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A Study Of The Development And Applications Of Number Theory

Kirtika
Assistant Professor
Jagannath University, Jaipur, Rajasthan

Abstract. This research primarily examines the evolution and practical uses of number theory. The focus is on tracing the historical development of this field and investigating its impact on various aspects of production and daily life. Originally centered on the study of integers, number theory has seen continual enhancement over time, thanks to the contributions of mathematicians from different eras. This progress has led to the establishment of a comprehensive and unified discipline. Serving as a fundamental field, number theory has far-reaching effects on other disciplines, serving as the cornerstone for many areas of study.

1. An Overview of Number Theory

The Concept of Number Theory

The notion of number theory is straightforward—it is a theory centered around numbers. This concept dates back over 3,000 years when the understanding of numbers and arithmetic first emerged. Initially referred to as arithmetic, the term evolved to "number theory" in the early twentieth century. Number theory, belonging to the realm of mathematics, serves as the cornerstone for various scientific and engineering disciplines. Essentially, within the broader spectrum of mathematics, number theory holds a pivotal position. Number theory primarily deals with the nature of integers. Questions in number theory are concise, and the key to solve these questions is unique factor decomposition. In addition, in the process of reconstructing the unique factorization, some new concepts, such as complex integers, ideal numbers and ideals, are introduced, which also provide new research methods for number theory.

The Subdivisions of Number Theory

Number theory can be categorized into several subdivisions, with elementary number theory, algebraic number theory, geometric number theory, and analytic number theory being among the more significant ones. Additionally, there are popular subdivisions like transcendental number theory and combinatorial number theory. The specific focuses and distinctions of these subdivisions are outlined in the following table.

Table 1 The focuses of these subdivisions and their differences

Subdivision	Explanation
Elementary number theory	Elementary number theory is a fundamental branch of number theory that relies on basic methods. Its essence lies in the application of properties related to divisibility, primarily focusing on divisible theory and congruence theory. Key results within this theory encompass well-known theorems such as the congruence theorem, Euler's theorem, the Chinese remainder theorem, and others.
Analytic number theory	Analytic number theory is a field that explores integers through the lens of calculus and complex analysis. It involves the use of analytic functions, such as the Riemann function ζ, to investigate the properties of integers and primes. These analytic tools provide a valuable perspective for understanding various aspects of number theory.
Algebraic number theory	Algebraic number theory is primarily focused on examining the characteristics of different rings of integers through an algebraic structural lens. It delves into the nature and properties of these integer rings from an algebraic standpoint.
Geometric number theory	Geometric number theory is a branch that investigates the distribution of integers through a geometric lens. It explores the spatial relationships and arrangements of integers using geometric principles and methods.
Computational number theory	Computational number theory is a field dedicated to addressing questions and problems within number theory by employing computer algorithms. It leverages computational methods to analyze, explore, and solve various issues and conjectures in the realm of number theory.

The Significance of Number Theory

[1] For an extended period, number theory primarily showcased fundamental mathematical properties and was often considered a discipline without immediate practical application value. However, with the significant scientific and technological advancements ushered in by the advent and growth of computers, number theory has found extensive and diverse applications. It has transcended its status as pure mathematics to become a mathematical discipline with tangible practical value. Presently, number theory is extensively and comprehensively employed in various fields, including computing, cryptography, physics, chemistry, biology, acoustics, electronics, communication, graphics, and even musicology.

This observation underscores the significance of number theory, as it demonstrates its broad applicability across various fields involving mathematics. This versatility has led to the emergence of a new discipline known as applied number theory. Consequently, number theory has evolved beyond being solely a pure discipline to becoming a substantial applied discipline. Considering the ongoing development trends and applications of number theory, it is evident that this ancient discipline is poised to remain vibrant and relevant in the future.

The Development of Number Theory and Algebra

[2] Over time, numerous questions in number theory have been proposed and subsequently resolved, drawing increasing attention to the field. Throughout its extensive history, various techniques and methods have emerged to solve these problems, leading to the formation of several theories. Algebraic number theory, in particular, has seen advancements alongside the expansion of number fields and practical applications. Renowned philosopher Bacon once remarked that history enhances human intellect, emphasizing the importance of exploring the early development of algebraic number theory. Domestic research on algebraic number theory primarily involves comprehensive discussions on its progress. This paper aims to investigate the origins of algebraic number theory by analyzing key issues in the development of two higher reciprocity laws and Fermat's theorem. Through the collection and collation of relevant data, it seeks to provide a new perspective on historical analysis, aiming for a more comprehensive and insightful examination.

1) The stage of Arithmetic: during the period from about 3,800 to the third century, arithmetic symbols were not uniform, and algebra was separated from geometry. The ancient Greeks made the greatest contribution to number theory, including some renowned achievements, such as Euclid's Euclidean algorithm in geometry which proposed that the number of prime numbers is infinite, and the fundamental theorem of arithmetic which was involved in elementary number theory.

2) The complete stage of number and equation theory: during the period from the 7th century to the 16th century, irrational and imaginary numbers were discovered.

a) The discovery of irrational numbers: Hipparsos of the Pythagorean school discovered the first irrational number, shocking the leaders of the school at that time. He proposed that all numbers could be expressed as the ratio of integers, which led to the first mathematical crisis.

b) Creation of arithmetic operators and solution to irrational equations: In India, the mathematician Brahmagupta introduced a group of symbols used to express concepts and describe operations in the 7th century, and Posgallo later put forward the concept of negative square root, the solution to irrational equations and the algorithm of irrational numbers in the 12th century, which fostered the study of algebrato a new stage.

c) Establishment of imaginary number theory: in the book The Great Art published in 1545 by the Milanese scholar Cardano (1501-1556), the general solution to the cubic equation was unveiled, which was known as Cardano's formula later. Cardano was the first mathematician to formulate the square rootof a negative number.

3) The stage of linear algebra: during the period from the 17th century to the 19th century, the toolsfor solving linear problems, matrices, determinants, and vectors emerged, which provided services to the industrial society.

4) The stage of abstract algebra: during the period from the 19th century to the present, the importance of form and technique to the algebra structure was highlighted, which offered services to the information society.

2. The Classical Questions and Conjectures in Number Theory

Mersenne Prime

Mersenne primes are derived from Mersenne numbers which refer to the positive integers of the form 2p-1, where if the exponent p is prime, p is usually defined as Mp. If a Mersenne number is prime, it iscalled a Mersenne prime; otherwise it is called a Mersenne number.

Prime numbers, also known as primes, refer to the numbers which are divisible only by 1 and themselves, such as 2, 3, 5 and so on. Euclid has proved that the number of primes is infinite with proofsby contradiction. In the infinite sequence 2n-1, Mersenne numbers and Mersenne primes only account for a small proportion, but Mersenne primes are infinite.

If the exponent n is prime, Mn is a prime number. However, when n is prime, Mp may not be prime (for example, M2=4-1=3 and M3=8-1=7 are prime, while M11=2047=23*89 is not a prime number). For the time being, 51 prime numbers have been discovered, of which the largest one is M82589933 with 24862048 digits. Nowadays, distributed network computing technology has become the latest method to discover primes.

Goldbach Conjecture

Goldbach conjecture is one of the oldest unsolved problems in number theory. It stated that every even integer greater than two can be written as the sum of two primes. Goldbach conjecture is associated with integer partition which is proposed by European number theorists at that time and focused on the question--“can you analyze integers as the sum of certain numbers with certain properties?” To be specific, the question is whether you can divide all integers into the sum of several complete squares, orthe sum of several complete cubes. Such a partition of a given even number into the sum of two prime numbers is called Goldbach analysis. Goldbach conjecture took a long time to develop. China’s mathematician Chen Jingrun proved that every sufficiently large even number can be written as the sum of some prime number and another number which is the product of two primes.

Based on Goldbach conjecture of even numbers, the conjecture that every odd integers greater than 7 can be written as the sum of three primes has been proposed, which is called the weak Goldbach conjecture. It has been proved in 2013.

Fibonacci Sequence

Fibonacci sequence, defined by Italian mathematician Leonardo Fibonacci, refers to a series of numbers in which beginning from the third number in the sequence, each number is the sum of the two preceding ones. The nth number in the sequence can be denoted by f(n), and its recursive sequence can be expressed as the following formula.

$$f(n) = f(n-1) + f(n-2)$$

Applications of Fibonacci Sequence:

1) Golden ratio: as the number of items in the sequence increases, the ratio of the former to the latter increases closer to the golden ratio.

2) Pascal triangle: the numbers on diagonals of Pascal triangle add to the Fibonacci sequence.

3) Area of a rectangle: the squares of the first few numbers in the Fibonacci sequence are treated as different small quadrilateral areas, and they can be combined into large quadrilateral areas.

The Significance of Mathematical Conjectures

Apart from the above conjectures, there are many other conjectures. Most of the mathematical conjectures are based on observation, verification, induction and generalization of a large number of facts. Such a method of abstracting the general and common properties from the special properties is an important driver of mathematical research. The expression and research of mathematical conjectures vividly reflect the application of dialectics in mathematics. Moreover, mathematical conjectures promote the study of mathematical methodology.

Furthermore, mathematical conjectures often play the role as the important indicator of mathematical development. Fermat conjecture gave birth to algebraic number theory, while Goldbach conjecture promoted the development of screening methods. Riemann conjecture proved the prime number theorem, while the Four-color conjecture was solved by computer, and thereby a new era of machine verification has been opened. Therefore, mathematical conjectures are not only the precious gemstones, but also the key driver in the development of mathematics.

3. Applications of Number Theory

Cryptography

With the development of network encryption technology, number theory has found its own place--cryptography. Professor Wang Xiaoyun who cracked the MD5 code a few years ago is from the number theory school of Shandong University. Because of the irregular appearance of prime factors in composite numbers, it is very difficult to decompose composite numbers into the product of prime numbers. At the same time, it is this difficulty that enlightens people to use it to design difficult codes.

When studying number theory, especially cryptography, we pursue deterministic algorithm rather than probabilistic algorithm, and we will only lower our requirements and apply probabilistic algorithmif there is no deterministic algorithm.

Computer Animation

Linear transformation is usually used to make images, and computer graphics are to build graphics on display devices through algorithms and programs, so linear transformation technology can be used to make computer animation. Computer graphics mainly consist of image representation, storage and computation. With the improvement of software capabilities, linear transformation technology is commonly used in computer animation.

Machine Translation

The main algorithm of machine translation is based on the statistical method, with the accuracy of 90%.In addition, this algorithm is also used in image search technology. The core of this method is that the language units of source language and target language can be represented by vectors, and the lexical vectors of different languages can be projected onto a two-dimensional plan for analysis. Experimental results show that the lexical vectors of different languages do have some relations similar to linear relations, so it is of significance to classify machine translation as a linear transformation.

Other Basic Fields

Number theory also plays a surprising role in other theories. In quantum theory, Hermite operator is oneof the most basic concepts. Apart from that, number theory is also widely used in non-mathematical disciplines, such as information science, theoretical physics, quantum chemistry, and so on.

4. Conclusion and Prospects

This paper mainly discusses the basic concept, theory, development process and applications of number theory. As a foundation of science and engineering disciplines, the development trend and level of mathematics has a profound influence on other disciplines. By reviewing the development of number theory and its applications, this paper aims to help readers acquire the origin and development of number theory, and its future trend in the combination of computer science. In today's society, with the rapid development of computer field, number theory or even mathematical discipline will make greater stridesin the future.

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