during hard or regular conditions. Through this forcing system, users receive information automatically without the need for active web access. Furthermore, this system offers the advantage of mobile accessibility, allowing users convenient access to fish tank status information.

Our major support are pointed as follows:

- 1) We introduce a resilient fish tank maintaining system based on things connected to internet, employing an Arduino cloud algorithm for device to detect physical signal as data processing on a cloud. This functionality is crucial for forecasting the fish tank system's condition. The system autonomously executes necessary actions when the water creature environment reaches hard states, such as heavy temperatures. It seriously transmits the device to detect physical signal data to the cloud for both normal and critical conditions, ensuring users receive automated updates through a push system without needing to access the web manually..

- 2) We present a critical analysis of the potential for information late signals occurring from the device to detect physical signal as a system to the cloud, cloud processing, and subsequent late signals in relaying information from the cloud to the device. This analysis involves monitoring the processes occurring on the cloud.
- 3) We propose an examination of the late signal latency in delivering information from the device to the end user, aiming to prevent the aquaculture conditions from reaching critical stages. Previous studies, and have not extensively addressed this aspect. The specifics of our analytical model, which relies on the late signal probability density function, are provided in Figure a.
- 4) This document also looks at late signal in cloud process, something other references missed.

The structure of the rest of this study is outlined follows: part 2 delves into suitable works. Following that, Part 3 elucidates the aquaculture environment and presents the test results. Part 4 entails the assessment of the system's performance. Lastly, our research is concluded in Part 5..

## II. RELATED WORK

### A. APPLICATION OF THINGS CONNECTED TO INTERNET

Internet things stands out as first of the good prominent topics in the relate signal sector, significantly shaping the landscape of the Internet. Recent research has explored IoT across diverse domains, including agriculture, environmental monitoring, urban development, sports, transportation, retail, disaster management, and energy. In agriculture, for instance, there's been investigation into the plant wall system, utilizing IoT for automating monitoring and control tasks. Similarly, in urban areas, IoT-driven air pollution monitoring systems have been studied, with projects spanning multiple countries and incorporating thousands of devices. Notably, in smart city initiatives, research delves into vertical IoT platforms, offering a multi-platform approach facilitating crossplatform communication. Sports research has explored applications such as cycling, employing IMU a device which is used to measure the physical signals s to provide cyclists with comprehensive exercise metrics. In transportation, studies on car parking have introduced reinforcement learning solutions for optimal parking spot selection. In retail, IoT innovations like smart shopping carts have tackled queue management by enabling wireless billing at checkout. Disaster management research has focused on flood prediction using IoT, leveraging climate a device which is used to measure the physical signals s and machine learning algorithms. Furthermore, in the energy sector, Smart Grid studies have investigated methods like edge computing and anomaly detection for detecting damaged Smart Meters.As

Item	Haiyunnisa [3]	Tseng [4]	Raju [5]	Angani [27]	FishTalk [6]	Proposed
1. Sensors	pH, DO, Water Level, Temp	pH, DO, Temp, Water Level	DO, Salt, NH3, Nitrite, Temp, pH, Alkalinity	Temp, DO, Ultrasonic, pH	pH, EC, DO, TDS, Water Level, Temp	Temp, DO, EC
2. Actuators	Heater, Light, Feeder, Air Pump	N	Light	Solenoid Valve, Pump	Feeder, Fan, Heater, Light, Air Pump, RO Filter	Heater, Fan, Relay, RO Filter
3. Actuators Controlled by Sensors	Yes (simple threshold)	N	No (light is always on at night)	Yes	Yes	Yes
4. Smart Feeder	No (manual)	N	N	N	Yes	Yes
5. Control Board	MSP430	Arduino UNO	Raspberry Pi 3	Raspberry Pi	ESP8266, ESP12F	ESP8266
6. MQTT Communication	N	N	N	N	N	Yes
7. Prediction Server	N	N	N	N	N	Yes
8. Sensor-server-actuator delay measurement	N	N	N	N	Yes	Yes
9. Server process delay and user delay measurement	N	N	N	N	N	Yes
10. Android-based mobile application	N	N	N	N	N	Yes
11. iOS-based mobile application	N	N	N	N	N	Yes
12. MQTT	N	N	N	N	N	Yes

numerous Things connected to internet products enter into commercial, a new concern arises: security risks. Unlike other IT domains, the field of IoT is still in its early stages, indicating that online risk assessment for IoT remains underdeveloped. Several studies have examined the efficacy of current risk analysis methods and explored strategies for optimizing risk assessment processes.

### B. SMART FISH TANK

Many research peoples thoroughged on things works on internet water culture using several devices. Chen *et al.* looked on the

**Box 1.** solutions for internet based fish tank.

Aquariums employ a quality control system for monitoring Dissolved Oxygen (DO) levels. The system utilizes a DO a device which is used to measure the physical signals for input and a microbubble device as actuators. Tseng. utilized hydro a device which is used to measure the physical signals s, Acidity a device which is used to measure the physical signals s, and Oxygen a device which is used to measure the physical signals s in their study..

Further investigations into Internet -based fish tank have employed more specialized device to detect physical signal , leading to broader insights. Raju and Varma similarly utilized a device which is used to measure the physical signals s to monitor fish tankenvironments, including pH, and temperature a device which is used to measure the physical signals s. Their system also integrates water quality a device which is used to measure the physical signals . Anyway, unlike some systems, theirs lacks automatically

functioning actuators. Instead, users receive alerts with instructions that they must manually execute.

Several studies have incorporated additional beneficial features into aquaculture systems. Angani et al. focused their research on recycling water within an fish tankto minimize wastage. Their study employed a device which is used to measure the physical signals s including dissolved oxygen Oxygen, acidity, warmth, and water height a device which is used to measure the physical signals s, plus tools like solenoid valves for controlling water movement and pumps.

Certain studies have undertaken a thorough assessment of IoT systems implemented in aquaculture. For instance, FishTalk utilized a range of a device which is used to measure the physical signals s including pH, EC, oxygen, Solids,heat/cold level,Aqua level. The system also employed various actuators such as a fish food, fan, water heating, photo source, air pressure, and

Water filter. Beyond it's comprehensiveness of the a device which is used to measure the physical signals and actuator setup, what is also noteworthy What sets this study apart from other investigations into smart

aquariums is its focus on measuring late signal. Through meticulous sensing and actuation, this research ensures a reliable and statistically grounded assessment of late signal. Utilizing Erlang and gamma distribution measurements, it is demonstrated that system late signals pose no threat to



the wellbeing of the fish. An area for further research lies in integrating temperature forecasting to further bolster the system's late signal safety. Box 1 provides a comprehensive comparison of the reviewed literature, emphasizing the unique contribution offered by this system.

### C. WATER HEAT/COOL PREDICTION

Scientists used computer programs to predict things like the temperature of water. For example, they used a method called partial least squares regression to predict the East Asia Winter Monsoon. Another study used four different types of tree-based models to forecast solar energy. In Pakistan, they even used something called quantile regression forest to predict heatwaves..

Also, other researchers have used prediction methods in various areas like energy, economics, and electricity. They've used a type of advanced computer network called Enhanced Convolutional Neural Networks (ECNN) to predict electricity prices and usage. For heating and cooling systems in neighborhoods, they use a method called online ensemble decision tree-neural network (DCNN) learning to make forecasts. Decision tree classification is employed for copper price forecasting, a methodology also applicable in economic analysis. Electric power load forecasting utilizes reduced error pruning tree, a type of decision tree..

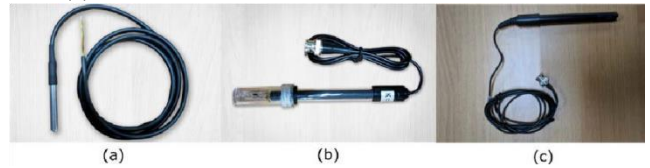
Several studies have employed machine learning for water temperature forecasting. Literature exists on utilizing the cloud model for this purpose, which outperforms RBF and SVM. In a prawn engineering culture pond, a mixed empirical mode-decomposition-back-propagation neural network method is employed. Urban water quality management forecasting utilizes a genetic algorithm-optimized long short-term memory approach. Additionally, Scientists are using a mix of support vector regression and a method inspired by fruit flies to guess how much water will be in rivers. They noticed that there's no special way to predict water levels just for fish farms, so they're trying to fix that.

### III. TESTING ENVIRONMENT

#### A. fish tanka device which is used to measure the physical signals S

This study uses three a device which is used to measure the physical signals s: a Waterproof Temperature a device which is used to measure the physical signals DS18B20, a TDS a device which is used to measure the physical signals , and a DFRobot DO a device which is used to measure the physical signals . The DS18B20 measures the water's temperature (shown in Pic 1a). The TDS a device which is used to measure the physical signals checks how clean the water is in the fish tank(shown in Pic 1b). The DFRobot DO a device which is used to measure the physical signals measures dissolved oxygen levels in the water.microcontrollers, is a low-power a device which is

used to measure the physical signals showcased in Pic 1 Part (c).



PIC 1. The water a device which is used to measure the physical signals s: (a) temperaturer (b) Solidsr (c)Oxygen.

#### B. FISH TANK DEVICES

For this study, we're using a few devices called actuators. The HB-100 Water Heater is made for aquariums that hold between 50 to 100 liters of water (shown in Pic 2a). The FS-120 Fan, , tailored for fish tankuse, measures  $172 \times 120 \times 120$  mm, operates at a frequency of 50/60 Hz, and consumes 15 W of power, shown in Pic 2 Part (b). A 5 V Relay is utilized to manage high-power devices with low-power microcontrollers. An RO filter cleans water in dirty aquariums, making it clearer and safer for fish. Automatic fish feeders, also known as feeders, are controlled by the system and dispense food when needed (shown in Pic 2c).



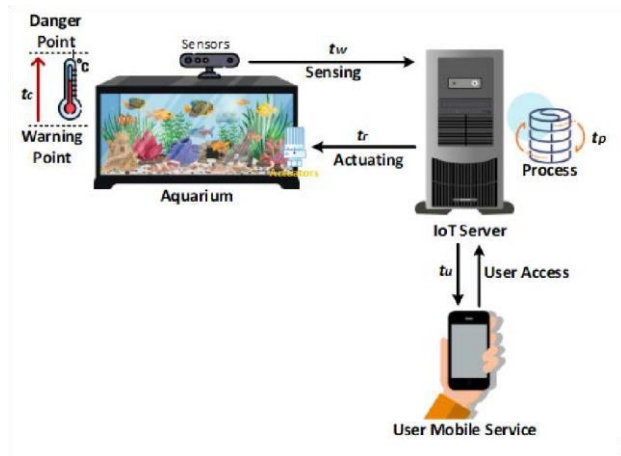
PIC 2. The aquaculture actuators: (a) Waterheater (b) Fan (c) Automatic fish feeder.

#### C. ENVIRONMENTAL THINGS CONNECTED WITH INTERNET

To achieved system adopts Internet of things Structure, comprising 3 layers the end device, cloud, & application. In this context, the fish tank serves as the final device, encompassing all a device which is used to measure the physical signals s, actuators, and relate signal components. Control of the fish tank is managed by a microcontroller, with the ESP8266 being the specific model utilized in this study. Communication between the final device device and both the cloud and final app is facilitated wirelessly through An MQTT broker is like a traffic controller for messages between devices. [29]. The Wi-Fi module integrated within the ESP8266 is responsible for wireless communication functionality.

A Python cloud is employed for this purpose, facilitating communication via MQTT between the cloud back end and the end device, the Cloud cloud receives temperature data, makes predictions, turns these suggestions into decisions, and then shares the results of those decisions. Prediction tasks are executed by a Cloud model, which undergoes training in accordance with established machine learning

principles. Data exchange with the cloud is mediated by an MQTT broker. The Application, operational on the user's smartphone, provides a user-friendly interface. An illustration depicting the entire system setup is presented in Pic 3 for reference..



PIC 3. The end device block diagram.

The device operates as following: Initially, the device which measure physical signal on the end device sends heat/cold data to the algorithm for process. Subsequently, both the cloud and the final app receive this information. The application presents heat/cold information as monitoring data. The cloud processes the received data, generating predictions that inform subsequent decision making. These decisions dictate which actuators should be activated and which should remain idle. The decisions are then relayed to cloud. The two final devices and the app receive this final decision. The end device adjusts the actuators based on the decision received, while the application displays the status of each actuator in accordance with the decision made.

#### D. DECISION TREE REGRESSION FORECASTING

The Decision Tree model emerges from the processing of training data. Among the well-known variants of Decision Trees, such as ID3, the training unfolds through several phases. Initially, the decision of each and every things is computed. Entropy represents measure of diverse within the data, with high values indicating greater diverse in it. The maximum Entropy value is 1. Subsequently, the information gain value is determined, revealing the feature that predominantly impacts the output. A higher Information Gain value signifies a more significant influence of features. Finally, the tree is structured based on the Information Gain and Entropy values of each feature.

The Decision Tree model undergoes testing using a separate dataset, known as testing data, which comprises collected data with corresponding outputs. It's crucial that the testing data differs from the training data. The performance of the Decision Tree model is assessed based on its accurate on generating result from this test data,

where accurate data is determined by done some comparison the model's result with the actual output of the testing data.

Decision Trees come in two forms: classification and regression. In the classification type, the decision tree typically takes event attributes as input. For example, it can predict someone's gender using details like weight,weight. Conversely, in cloud model, the input details typically includes sequences of events over time or continuous values.

In the suggested system, Decision Tree Regression (DTR) is employed to forecast upcoming temperature data, enhancing system responsiveness beyond traditional methods. DTR belongs to the category of supervised learning within machine learning, relying on labeled data for training. Real-time temperature data spanning approximately 24 hours is utilized for both training and testing purposes, with an 80% portion allocated for training and the remaining 20% for testing. The trained model is exportable for future applications. Algorithm 1 elucidates the Decision Tree Regression process. Within the algorithm, SSE represents the Sum of Squared Errors, calculate signald as the sum of squared differences between array members and their average value..

The IOT model trains & produces a model, which is then moved and put into the assumption medium. Here, the model takes in data as input. It processes this data to guess the temperature. The guessed temperature is then put into one of three groups: LOW, HIGH, or NEUTRAL. If it's LOW, the heater turns on. If it's HIGH, the fan turns on. NEUTRAL means neither turns on. These results are return to the final Device to control the temperature. They communicate using MQTT, where Subscribe gets data from the MQTT Broker, and Publish sends data to it. The whole process is detailed in Algorithm 2. Tests have been done using a temperature dataset collected over a week.

#### Algorithm 1: AI Modeling

**Data:** Dataset

**Result:** Regressor

Data: Temperature

Result: Decision

```

1 Procedure PredictionServer(Temperature)
2   Temperature = mqttt.Subscribe();
3   Prediction = DTRModel.Predict(Temperature);
4   if Prediction < MinTemperature then
5     Decision = "LOW";
6   elseif Prediction > MaxTemperature then
7     Decision = "HIGH";
8   else
9     Decision = "NEUTRAL";
10  mqttt.Publish(Decision);
    
```

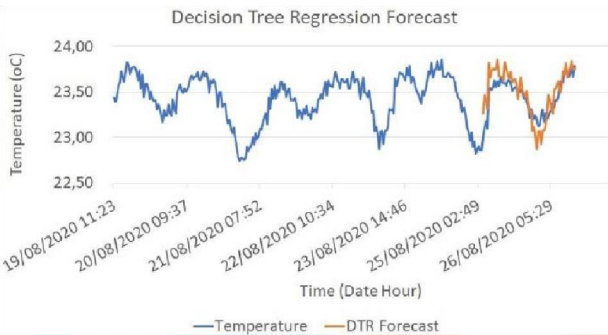


FIGURE 4. DTR forecast.

In our system, we look at how temperatures change over time, shown in Figure 4. We use the data we collect during tests to guess what the next temperature will be. Then, we compare our guesses with the real temperatures. We use three ways to check how accurate our guesses are: RMSE, MAPE, and R2. These help us see if our guesses are close to the real temperatures.

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \tilde{y}_i)^2} \quad (1)$$

$$MAPE = \frac{1}{n} \sum_{i=1}^n |PE_i| \quad (2)$$

FIGURE 5. R squared value.

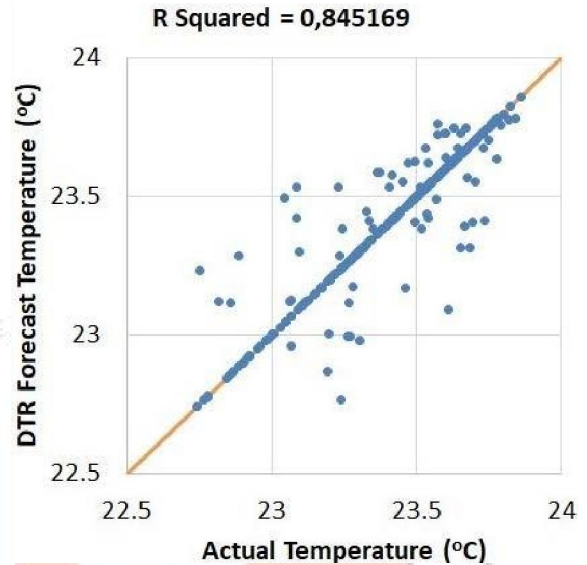
The RMSE and MAPE measurement results indicate that the difference between predictive data and real data is low—the closer to 0, the RMSE and MAPE value, the better. Meanwhile, the R<sup>2</sup> value is 84.52%, which shows the closeness of the prediction variable with the real value variable. It shows that the predicted value strongly correlate signals with the real value.

where n is number of data, PE<sub>i</sub> is the percentage of error of two variables, and

$$R^2 = \left( \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{(n \sum x^2 - (\sum x)^2)(n \sum y^2 - (\sum y)^2)}} \right)^2 \quad (3)$$

where n is number of dataset, x is first variable, and y is second variable in the context.

The calculation results obtained show the RMSE value of 0.15, MAPE value of 0.52% and R<sup>2</sup> of 84.52% which is shown in Figure 5.



E. APPLICATION

In the IoT architecture, users control applications. We used Android Studio for app development in our system. The app lets users check the fish tank and actuator status. Diagram 6 shows what the app looks like, which has two parts: the main what's happening on it and adjust things.

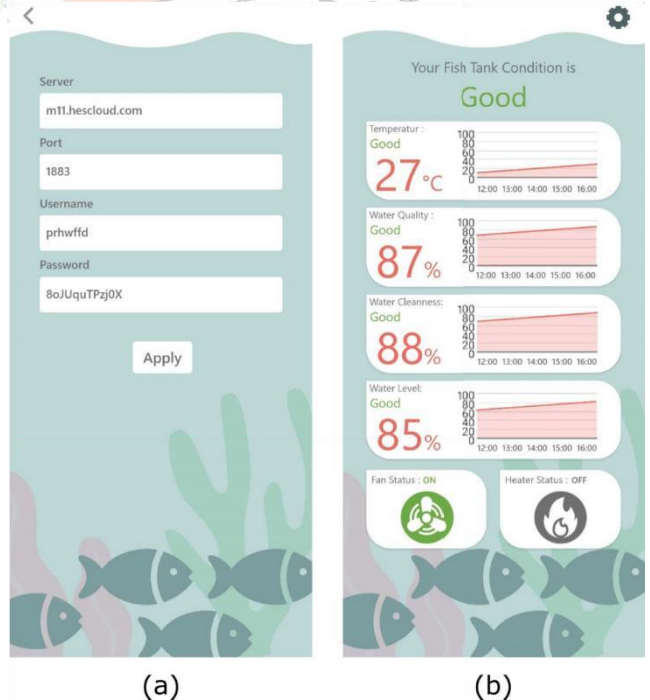


FIGURE 6. App front page. (a) Settings. (b) Maintaining.



The Monitor system UI shows fish tank values and actuator status. It tracks temperature, water quality, cleanliness, and water level, along with Fan and Heater statuses.

Each fish tank value has a status displayed. When all values are within the threshold range, "Good" text appears for each value. In the Monitoring UI, a big "Good" text is displayed at the top. If any value goes above a certain range, it changes to "Bad," and the big text switches to "Bad" too, and a notification is sent to the user. In the Settings, users can connect to the MQTT cloud by entering the cloud IP address, port number, username, and password..

The protection median is established with a username and password to securely access the Cloud server. All information retrieved from the application consists of the data published on the cloud server by either the end device or the cloud. Monitoring data is transmitted by the end device, while actuator states are sent by the cloud.

**F. TESTING**

Testing was conducted to assess the performance of the regression tree forecasting. Betta fish were introduced into the testing environment, where their water conditions served as the testing parameter. As betta fish thrive in water temperatures ranging from The temperature should stay between 25°C and 27°C, that's the rule. were tailored accordingly. The methodology unfolded as follows: the IoT system was established and activated, initiating data collection through a device which is used to measure the physical signals *s*, activating the computer network system for communication, and feeding the sensed data into the forecasting system. Subsequently, testing data was applied to the model, generating prediction data as output. This census data was used to decide what actions the actuators should take at the predicted time. yielding decision data as output. Performance evaluation ensued, with accuracy serving as the measurement parameter. Accuracy was determined by comparing Comparing forecasted decisions with actual decisions. resulting in a recorded accuracy of 99%..

**IV. CHECKING AND PERFORMING SOME EVALUATION**

**A. INTERNET SYSTEM late signals AND MEASUREMENT**

Some system underwent prior testing, employing an empirical approach with a connected fish tanksystem within the IoT infrastructure. MQTT protocol was utilized over a Wi-Fi network for this test. Findings indicate the system's capability to effectively monitor and manage the fish tankin accordance with user specifications. The cloud demonstrates prompt responsiveness to alterations in the fish tankenvironment, efficiently relaying instructions to the actuator as per cloud operations. Moreover, user-received information remains within the system acceptable variables..

In addition to practical experimentation, this Part delves into the creation of an analytical framework aimed at gauging the most severe potential system breakdown in the aquarium's monitoring and control, from the user's viewpoint. Various time-relate signald variables are employed in this scenario. These variables encompass the time taken for a device which is used to measure the physical signals data transmission to the cloud (*t<sub>w</sub>*), serving as a system alert to the user, and the cloud's processing duration (*t<sub>p</sub>*), denoting the time required for the cloud to interpret received information and formulate signal subsequent actions. Additionally, the timeframe for data transmission from the cloud to the actuator (*t<sub>r</sub>*) in response to cloud-received information is scrutinized. Further, the interval between data transmission from the cloud to the user (*t<sub>u</sub>*) and the time lapse between the warning and critical points (*t<sub>c</sub>*) are examined. These timeframes are pivotal in understanding the duration required for the aquarium's water temperature to escalate signal from the initial warning stage to a critical state.

Figure 7 illustrates this process

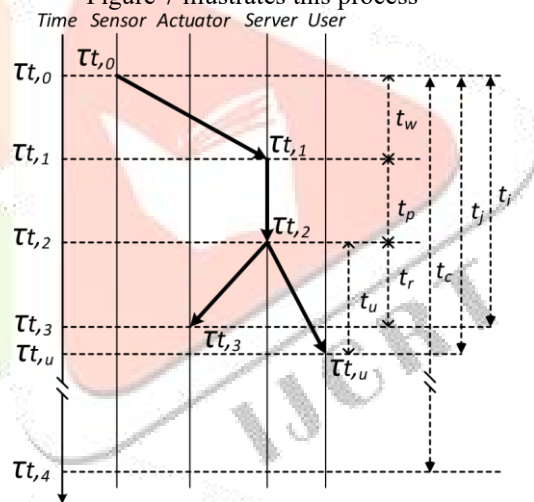


FIGURE 7. Time diagram.

The time picture shows how information flows a device which is used to measure the physical signals to both the actuator and the user. When the a device which is used to measure the physical signals first detects a problem (like a sudden rise in temperature from a malfunctioning heater), it sends a alert to the cloud. The cloud gets this warning and processes it, then decides what to do and sends messages to both the actuator and the user. The actuator is activated at ( $\tau t,3$ ), for instance, shutting down the heater. The time interval from the system's detection of a potential critical state ( $\tau t,0$ ) to the activation of the actuator ( $\tau t,3$ ) is denoted as *t<sub>i</sub>*. Concurrently, the cloud transmits information to the users, reaching them by  $\tau t,u$ . The duration from the system's detection of a potential critical state ( $\tau t,0$ ) to the user application receiving the information ( $\tau t,u$ ) is represented as

tj. Meanwhile, the aquarium's condition continues to approach critical levels until reaching  $\tau_4$ , at which point critical thresholds are exceeded. This triggers controlled measures, enabling the system to restore normal conditions before criticality ensues.

Analytical models delve into the most adverse scenarios concerning IoT network connections and their impact on fish tank monitoring systems. These models are constructed by examining empirical late signal data generated during communication processes among IoT system entities. The late signal data is estimated using a Probability Density Function, which is checked for accuracy with the Kolmogorov-Smirnov normality test. The final model is based on real testing results and empirical data. This model holds significance in ensuring the smooth operation of the system, as it accounts for potential system failures stemming from message late signals and packet loss between IoT entities. Theoretical validation proves more dependable than merely relying on empirical data derived from laboratory tests. By computing these worst-case scenarios, the system instills a certain level of confidence in overcoming challenges within the IoT communication network. The focal point of concern lies in unregulated signal late signals stemming from network congestion or malfunctioning functions, leading to packet loss and hindering proper information transmission among IoT entities.

This assessment focuses on two important areas: how quickly the actuator responds and how fast information reaches the user compared to when critical conditions occur. We use certain key measurements to judge this, like how long it takes for data to go from the a device which is used to measure the physical signals to the cloud ( $t_w$ ), how long the cloud takes to process it ( $t_p$ ), how long it takes for the cloud to tell the actuator what to do ( $t_r$ ), and how long it takes for the cloud to tell the user ( $t_u$ ). Unfortunately, previous studies didn't look at how long it takes for the cloud to process data or for information to get to the user, even though they're really important. But luckily, in our research, we've looked at all the late signal factors, including how long the cloud takes to process data and how long it takes for messages to reach the user from the cloud.

$$\tau_{t,3} = t_w + t_p + t_r \quad (4)$$

and

$$\tau_{t,u} = t_w + t_p + t_u \quad (5)$$

The assessing procedure presupposes the activity of the cloud, engaged in multiple concurrent tasks. The cloud's workload is crucial to emulate signal a realistic operational environment, accounting for various ongoing tasks. The time it takes to process data is important because it affects how long everything takes, which the previous system didn't consider. So, our evaluation looks at what happens

when things aren't going well. such as network disruptions leading to prolonged data transmission late signals. Through analysis, this study verifies that even under such unfavorable circumstances, the system can maintain effective control processes, ensuring satisfactory performance despite challenges.

The next scenario concerns the dissemination of information to users across varied locations. In specific circumstances, users may find themselves in complex situations where they lack the time to monitor the fish tanksystem actively. Consequently, it becomes imperative for the system to relay this information to the user. An advantageous feature of our developed system, not previously addressed in literature, is its implementation of a push information mechanism for users. Various time elements must be taken into account: the late signal in a device which is used to measure the physical signals - to cloud data transmission ( $t_w$ ), cloud processing time ( $t_p$ ), and the late signal in information reaching the user from the cloud ( $t_u$ ). The collective time span is juxtaposed with the likelihood of an fish tank encountering a hard times. Assessing this situation encompasses both normal network conditions and scenarios of poor network connectivity.

#### B. ASSESING THE EFFECT OF late signal RECEPTION IN SERVER

Here's the check of the test output for late signals in send of data across the IoT network from one device to another. For instance,  $t_w$  represents the late signaled in transmitting data from the a device which is used to measure the physical signals to the cloud, while  $t_p$  is the process late signal.

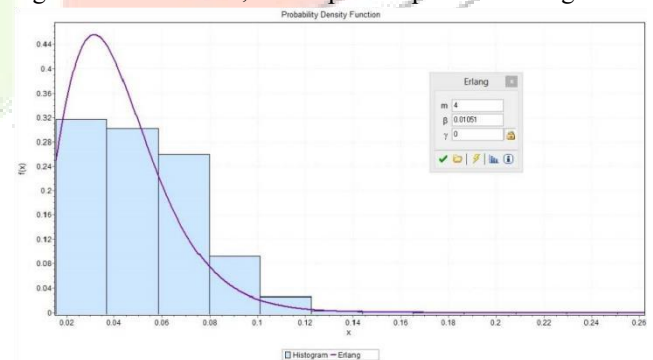


FIGURE 8. The late signal  $t_p$  graph.

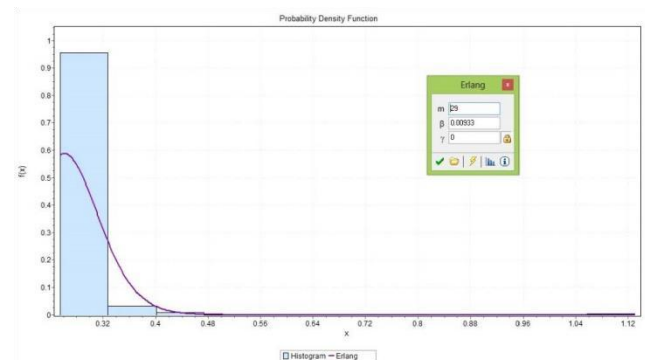


FIGURE 9. The late signal  $t_r$  graph.



We've measured four types of late signals:  $t_w$  for a device which is used to measure the physical signals -to-cloud,  $t_p$  for cloud processing,  $t_r$  for cloud-to-actuator, and  $t_u$  for cloud-to-user. Each of these late signals is represented by some alphabets with some functions  $f_w(t_w)$ ,  $f_p(t_p)$ ,  $f_r(t_r)$ , and  $f_u(t_u)$ . We've collected data from 3000 measurements for  $t_w$ , 3041 for  $t_p$ , 3000 for  $t_r$ , and 3022 for  $t_u$ , all relate signal to Wi-Fi transmission late signals. From these measurements, we've calculate signal the expected values and variances using Erlang distributions. For example, the expected value for  $t_w$  is 0.27704 ms, with a variance of 0.00135. Similarly, the expected value for  $t_p$  is 0.05223 ms, with a variance of 0.0005491. The expected value for  $t_r$  is 0.28036 ms, with a variance of 0.00131, and for  $t_u$ , it's also 0.28036 ms, with a variance of... 00131. The histogram for each PDF with an  $m$  and the scale parameter  $\beta$  is formulate signald as follows.

$$f_E(t, m, \beta) = \frac{\beta^m t^{m-1} e^{-\beta t}}{(m-1)!} \quad (6)$$

and

$$\int_0^\infty f_E(t, m, \beta) dt = 1 \quad (7) \quad \tau=0 \quad k=0 \quad k!$$

where  $E[t] = \frac{m}{\beta}$  and  $V[t] = \frac{m}{\beta^2}$

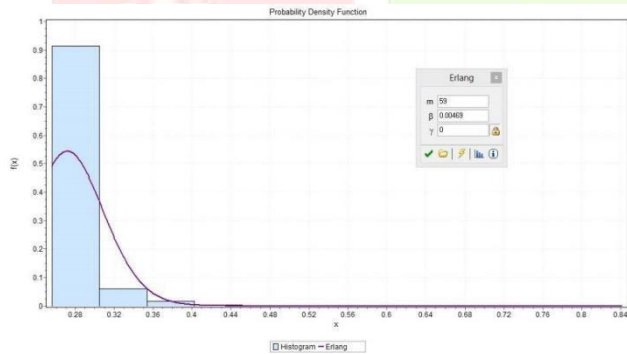


FIGURE 10. The late signal  $t_u$  graph.

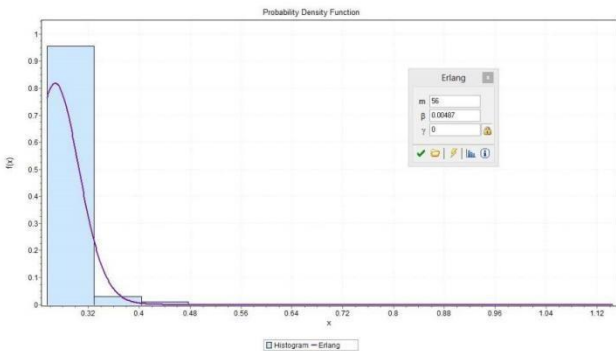


FIGURE 11. The late signal  $t_w$  histogram.

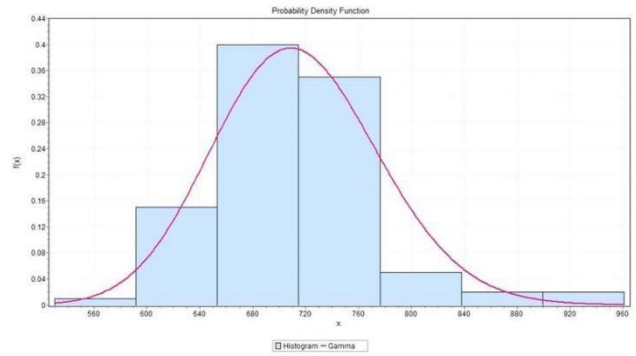


FIGURE 12. The late signal  $t_c$  graph.

From derivation (6), we can estimate  $f_w(tN,w)$  using  $f_E(tN,w,mN,w,\beta N,w)$ , where the variable is  $mN,w = 56$  and the scale variable is  $\beta N,w$ .

As discussed earlier, the monitoring and control of the fish tanksystem involve four time units:  $t_w$ ,  $t_p$ ,  $t_r$ , and  $t_u$ . For instance, if the water temperature gradually rises due to factors like heater activation or external heat, reaching a critical temperature, say  $25^\circ\text{C}$ , detrimental to the fish, timely information transmission becomes crucial. At  $24^\circ\text{C}$ , the a device which is used to measure the physical signals detects the approaching critical conditions  $(\tau,0)$ , signaling the cloud with a Travel time is the duration it takes to journey from one location to another.  $t_w$ . Upon cloud reception at  $\tau,1$ , data processing takes  $t_p$  time, followed by a response sent at  $\tau,2$ . Subsequently, the output attained the fish tank within  $t_r$ , and the actuator at  $\tau,3$ . Concurrently, the cloud notifies the user via a mobile app withThe time it takes to reach the critical point,  $t_c$ , from the initial detection at  $\tau,0$  to the actuator at  $\tau,4$ , must be longer than the sum of the a device which is used to measure the physical signals -to-cloud late signal ( $t_w$ ), cloud processing time ( $t_p$ ), and cloud-to-actuator late signal ( $t_r$ ), or  $\tau,4 > \tau,3$  and the cloud-to-user late signal ( $t_u$ ), or  $\tau,u$ . This dual control system, activating the actuator and alerting the user, ensures effective fish tankcondition management. Empirical testing of temperature rise time to critical points yielded 100 data points, represented by a histogram (Figure 12), with a measured value  $t_c^*$ . Approximating  $t_c^*$  with a Gamma distribution revealed  $E[t_c^*] = 712.14s$  and  $V[t_c^*] = 3.902$ , yielding shape parameter  $\alpha = 130.69$  and scale parameter  $\hat{\mu} = 5.464$ . The PDF curve estimate was validated through the Kolmogorov-Smirnov suitability test. It's important to keep the system under control before it reaches a critical point that could harm the fish. We call this critical time  $t_c$ . The time it takes to get information and activate the actuator is  $t_i$ , and the time to inform the user is  $t_j$ . We need a model to figure out how likely it is to reach this critical point.A lower probability indicates a more favorable system. Thus, to avert the system from entering a hazardous state for the fish, it is imperative to guarantee that  $\tau,4 > \tau,3$  or  $t_c > t_i$ , and  $\tau,4 > \tau,u$  or  $t_c > t_j$ . Further details are provided in

Appendix B.

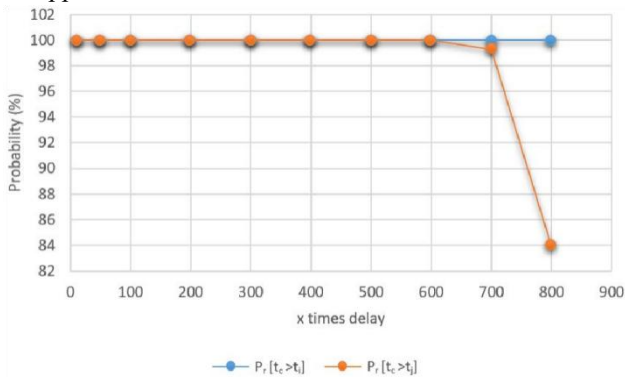


FIGURE 13. Impact of network late signal to probability of Pr[tc > ti] and Pr[tc > tj].

$$Pr[tc > ti] = 1 - \sum_{k=0}^{+m_p+m_r-1} \binom{\alpha+k-1}{k} \left[ \frac{\beta^k \mu^\alpha}{(\beta+\mu)^{\alpha+k}} \right]$$

$$Pr[tc > tj] = 1 - \sum_{k=0}^{+m_p+m_u-1} \binom{\alpha+k-1}{k} \left[ \frac{\beta^k \mu^\alpha}{(\beta+\mu)^{\alpha+k}} \right]$$

In verifying the developed analytical model, testing involved sample cutted data to validate the both equations. Then numerous calculations involving time variables tC, ti, and tj, it was concluded that the probability is exceedingly small, less than 0.01%. This empirical evidence serves as a fitting confirmation of the developed analytical model within the test environment, affirming the system's effectiveness in monitoring and controlling the fish tank within an IoT context. Subsequently, the next phase of testing aims to anticipate adverse network conditions, assuming a significant increase in late signal with expected values of E[ti] and E[tj], along with their respective variances V[ti] and V[tj]. By substantially augmenting late signal values, as depicted in Figure 13, the test results indicate that the new system remains resilient even when subjected to an 800-fold increase in late signal, with probabilities Pr[tc > ti] = 99.9% and Pr[tc > tj] = 84.025%.

This underscores the system's robustness, demonstrating its ability to maintain control despite deteriorating network conditions marked by substantial message delivery late signals within the IoT framework.

Thus far, this Part has provided empirical and mathematical demonstrations of system testing. Both methodologies consistently confirm the system's ability to effectively monitor and control the fish tank through the IoT network, even under adverse network conditions. Despite

the identified accomplishments, several limitations are evident. Primarily, the system is tailored for a specific fish tank size and environmental temperature range. Moreover, external air conditions exert an influence on system performance.

Our examinations have confirmed that the suggested system operates with a discernible late signal, yet opportunities for enhancement remain evident. The forefront concepts introduced include edge computing, a paradigm within cloud-enabled IoT frameworks wherein intelligent processing migrates from centralized cloud clouds to local end devices. This notion necessitates the development of streamlined intelligent model solutions. Numerous investigations into the quantization of machine learning models have already been conducted and can be leveraged for adoption.

V. CONCLUSION

Our study introduces a new system that uses technology to make aquariums better. It predicts changes in the environment and can handle late signals. It uses a special algorithm called decision tree regression (DTR) to make predictions. The system has different parts like a device which is used to measure the physical signals s for temperature, TDS, and dissolved oxygen, as well as a heater, a fan, e-relay, and a purifier. It also has a computer cloud and an app for your phone. They talk to each other using something called MQTT. Our tests show that the system is good at sending data from a device which is used to measure the physical signals s to the cloud, processing it, and then making things happen with the actuators. It's also good at telling users when something's wrong. We made a plan to make sure data moves smoothly between all the parts of the system, even if there are big late signals in the network. We collected a lot of data and found that the system is very unlikely to have a problem, even if there are big late signals. Even when there are late signals, the system still works well, especially in places where the temperature and water level are stable. In the end, our research gives us a strong system for controlling aquariums using technology, and it helps us understand how clouds work, which hasn't been studied much before.

APPENDIX A

$$Pr[tc > t_w + t_p + t_r]$$

$$\int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty f_C(t_C) f_C(t_C) f_C(t_C) f_C(t_C) f_C(t_C)$$

$$= \int_{t_C=0}^\infty \int_{t_r=0}^\infty \int_{t_p=0}^\infty \int_{t_w=0}^\infty f_C(t_C)$$

$$\times f(E)(t_r, m_r, \beta_r) \times f(E)(t_p, m_p, \beta_p)$$

$$\times f(E)(t_w, m_w, \beta_w) dt_w dt_p dt_r dt_C \tag{8}$$

In equation (8), if  $\beta_w = \beta_p = \beta_r$ , it is assumed that  $\beta_r > \beta_p > \beta_w$ , we have

$$\int_0^\infty \int_0^\infty f(E)(t_w, m_w, \beta_w) dt_w$$

$t_w=0$

$i$

$= 1 -$

$$\frac{\beta_w^k (t_C - t_p)^k e^{-\beta_w (t_C - t_p)}}{k!}$$

$$\sum_{k=0}^{m_w-1} \left[ \dots \right]$$

$$= 1 - \sum_{k=0}^{m_w-1} \left[ \frac{\beta_w^k (t_C - t_p)^k e^{-\beta_w (t_C - t_p)}}{k!} \right] \sum_{i=0}^k \binom{k}{i} \dots$$

$$= 1 - \sum_{k=0}^{m_w-1} \sum_{i=0}^k \binom{k}{i} \frac{\beta_w^k (t_C - t_p)^k e^{-\beta_w (t_C - t_p)}}{k!} \dots \tag{9}$$

Substitute (8) and (9) in above

$P_r[t_C > t_w + t_p + t_r]$

$Z \infty Z_{t_C} Z_{t_C - t_r}$

$= \int_{t_C=0}^{\infty} \int_{t_r=0}^t \int_{t_p=0}^{t-t_r} f_C(t_C) f(E)(t_r, m_r, \theta_r) \dots$

$\times f(E)(t_p, m_p, \theta_p) dt_p dt_r dt_C$

$Z \infty Z_{t_C} Z_{t_C - t_r}$

$- \int_{t_C=0}^{\infty} \int_{t_r=0}^t \int_{t_p=0}^{t-t_r} f_C(t_C) f(E)(t_r, m_r, \theta_r) \dots$

$\times f(E)(t_p, m_p, \theta_p)$

$$\times \left[ \sum_{k=0}^{m_w-1} \frac{\beta_w^k}{k!} \sum_{i=0}^k \binom{k}{i} t_C^{k-i} e^{-\beta_w t_C} (-t_p)^i e^{\beta_w t_p} \right] \tag{10}$$

$P_r[t_C > t_w + t_p + t_r] = M - N$

Equation formula M

$Z \infty Z_{t_C} Z_{t_C - t_r}$

$$M = \int_{t_C=0}^{\infty} \int_{t_r=0}^t f_C(t_C) f(E)(t_r, m_r, \theta_r) \dots \tag{11}$$

From equation (11), if  $\theta_p = \beta_w$  and it is assumed that  $\theta_p >$

$\beta_w$ , then  $Z_{t_C - t_r} f(E)(t_p, m_p, \theta_p) dt_p$

$t_p=0$

$$= 1 - \sum_{k=0}^{m_p-1} \frac{\beta_p^k (t_C - t_r)^k e^{-\beta_p (t_C - t_r)}}{k!} \dots$$

$$= 1 - \sum_{k=0}^{m_p-1} \frac{\beta_p^k e^{-\beta_p (t_C - t_r)}}{j!} \sum_{h=0}^k \binom{k}{h} \dots$$

1

$$= 1 - \sum_{k=1}^{m_p} \left( \frac{\beta_p^k}{k!} \right) \sum_{i=0}^k \binom{k}{i} \dots \tag{12}$$

Substitute (12) and (11)

$Z \infty Z_{t_C}$

$$= \int_{t_C=0}^{\infty} \int_{t_r=0}^t \dots \sum_{k=0}^{m_p-1} \left( \frac{\beta_p^k}{k!} \right) \sum_{i=0}^k \binom{k}{i} \dots$$

$\times f_C(t_C) f(E)(t_r, m_r, \theta_r)$

(13)

$M = P - Q$

with

$\int_{t_C=0}^{\infty} \int_{t_r=0}^t f_C(t_C) f(E)(t_r, m_r, \theta_r) \dots$

$$P = \int_{t_C=0}^{\infty} \int_{t_r=0}^t f_C(t_C) f(E)(t_r, m_r, \theta_r) \dots$$

$Z \infty Z_{t_C}$

$$P = \int_{t_C=0}^{\infty} \int_{t_r=0}^t f_C(t_C) f(E)(t_r, m_r, \theta_r) dt_r dt_C \tag{14}$$

and

Q is the remainder.

$$\left( \frac{\beta_r^k}{k!} \right) \int_{t_C=0}^{\infty} \dots f_C(t_C) e^{-\beta_r t_C} dt_C \tag{15}$$

$$Z \infty = (-1)^k \left[ \frac{d^{(k)} t_C(x)}{d x^k} \right] f^k(t) e^{x t} dt \tag{16}$$



$$= 1 - \sum_{k=0}^{\infty} \left[ \frac{(-1)^k}{k!} \left( \frac{d}{dx} \right)^k f_C(x) \right]_{x=\beta_r} \quad (17)$$

$$Q = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} f_C(x) \Big|_{x=\beta_r}$$

$$= \int_{t_C=0}^{\infty} P \int_{t_r=0}^{t_C=0} f_C(t) f(E)(t_r, m_r, \beta_r) \left[ \sum_{k=0}^{m_p-1} \frac{(\beta_p^k)}{k!} \sum_{i=0}^k \binom{k}{i} t_C^{k-i} e^{-\beta_p t_C} (-t_r)^i e^{\beta_p t_r} \right] dt_r dt_C$$

$$= \sum_{k=0}^p \binom{k}{i} \sum_{i=0}^{m-1} \int_0^{\infty} Q_{m-1-k} f_C(t_C)$$

$$\times \left[ \frac{\beta_r^{m_r} (m_r + i - 1)!}{(\beta_r - \beta_p)^{m_r + i} (m_r - 1)!} \right] \times \left[ 1 - \sum_{l=0}^{m_r+i-1} \frac{(\beta_r - \beta_p)^l t_C^l e^{-(\beta_r - \beta_p)t_C}}{l!} \right] dt_C$$

$$= \sum_{k=0}^{m_p} \binom{k}{i} \sum_{i=0}^k Q_{m-1-k} (-1)^i \left[ \frac{\beta_r^{m_r} (m_r + i - 1)!}{(\beta_r - \beta_p)^{m_r + i} (m_r - 1)!} \right] \times (R - S) \quad (20)$$

with

$$= \int_{t_C=0}^{\infty}$$

X Q

$$t_C k - i e^{-\beta_p t_C} (-t_r)^i e^{\beta_p t_r} dt_r dt_C$$

$$(-1)^i \sum_{i=0}^k \binom{k}{i} (-1)^i \int_0^{\infty}$$

$$\times f(E)(t_r, m_r, \beta_r) \quad t_r=0 \text{ and } m_p$$

$$= \sum_{k=0}^p \sum_{i=0}^k \int_{t_C=0}^{\infty} f_C(t_C) t_C^{m+i-1} e^{-\beta_p t_C} dt_C$$

$$R \int_{t_C=0}^{\infty} f_C(t_C) t_C^{m+i-1} e^{-\beta_p t_C} dt_C \quad (21)$$

$$S = \sum_{l=0}^{\infty} \frac{(\beta_r - \beta_p)^l t_C^l e^{-(\beta_r - \beta_p)t_C}}{l!}$$

$$= \int_{t_r=0}^{t_c} h \frac{\beta_r^{m_r} t_r^{m_r+i-1} e^{-\beta_r t_r}}{(m_r-1)!} i^i e^{\beta_p t_r} dt_r$$

$$= \int_{t_r=0}^{t_c} h \frac{\beta_r^{m_r} t_r^{m_r+i-1} e^{-(\beta_r-\beta_p)t_r}}{(m_r-1)!} i^i dt_r$$

$$f(E)(t_r, m_r, \beta_r) t_r i e^{\beta_p t_r} dt_r dt_c$$

Based on (18) it is obtained

$$f(E)(t_r, m_r, \beta_r) t_r i e^{\beta_p t_r} dt_r$$

$$\times \int_{t_r=0}^{t_c} \left[ \frac{(\beta_r - \beta_p)^{m_r+i} t_r^{m_r+i-1} e^{-(\beta_r - \beta_p)t_r}}{(m_r+i-1)!} \right] dt_r$$

$$= \left[ \frac{\beta_r^{m_r} (m_r+i+1)!}{(\beta_r - \beta_p)^{m_r+i} (m_r-1)!} \right]$$

$$\times \left[ 1 - \sum_{l=0}^{m_r+i-1} \frac{(\beta_r - \beta_p)^l t_c^l e^{-(\beta_r - \beta_p)t_c}}{l!} \right] \quad (19)$$

Equation (21) in a different form

$$R = \int_{t_c=0}^{\infty} f_c(t_c) t_c^{k-i} e^{-\beta_p t_c} dt_c$$

$$= (-1)^{k-i} (23) \left[ \frac{d^{k-i} f_c}{dx^{k-i}} \right] \Big|_{x=\beta_p}$$

Equation (22)

$$= \sum_{l=0}^{m_r} \left[ \frac{(\beta_r - \beta_p)^l}{l!} \right] (-1)^{k+l-i} \left[ \frac{d^{k+l-i} f_c}{dx^{k+l-i}} \right] \Big|_{x=\beta_p}$$

Based on equations (20), (23) and (24),

$$= \sum_{m=1}^p \left[ \frac{(-\beta_p)^k}{k!} \right] \sum_{i=0}^k \binom{k}{i} \left[ \frac{\beta_r (m_r+i-1)!}{(\beta_r - \beta_p)^{m_r+i} (m_r-1)!} \right]$$



$$\begin{aligned}
 & \left\{ \left[ \frac{d^{(k-i)} f_C^\alpha(x)}{d_x^{(k-i)}} \right] \Big|_{x=\beta} \right\}_{i=0}^{k=0} \times \dots \times \left[ \frac{(\beta_p - \beta_r)^l}{l!} \right]_{l=0}^{i-1} \left[ \frac{d^{(k+l-i)} f_C^\alpha(x)}{d_x^{(k+l-i)}} \right] \Big|_{x=\beta} \\
 & = \dots \times \left[ \frac{(-\beta_p) \beta_r}{+} \right]_{i=0}^{i-1} \dots \times \left\{ \left[ \frac{d^{k-i} f_C^\alpha(x)}{d_x^{(k-i)}} \right] \Big|_{x=\beta} \right\}_{i=0}^{r+i-1} \times \dots \times \sum_{l=0}^{r+i-1} \left[ \dots \right] \left[ \frac{(\beta_p - \beta_r)^m}{m!} \right] \left[ \frac{d^{(k+l-i)} f_C^\alpha(x)}{d_x^{(k+l-i)}} \right] \Big|_{x=\beta} \\
 & \text{Based on equations (13), (17) and (25), we obtain} \\
 & \text{Substitute (19) and (18)} \\
 & M = P - Q \quad Z_{tC-tr} \\
 & = P_r [tC > tr + tp]
 \end{aligned}$$

Based on equations (13), (17) and (25), we obtain  
Substitute (19) and (18)



$$\begin{aligned}
 &= 1 - \sum_{k=0}^{m_r-1} \left[ \frac{(-\beta_r)^k}{k!} \right] \left[ \frac{d^{(k)} f_C^\alpha(x)}{d_x^k} \right] \Big|_{x=\beta} \\
 &= \sum_{t_C=0}^{m_w-1} \sum_{i=0}^k (-1)^i \int_{t_C=0}^{t_C} \int_{t_r}^{m_w-1} \frac{m_w-1}{k} \dots \\
 &\quad \times \sum_{k=0}^{m_r} \frac{N f_C(t_C)}{(\beta_r - \beta_p)^{m_r} i^{(k-i)}} \times f(E)(t_r, m_r, \beta_r) t_C k - i e^{-\beta_w t_C} \\
 &\quad \times \int_{t_C=0}^{t_C-t_r} f(E)(t_p, m_p, \beta_p) t_p^i e^{\beta_w t_p} dt_p dt_r dt_C \quad (27) \\
 &\quad \times \sum_{t_C=0}^{t_C-t_r} f(E)(t_p, m_p, \beta_p) t_p^i e^{\beta_w t_p} dt_p t_p=0 \\
 &\quad = \int_{t_p=0}^{t_C-t_r} \left[ \frac{\beta_p^{m_p} t_p^{m_p-1} e^{-\beta_p t_p}}{(m_p-1)!} \right] t_p^i e^{\beta_w t_p} dt_p \\
 &\quad = \int_{t_p=0}^{t_C-t_r} \left[ \frac{\beta_p^{m_p} (m_p+i-1)!}{(\beta_p - \beta_w)^{m_p+i} (m_p-i)!} \right] \frac{dt_p}{(m_p-1)!} \\
 &\quad \times \int_{t_p=0}^{t_C-t_r} \left[ \frac{(\beta_p - \beta_w)^{m_p+i} t_p^{m_p+i-1} e^{-(\beta_p - \beta_w) t_p}}{(m_p+i-1)!} \right] dt_p t_p=0
 \end{aligned}$$

From equation (10), we obtain

$$\begin{aligned}
 P_r[t_C > t_w + t_p + t_r] &= M - N \\
 N &= \int_{t_C=0}^{\infty} \int_{t_r=0}^{t_C} \int_{t_p=0}^{t_C-t_r} f_C(t_C) f(E)(t_r, m_r, \beta_r) \\
 &\quad \times f(E)(t_p, m_p, \beta_p) \\
 &\quad \times \left[ \sum_{k=0}^{m_w-1} \left( \frac{\beta_w^k}{k!} \right) \sum_{i=0}^k \binom{k}{i} t_C k - i e^{-\beta_w t_C} (-t_p)^i e^{\beta_w t_p} \right] dt_p dt_r dt_C
 \end{aligned}$$

$$\begin{aligned}
 &= \int_{t_p=0}^{t_C-t_r} \left[ \frac{\beta_p^{m_p} (m_p+i-1)!}{(\beta_p - \beta_w)^{m_p+i} (m_p-i)!} \right] \frac{dt_p}{(m_p-1)!} \\
 &\quad \times \int_{t_p=0}^{t_C-t_r} \left[ \frac{(\beta_p - \beta_w)^{m_p+i} t_p^{m_p+i-1} e^{-(\beta_p - \beta_w) t_p}}{(m_p+i-1)!} \right] dt_p t_p=0
 \end{aligned}$$

for example

$$\begin{aligned}
 + i - 1) &= \left[ \frac{\beta_p^{m_p} (m_p+i-1)!}{(\beta_p - \beta_w)^{m_p+i} (m_p-i)!} \right] \frac{1}{dx^k} \left[ \frac{d^{(k-i)} f_C^\alpha(x)}{d_x^k} \right] \\
 &\left[ 1 - \sum_{g=0}^{m_p+i-1} \frac{(\beta_p - \beta_w)^g (t_C - t_r)^g e^{-(\beta_p - \beta_w)(t_C - t_r)}}{g!} \right]
 \end{aligned}$$

combined with (27)

$$= \sum_{k=0}^{\infty} \left(\frac{\beta_w}{k!}\right) \sum_{i=0}^k (-1)^i \int_{t_r}^{m_w-1} \int_{t_c}^{m_w-1} f_c(t_c) dt_c \times \dots$$

$$= \frac{\beta_r^{m_r} (m_r+i-1)!}{(\beta_r-\beta_p)^{m_r+i} (m_r-1)!} \times \left[ 1 - \sum_{l=0}^{m_r+i-1} \frac{(\beta_r-\beta_p)^l t_c^l e^{-(\beta_r-\beta_p)t_c}}{l!} \right] \quad (29)$$

Obtaining equation N

$$= \sum_{k=0}^{m_w} \left(\frac{\beta_w^k}{k!}\right) \sum_{i=0}^k (-1)^i \int_{t_c}^{m_w-1} \left[ \frac{d}{dx^k} f_c(x) \right]_{x=\beta_r} \times \dots$$

$$\times \left[ \frac{\beta_r^{m_r} (m_r+i-1)!}{(\beta_r-\beta_p)^{m_r+i} (m_r-1)!} \right] \times \left[ 1 - \sum_{g=0}^{m_p+i-1} \frac{(\beta_p-\beta_w)^g (t_c-t_r)^g e^{-(\beta_p-\beta_w)(t_c-t_r)}}{g!} \right]$$

$$\times \int_{t_c}^{\infty} f_c(t_c) dt_c \times \dots$$

$$\times \left[ 1 - \sum_{g=0}^{m_p+i-1} \frac{(\beta_p-\beta_w)^g (t_c-t_r)^g e^{-(\beta_p-\beta_w)(t_c-t_r)}}{g!} \right]$$

$$\times \sum_{k=0}^{m_w-1} \left(\frac{\beta_w^k}{k!}\right) \sum_{i=0}^k (-1)^i \times T \times O$$

$$\times \left[ \frac{\beta_p^{m_p} (m_p+i-1)!}{(\beta_p-\beta_w)^{m_p+i} (m_p-1)!} \right] \times \left[ 1 - \sum_{g=0}^{m_p+i-1} \frac{(\beta_p-\beta_w)^g (t_c-t_r)^g e^{-(\beta_p-\beta_w)(t_c-t_r)}}{g!} \right]$$

$$\times \int_{t_c}^{\infty} f_c(t_c) dt_c \times \dots$$

