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ODD-PRIME HARMONIOUS LABELING

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Abstract: The present paper investigates new idea of odd-prime harmonious labeling of some new graphs. We prove that the graphs Cycle C_4 , Path P_2, Star S_4, Tree, Firecracker, Fork, Ladder L_3, Bistar $B(2,2)$, Claw $K_{1,3}$ graphs are odd-prime harmonious . A graph that calls for an odd prime harmonious labeling is called an odd- prime harmonious graph.

Keywords: Harmonious labeling, Odd Harmonious labeling, Prime harmonious labeling, Odd-Prime Harmonious labeling.

1. INTRODUCTION:

The paper written by Leohard Teuler was published in 1736 is widely regarded as the first paper in the history of Graph Theory.The concept of Graph labeling was first introduced in the mid 1960s. In the intervening 50 years nearly 200 graph labeling techniques have been studied in over 2000 papers. Lots of research work is been carried out in the labeling of graphs in past few work since the first initiated by A. Rosa [9] in his 1967 paper. Harmonious labelling was obviously the result of Graham and Sloane's research [5]. But in the 2013, Dushyant Tanna [4] has introduced several harmonious labeling techniques. The concept of odd harmonious labeling was due to Liang and Bai [7] in 2014. The Concept of prime harmonious labeling was introduced and studied by P. Deepa, S. Uma Maheswari and K. Indirani [3] in 2016. Due to development of information technology and solving typical algorithms of colouring problems - graph labeling is extremely useful. In this paper we have introduced Odd-Prime Harmonious labeling and some of its properties.

2. PRELIMINARIES:

Definition 2.1:
The square graph C_4 is essentially a cycle graph with four vertices, forming a square.

Definition 2.2:
The path graph consists of a string of edges leading from a source node to a destination node.

Definition 2.3:
The star graph of order n, denoted by S_n is a simple graph with n vertices.

Definition 2.4:
A Tree is a connected, acyclic, and undirected graph where any two vertices are connected by exactly one path.

Definition 2.5: [11]
An (n,k)- firecracker is a graph obtained by the concatenation of nk-stars by linking one leaf from each.

Definition 2.6: [10]

The Fork graph, sometimes also called the chair graph, it is a 5-vertex graph. It could perhaps also be known as the
'h-graph'.

Definition 2.7: [4]

The ladder graph denoted as L_n for $n \geq 2$, which is the Cartesian product of the path graph P_2 and the path graph P_n.

It indeed has 2n vertices and 3n-2 edges. Each vertex in P_n is paired with each vertex in P_n, creating a grid-like structure.

Definition 2.8: [3]

A Bistar graph, denoted as B(n,n) which is formed by connected the center vertices of two complete graphs, K(1,n) with an edge.

Definition 2.9: [8]

A Claw graph is a complete bipartite graph $K_{1,3}$ which also resembles a star graph. A claw-free graph ensures that no induced subgraph within it forms this specific structure.

Definition 2.10: Harmonious Labeling: [5]

Let G be a graph with q edges. A function f is called harmonious labeling of graph G if
f: V → {0,1,2,...q-1} is injective and the induced function f^*:E → {0,1,2,...q} defined as f^*(e= uv) = (f(u)+f(v))(mod q) is bijective. A Graph which admits harmonious labeling is called harmonious graph.

Definition 2.11: Odd Harmonious Labeling: [7]

A graph G is said to be odd harmonious if there exist an injection f: V(G) → {0,1,2,…2q-1} such that the induced function f^* : E(G) → {1,3,…2q-1} defined by f^* (uv) = f(u)+f(v) is a bijection.

Definition 2.12: Prime Harmonious Labeling: [3]

Let G be a graph with q edges. A function f is called prime harmonious labeling of graph G if
f: V(G) → {0,1,2,…2q-1} is injective and the induced function f^* :E(G) → {0,1,2,…2q} is defined by
f^*(e=uv)= G.C.D [f(u),f(v)]= 1 and f^*(e= uv)= [f(u)+f(v)][mod q].

3. Main Result

Definition 3.1: Odd-Prime Harmonious Labeling:

Let G be a graph with q edges. A function f is called prime harmonious labeling of graph G if f: V →{0,1,2,3,...2q-1}
is injective and the induced function f^*: E(G)→ {1,3,...2q-1} is defined by f^*(e=uv)= G.C.D [f(u),f(v)]= 1 and
f^*(e= uv)= [f(u)+f(v)][mod q]. A graph which satisfies the conditions of prime labeling and harmonious labeling is called a prime harmonious labeling. A graph which admits a prime harmonious labelingis called a prime harmonious graph.

In this paper we prove that the Square C_4, Path, Star K(1,4), Tree, Firecracker, Fork, Ladder L_3, Bistar B(2,2),Claw graph are odd-prime harmonious labeling.

Theorem 3.2:

The square graph C_4 is odd prime harmoniously labelled.

Proof:

Let us consider the square graph.

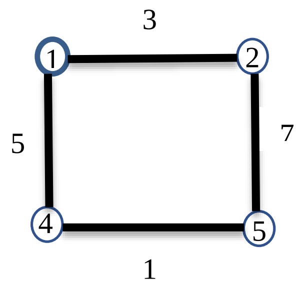

Figure I

The condition for the odd prime harmonious labeling graph is $f^{*}(e=uv)$= G.C.D [f(u),f(v)]= 1 and f(e= uv)=[f(u)+f(v)][mod q].

The number of edges in square graph is q = 4.

The labeling for vertex are f:V(G)→ {0,1,2,3,4,5,6,7}.

The labeling for edges are f^{*}:E(G) → {1,3,5,7}.

The G.C.D of (1,2);(2,5);(5,4);(1,4) are 1.

Using the above condition the edges can be labelled as 1,3,5,7.

Thus the square graph is odd-prime harmoniously labelled.

Theorem 3.3:

The path graph P2 is odd prime harmoniously labelled.

Proof:

Let us consider the path graph.

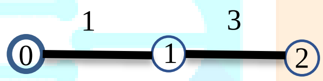

Figure II

The condition for the odd prime harmonious labeling graph is $f^{*}(e=uv)$= G.C.D [f(u),f(v)]= 1 and

f(e= uv)=[f(u)+f(v)][mod q].

The number of edges in path graph is q = 2.

The labeling for vertex are f:V(G)→ {0,1,2,3}.

The labeling for edges are f^{*}:E(G) → {1,3}.

The G.C.D of (0,1);(1,2) are 1.

Using the above condition the edges can be labelled as 1,3.

Thus the path graph is odd-prime harmoniously labelled.

Theorem 3.4:

The Star graph S4 is odd prime harmoniously labelled.

Proof:

Let us consider the star S4 graph.

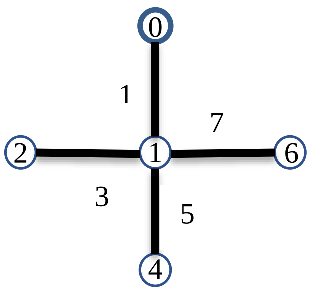

Figure III

The condition for the odd prime harmonious labeling graph is $f^*(e=uv)$= G.C.D [f(u),f(v)]= 1 and f(e= uv)=[f(u)+f(v)][mod q].

The number of edges in path graph is q = 4.

The labeling for vertex are f:V(G)→ {0,1,2,3,4,5,6,7}.

The labeling for edges are f^*:E(G) → {1,3,5,7}.

The G.C.D of (0,1);(1,2);(1,4);(1,6) are 1.

Using the above condition the edges can be labelled as 1,3,5,7.

Thus the Star S4 graph is odd-prime harmoniously labelled.

Theorem 3.5:

The Tree graph is odd prime harmoniously labelled.

Proof:

Let us consider tree graph.

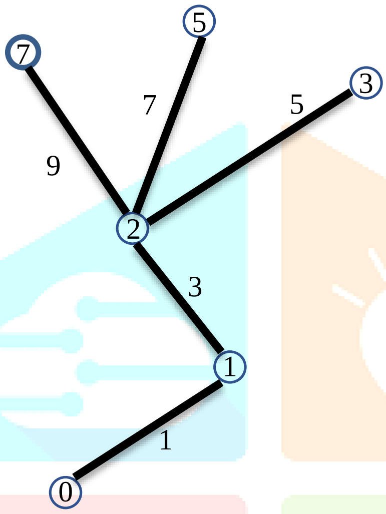

Figure IV

The condition for the odd prime harmonious labeling graph is $f^*(e=uv)$= G.C.D [f(u),f(v)]= 1 and f(e= uv)=[f(u)+f(v)][mod q].

The number of edges in path graph is q = 5.

The labeling for vertex are f:V(G)→ {0,1,2,3,4,5,6,7,8,9}.

The labeling for edges are f^*:E (G) → {1,3,5,7,9}.

The G.C.D of (0,1);(1,2);(2,7);(2,5);(2,3) are 1.

Using the above condition the edges can be labelled as 1,3,5,7,9.

Thus the Tree graph is odd-prime harmoniously labelled.

Theorem 3.6:

The Firecracker graph is odd prime harmoniously labelled.

Proof:

Let us consider the Firecracker graph.

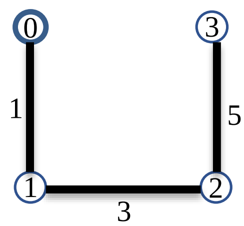

Figure V

The condition for the odd prime harmonious labeling graph is $f^*(e=uv)$= G.C.D [f(u),f(v)]= 1 and f(e= uv)=[f(u)+f(v)][mod q].
The number of edges in path graph is q = 3.
The labeling for vertex are f:V(G)→ {0,1,2,3,4,5}.
The labeling for edges are f^*:E(G) → {1,3,5}.
The G.C.D of (0,1);(1,2);(2,3) are 1.
Using the above condition the edges can be labelled as 1,3,5.
Thus the Firecracker graph is odd-prime harmoniously labelled.

Theorem 3.7:
The Fork graph is odd prime harmoniously labelled.
Proof:
Let us consider the Fork graph.

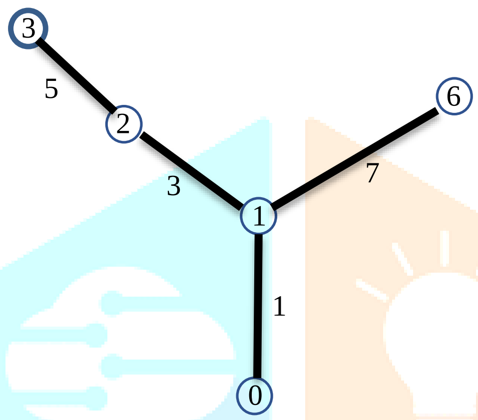

Figure VI

The condition for the odd prime harmonious labeling graph is $f^*(e=uv)$= G.C.D [f(u),f(v)]= 1 and f(e= uv)=[f(u)+f(v)][mod q].
The number of edges in path graph is q = 4.
The labeling for vertex are f:V(G)→ {0,1,2,3,4,5,6,7}.
The labeling for edges are f^*:E(G) → {1,3,5,7}.
The G.C.D of (0,1);(1,2);(2,3);(1,6) are 1.
Using the above condition the edges can be labelled as 1,3,5.
Thus the Fork graph is odd-prime harmoniously labelled.

Theorem 3.8:
The Ladder graph L3 is odd prime harmoniously labelled.
Proof:
Let us consider the Ladder L3 graph.

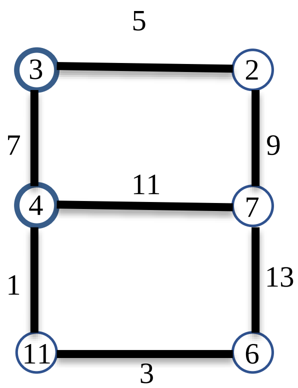

Figure VII

The condition for the odd prime harmonious labeling graph is $f^*(e=uv)$= G.C.D [f(u),f(v)]= 1 and f(e= uv)=[f(u)+f(v)][mod q].

The number of edges in path graph is q = 7.

The labeling for vertex are f:V(G) → {0,1,2,3,4,5,6,7,8,9,10,11,12,13}.

The labeling for edges are f*:E(G) → {1,3,5,7,9,11,13}.

The G.C.D of (2,3);(3,4);(2,7);(4,7);(4,11);(7,6);(11,6) are 1.

Using the above condition the edges can be labelled as 1,3,5,7,9,11,13.

Thus the Ladder L4 graph is odd-prime harmoniously labelled.

Theorem 3.9:

The Bistar $B_{(2,2)}$ is odd prime harmoniously labelled.

Proof:

Let us consider the Bistar $B_{(2,2)}$ graph.

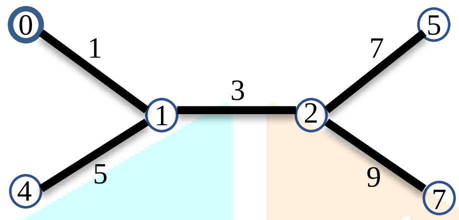

Figure VIII

The condition for the odd prime harmonious labeling graph is f*(e=uv)= G.C.D [f(u),f(v)]= 1 and f(e= uv)=[f(u)+f(v)][mod q].

The number of edges in path graph is q = 5.

The labeling for vertex are f:V(G)→ {0,1,2,3,4,5,6,7,8,9}.

The labeling for edges are f*:E(G) → {1,3,5,7,9}.

The G.C.D of (0,1);(1,2);(1,4);(2,5);(2,7) are 1.

Using the above condition the edges can be labelled as 1,3,5,7,9.

Thus the Bistar B(2,2) graph is odd-prime harmoniously labelled.

Theorem 3.10:

The Claw graph $K_{1,3}$ is odd prime harmoniously labelled.

Proof:

Let us consider the Claw $K_{1,3}$ graph.

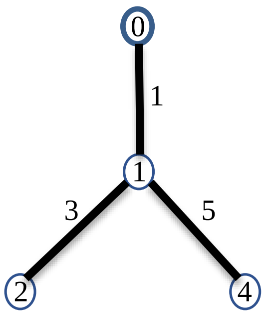

Figure IX

The condition for the odd prime harmonious labeling graph is f*(e=uv)= G.C.D [f(u),f(v)]= 1 and f(e= uv)=[f(u)+f(v)][mod q].

The number of edges in path graph is q = 3.

The labeling for vertex are f:V(G)→ {0,1,2,3,4,5}.The labeling for edges are f*:E(G) → {1,3,5}.

The G.C.D of (0,1);(1,2);(1,4) are 1.

Using the above condition the edges can be labelled as 1,3,5. Thus the Claw $K_{1,3}$ graph is odd-prime harmoniously labelled.

4. PROPERTIES OF ODD PRIME HARMONIOUS LABELING:

4.1. Odd prime harmonious labeling is not unique.

Consider the following example where two different odd prime harmonious labeling of the same graph are shown in the following figure.

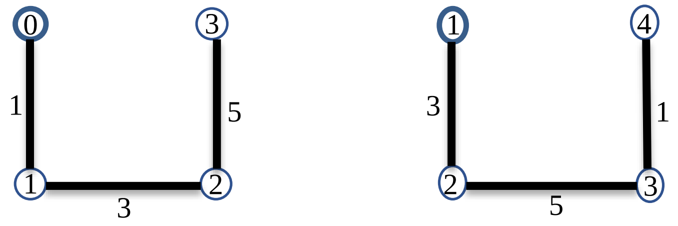

Figure X

4.2 Any vertex in an odd prime harmonious graph can be assigned the label 0 but, we have to see the adjacent vertex should only be 1.

4.3 The Cycle C_n (n>3) is odd prime harmonious labeling.

4.4 The Star S_n (n>2) is odd prime harmonious labeling.

4.5 The Path P_n (n>2) is odd prime harmonious labeling.

4.6 The Bistar $B_{(n,n)}$ (n>2) is odd prime harmonious labeling.

5. CONCLUSION:

The harmonious labeling is one of the most important labeling techniques. Here we introduce the new concept of odd prime harmonious labeling of some new graphs and it is very interesting to investigate a graph family which admits odd prime harmonious labeling. In general, all the graphs are not odd prime harmonious labeling.

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