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# Star Generalized Alpha Closed Sets In Pythagorean Fuzzy Topological Spaces

**S.Pavithra[1] , Dr.M.Trinita Pricilla[2]**
PG Student[1], Assistant Professor[2]
Department of Mathematics,
Nirmala College For women, Coimbatore, Tamil Nadu, India

**Abstract:** In this paper a Pythagorean Fuzzy star generalized α-closed sets and a Pythagorean Fuzzy star generalized α-open sets are introduced. Some of its properties are also analyzed. Also we have provided some applications of Pythagorean Fuzzy star generalized α-closed sets namely Pythagorean Fuzzy ${}_{\alpha}T_{1/2}$ space and Pythagorean Fuzzy ${}_{*g\alpha}T_{1/2}$ space.

**Key Words:** Pythagorean Fuzzy topology, Pythagorean Fuzzy star generalized alpha closed sets, Pythagorean Fuzzy star generalized alpha open sets, Pythagorean Fuzzy ${}_{\alpha}T_{1/2}$ space and Pythagorean Fuzzy ${}_{*g\alpha}T_{1/2}$ space.

## 1. INTRODUCTION

The concept of Fuzzy sets was introduced by Zadeh in 1965. After the introduction of Intuitionistic Fuzzy set by Atanassov in 1986, R.R. Yager initiated Intuitionistic Fuzzy set and presented a new set called Pythagorean Fuzzy set. In 1991, A.S. Binshahan introduced and investigated the notions of Fuzzy pre-open and Fuzzy pre-closed sets, In 2003, T. Fukutake, R.K. Saraf, M. Caldas and S. Mishra introduced generalized pre-closed Fuzzy sets in Fuzzy topological space, P. Rajarajeswari and L. Senthil Kumar introduced generalized pre-closed sets and Intuitionistic Fuzzy topological spaces. In this paper we have introduced Pythagorean Fuzzy star generalized α-closed sets and some of its characterizations are discussed.

## 2. PRELIMINARIES

**Definition 2.1:** A Pythagorean Fuzzy set (PFS in short) A in X is an object having the form A = < a, $\lambda_A$ (a),$\mu_A$ (a) > /a ∈ X where the functions $\lambda_A(a) : X \to [0,1]$ and $\mu_A(a) : X \to [0,1]$ denote the degree of membership (namely $\lambda_A(a)$) and the degree of non-membership (namely $\mu_A(a)$) of each element a ∈ X to set A respectively,
$0 \le \lambda_A(a)^2 + \mu_A(a)^2 \le 1$ for each a ∈ X.

**Definition 2.2:** Let A and B be PFSs of the form A = < a,$\lambda_A(a)$,$\mu_A(a)$ > /a ∈ X and B = < a, $\lambda_B(a)$, $\mu_B(a)$ > /a ∈ X. Then

1) A ⊆ B if and only if $\lambda_A(a) \le \lambda_B(a)$ and $\mu_A(a) \ge \mu_B(a)$ for all a ∈ X

2) A = B if and only if A ⊆ B and B ⊆ A

3) $A^C$ = {< a,$\mu_A(a)$,$\lambda_A(a)$ > /a ∈ X}

4) A∩B = {< a,$\lambda_A(a) \wedge \lambda_B(a)$,$\mu_A(a) \vee \mu_B(a)$ > /a ∈ X}

5) $A \cup B = \{< a, \lambda_A(a) \vee \lambda_B(a), \mu_A(a) \wedge \mu_B(a) > / a \in X\}$

For the sake of simplicity, we shall use the notation $A = < a, \lambda_A(a), \mu_A(a) >$ instead of $A = \{ < a, \lambda_A(a), \mu_A(a) > / a \in X\}$. The Pythagorean Fuzzy sets $0 = \{< a,0,1 > / a \in X\}$ and $1 = \{< a,0,1 > / a \in X\}$ are respectively the empty set and the whole set of X.

**Definition 2.3:** A Pythagorean Fuzzy topology (PFT in short) by subsets of a non - empty set X is a family of Pythagorean Fuzzy sets satisfying the following axioms.

1) $0,1 \in \tau$
2) $G_1 \cap G_2 \in \tau$ for every $G_1, G_2$ and
3) $\cup G_i$ for any arbitrary family $\{G_i | i \in J\}$

In this case the pair $(X,\tau)$ is called a Pythagorean Fuzzy topological space (PFTS in short) and any Pythagorean Fuzzy set G in $\tau$ is called a Pythagorean Fuzzy open set (PFOS in short) in X. The complement $A^C$ of a Pythagorean Fuzzy open set A in a Pythagorean Fuzzy topological space $(X,\tau)$ is called a Pythagorean Fuzzy closed set (PFCS in short).

**Definition 2.4:** Let $(X,\tau)$ be a PTFS and $A = \{< a, \lambda_A(a), \mu_A(a) > / a \in X\}$ be Pythagorean Fuzzy set in X. Then the interior and the closure of A are denoted by PFint(A) and PFcl(A) and are defined as follows.

$PFint(A) = \cup\{G | G$ is a PFOS in X and $G \subseteq A\}$

$PFcl(A) = \cap\{K | K$ is a PFCS in X and $A \subseteq K\}$

Also, it can be established that PFcl(A) is a PFCS if and only if PFcl(A) = A and PFint(A) is a PFOS if and only if PFint(A) = A. We say that A is PF-dense if PFcl(A) = X.

**Definition 2.5:** A Pythagorean Fuzzy set $A =< a, \lambda_A(a), \mu_A(a) >$ in a Pythagorean Fuzzy topological space $(X,\tau)$ is said to be a

- Pythagorean Fuzzy semi closed set (PFSCS in short) if $PFint(PFcl(A)) \subseteq A$
- Pythagorean Fuzzy semi open set (PFSOS in short) if $A \subseteq PFcl(PFint(A))$
- Pythagorean Fuzzy α-closed set (PFαCS in short) if $PFcl(PFint(A)) \subseteq A$
- Pythagorean Fuzzy α-open set (PFαOS in short) if $A \subseteq PFint(PFcl(A))$
- Pythagorean Fuzzy β-closed set (PFβCS in short) if $PFcl(PFint(PFcl(A))) \subseteq A$
- Pythagorean Fuzzy β-open set (PFβOS in short) if $A \subseteq PFint(PFcl(PFint(A)))$
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**Definition 2.6:** Let A be a PFS of a PFTS $(X,\tau)$. Then the Pythagorean Fuzzy semi-interior of A (PFsint(A) in short) and the Pythagorean Fuzzy semi-closure of A (PFscl(A) in short) is defined as

$PFsint(A) = \cup\{K | K$ is an PFSOS in X and $K \subseteq A\}$

$PFscl(A) = \cap\{K | K$ is an PFSCS in X and $A \subseteq K\}$

**Definition 2.7:** Let A be a PFS in $(X,\tau)$, then

1) $PFscl(A) = A \cup PFint(PFcl(A))$
2) $PFsint(A) = A \cap PFcl(PFint(A))$

**Definition 2.8:** A PFS $A =< a, \lambda_A(a), \mu_A(a) >$ in an PFTS $(X,\tau)$ is said to be a

- Pythagorean Fuzzy regular open set (PFROS) if $A = PFint(PFcl(A))$
- Pythagorean Fuzzy regular closed set (PFRCS) if $A = PFcl(PFint(A))$

**Definition 2.9:** A PFS A of a PFTS $(X,\tau)$ is a Pythagorean Fuzzy generalized closed set (PFGCS in short) if $PFcl(A) \subseteq U$ whenever $A \subseteq U$ and U is a PFOS in X.

**Definition 2.10:** Let a PFS A of a PFTS $(X,\tau)$. Then the Pythagorean Fuzzy β closure of A (PFβcl in short) and the Pythagorean Fuzzy β interior of A (PFβint in short) is defined as

$PF\beta cl(A) = \{K | K$ is a Pythagorean Fuzzy β closed set in X and $A \subseteq K\}$

$PF\beta int(A) = \{K | K$ is a Pythagorean Fuzzy β open set in X and $K \subseteq A\}$

**Definition 2.11:** Let A be a PFS in (X,τ), then

1) PFβcl(A) = A ∪ PFcl(PFint(PFcl(A)))
2) PFβint(A) = A ∩ PFint(PFcl(PFint(A)))

**Definition 2.12:** A PFS A of a PFTS (X,τ) is said to be a Pythagorean Fuzzy β generalized closed set (PFβGCS in short) if PFβcl(A) ⊆ U whenever A ⊆ U and U is a PFOS in X.

**Definition 2.13:** Let (X,τ) be a PFTS and A =< a,$\lambda_A$(a),$\mu_A$(a) > be a PFS in X. The α-interior of A is denoted by PFαint(A) and is defined by the union of all Fuzzy α-open sets of X which are contained in A. The intersection of all Fuzzy α-closed sets containing A is called the α-closure of A and is denoted by (PFαcl(A).

PFαint(A) = ∪{G | G is a Pythagorean Fuzzy αopen set in X and G ⊆ A}

PFαcl(A) = ∩{K | K is a Pythagorean Fuzzy α-closed set in X and A ⊆ K}

**Definition 2.14:** If A is a PFS in X, then PFαcl(A) = A ∪ PFcl(PFint(A)).

**Definition 2.15:** If A is a PFS in (X,τ), we have X − PFint(A) = PFcl(X − A) and X − PFcl(A) = PFint(X − A).

## 3. PYTHAGOREAN FUZZY STAR GENERALIZED ALPHA CLOSED SETS

**Definition 3.1:** A PFS A is said to be Pythagorean Fuzzy star generalized α-closed set (PF*GαCS in short) in (X,τ) if PFαcl(A) ⊆ U whenever A ⊆ U and U is a PFαOS in X. The family of all PF*GαCSs of an PFTS (X,τ) is denoted by PF*GαC(X).

**Example 3.2:** Let X = {a,b} and let τ = {0,U,1} be a Pythagorean Fuzzy topology on X, where U = {< a,0.3,0.7 >,< b,0.4,0.6 >}. Then the Pythagorean Fuzzy set A ={< a,0.3,0.7 >,< b,0.3,0.6 >} is a Pythagorean Fuzzy star generalized α-closed set in X.

**Theorem 3.3:** Every PFCS is a PF*GαCS but not conversely.

Proof: Let A be a PFCS in X and let A ⊆ U and U is an PFαOS in (X,τ). Since PFαcl(A) ⊆ PFcl(A) and A is a PFCS in X, PFαcl(A) ⊆ PFcl(A) = A ⊆ U. Therefore A is a PF*GαCS in X.

**Example3.4:** Let X = {a, b} and let τ = {0, U, 1} be a PFT on X, where U = {< a, 0.3, 0.7 >, < b, 0.4, 0.6 >}.Then the PFS A = {< a, 0.3, 0.7 >, < b, 0.4, 0.6 >} is a PF*GαCS in X but not an PFCS in X.

**Theorem 3.5:** Every PFβCS is a PF*GαCS but not conversely.

Proof: Let A be a PFβCS in X and let A ⊆ U and U is a PFαOS in (X,τ).By hypothesis, PFcl(PFint(PFcl(A))) ⊆ A. Since A ⊆ PFcl(A), PFcl(PFint(A)) ⊆ PFcl(PFint(PFcl(A))) ⊆ A. Hence PFαcl(A) ⊆ A ⊆ U. Therefore A is a PF*GαCS in X.

**Example 3.6:** Let X = {a, b} and let τ = {0, U, 1} be a PFT on X, where U = {< a, 0.3, 0.7 >, < b, 0.4, 0.6 >}. Then the PFS A = {< a, 0.2, 0.8 >, < b, 0.2, 0.7 >} is a PF*GαCS in X but not a PFβCS in X since P F cl(P F int(P F cl(A))) = {< a, 0.6, 0.4 >, < a, 0.7, 0.2 >} ⊈ A.

**Theorem 3.7:** Every PFGCS is a PF*GαCS but not conversely.

Proof: Let A be a PFGCS in X and let A ⊆ U and U is a PFαOS in (X,τ). Since PFαcl(A) ⊆ PFcl(A) and by hypothesis, PFαcl(A) ⊆ U. Therefore A is a PF*GαCS in X.

**Example 3.8:** Let X = {a,b} and let τ = {0,U,1} be a PFT on X, where U = {< a,0.3,0.7 >,< b,0.4,0.6 >}. Then the PFS A = {< a,0.2,0.8 >,< b,0.3,0.7 >} is a PF*GαCS in X but not a PFGCS in X since A ⊆ U But PFcl(A) = {< a,0.7,0.3 >,< b,0.6,0.4 >} ⊈ U.

**Theorem 3.9:** Every PFRCS is a PF*GαCS but not conversely.

Proof: Let A be a PFRCS in X. Then we know that A = PFcl(PFint(A)). This implies PFcl(A) = PFcl(PFint(A)). Then, PFcl(A) = A. Therefore, A is a PFCS in X. Hence we know that, A is a PF*GαCS in X.

**Example 3.10:** Let X = {a,b} and let τ = {0,U,1} be a PFT on X, where U = {< a,0.7,0.3 >,< b,0.8,0.1 >}. Then the PFS A = {< a,0.4,0.6 >,< b,0.3,0.7 >} is a PF*GαCS but not a PFRCS in X since PFcl(PFint(A)) = 0 ≠ A.

**Theorem 3.11:** Every PFαCS is a PF*GαCS but not conversely.

Proof: Let A be a PF*GαCS in X and let A ⊆ U and U is a PFαOS in (X,τ).We know that, PFcl(PFint(A)) ⊆ A. This implies PFαcl(A) = A ∪ PFcl(PFint(A)) ⊆ A Therefore, PFαcl(A) ⊆ U. Hence, A is a PF*GαCS in X.

**Example 3.12:** Let X = {a,b} and let τ = {0,U,1} be a PFT on X, where U = {< a,0.5,0.5 >,< b,0.8,0.2 >}. Then the PFS A = {< a,0.7,0.3 >,< b,0.8,0.2 >} is a PF*GαCS but not a PFαCS in X since PFcl(PFint(A)) = 1 ⊈ A.

**Theorem 3.13:** Every PFβGCS is a PF*GαCS but not conversely.

Proof: Let A be a PFβCS in X and let A ⊆ U and U is a PFαOS in (X,τ). We know that, A ∪ PFcl(PFint(PFcl(A))) ⊆ U. This implies PFcl(PFint(PFcl(A))) ⊆ U and PFcl(PFint(A)) ⊆ U. Therefore PFαcl(A) = A ∪ PFcl(PFint(A)) ⊆ U. Hence A is a PF*GαCS in X.

**Example 3.14:** Let X = {a,b} and let τ = {0,U,1} be a PFT on X, where U = {< a,0.6,0.4 >,< b,0.5,0.5 >}. Then the PFS A = {< a,0.5,0.5 >,< b,0.4,0.6 >} is a PF*GαCS but not a PFβGCS in X since (PFβcl(A)) = 1 ⊈ U.

**Remark 3.15:** Pythagorean Fuzzy semi-closed set and PF*GαCS are independent to each other.

**Example 3.16:** Let X = {a,b} and let τ{0,U,1} be a PFT on X, where U ={< a,0.2,0.8 >,< b,0.5,0.5 >}. Then the PFS A = U is a PFSCS but not a PF*GαCS in X since A ⊆ U but PFαcl(A) = {< a,0.8,0.2 >,< b,0.5,0.5 >} ⊈ U.

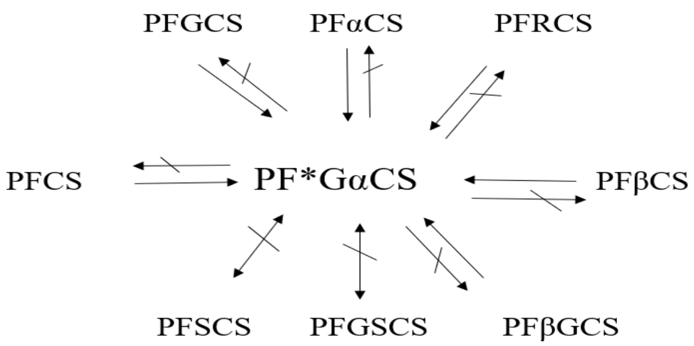
**Example 3.17:** Let X = {a,b} and let τ = {0,U,1} be a PFT on X, where U = {< a,0.7,0.3 >,< b,0.4,0.6 >}. Then the PFS A = {< a,0.6,0.4 >,< b,0.3,0.7 >} is a PF*GαCS but not a PFSCS in X since PFint(PFcl(A)) ⊈ A.

**Remark 3.18:** PFGSCS and PF*GαCS are independent to each other.

**Example 3.19:** Let X = {a,b} and let τ = {0,U,1} be a PFT on X, where U = {< a,0.1,0.9 >,< b,0.5,0.5 >}. Then the PFS A = U is a PFGSCS but not a PF*GαCS in X since A ⊆ U but PFαcl(A) ={< a,0.9,0.1 >,< b,0.5,0.5 >} ⊈ U.

**Example 3.20:** Let X = {a,b} and let τ = {0,U,1} be a PFT on X, where U = {< a,0.8,0.2 >,< b,0.6,0.4 >}. Then the PFS A = {< a,0.4,0.6 >,< b,0.3,0.7 >} is a PF*GαCS but not a PFSGCS in X since PFscl(A) = 1 ⊈ U.

**Remark 3.21:** From the above theorems and examples we have the following implications.




In this diagram by "A → B" we mean A implies B, “A ↚ B” means B does not imply A and "A ↮ B" means A and B are independent of each other. None of them is reversible.

**Remark 3.22:** The union of any two PF*GαCSs is not a PF*GαCS in general as seen in the following example.

**Example 3.23:** Let X = {a,b} be a PFTS and let τ = {0,U,1} be a PFT on X, where U = {< a,0.7,0.3 >,< b,0.9,0.1 >}. Then the PFSs A = {< a,0.2,0.8 >,< b,0.9,0.1 >},B = {< a,0.7,0.3 >,< b,0.8,0.2 >} are PF*GαCSs but A ∪ B is not a PF*GαCS in X.

## 4. PYTHAGOREAN FUZZY STAR GENERALIZED ALPHA OPEN SETS

**Definition 4.1:** A PFS A is said to be a PF*GαOS in (X,τ) if the complement $A^C$ is a PF*GαCS in X. The family of all PF*GαOSs of a PFTS (X,τ) is denoted by PF ∗ GαO(X).

**Example 4.2:** Let X = {a,b} and let τ = {0,U,1} be a PFT on X, where U = {< a,0.8,0.1 >,< b,0.7,0.2 >}. Then the PFS A = {< a,0.9,0.1 >,< b,0.8,0.2 >} is a PF*GαOS in X.

**Theorem 4.3:** For any PFTS (X,τ), we have the following:

(i) Every PFOS is a PF*GαOS.

(ii) Every PFSOS is a PF*GαOS.

(iii) Every PFβOS is a PF*GαOS.

(iv) Every PFαOS is a PF*GαOS.

Proof: It is obvious.

**Remark 4.4:** The converse of the above statements need not be true which can be seen from the following examples.

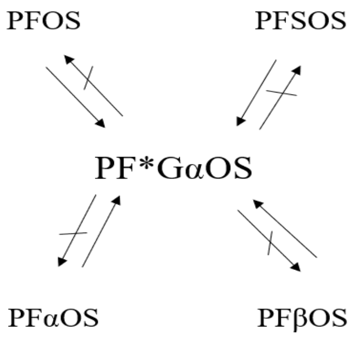
**Example 4.5:** Let X = {a,b} and let τ = {0,U,1} be a PFT on X, where U = {< a,0.4,0.6 >,< b,0.5,0.5 >}. Then the PFS A = {< a,0.7,0.3 >,< b,0.9,0.1 >} is a PF ∗ GαOS but not a PFOS in X.

**Example 4.6:** Let X = {a,b} and let τ = {0,U,1} be a PFT on X, where U = {< a,0.3,0.7 >,< b,0.2,0.8 >}. Then the PFS A = {< a,0.8,0.2 >,< b,0.9,0.1 >} is a PF ∗ GαOS but not a PFSOS in X.

**Example 4.7:** Let X = {a,b} and let τ = {0,U,1} be a PFT on X, where U = {< a,0.6,0.4 >,< b,0.5,0.5 >}. Then the PFS A = {< a,0.7,0.3 >,< b,0.5,0.5 >} is a PF ∗ GαOS but not a PFβOS in X.

**Example 4.8:** Let X = {a,b} and let τ = {0,U,1} be a PFT on X, where U={< a,0.5,0.5 >,< b,0.6,0.4 >}. Then the PFS A = {< a,0.6,0.4 >,< b,0.5,0.5 >} is a PF ∗ GαOS but not a PFαOS in X.

**Remark 4.9:** From the above theorem and examples we have the following diagrammatic representation.




In this diagram by "A → B" we mean A implies B and "$A \nleftarrow B$" means B does not imply A.

**Theorem 4.10:** Let (X,τ) be a PFTS. If A ∈ PF ∗ GαO(X) then V ⊆ PFint(PFcl(A)) whenever V ⊆ A and V is a PFαCS in X.

Proof: Let A ∈ PF ∗ GαO(X). Then $A^C$ is a PF*GαCS in X. Therefore PFαcl($A^C$) ⊆ U whenever $A^C$ ⊆ U and U is a PFαOS in X. That is PFcl(PFint($A^C$)) ⊆ U. This implies that, $U^C$ ⊆ PFint(PFcl(A)). whenever $U^C$ ⊆ A and is a PFαCS in X. Replacing $U^C$ by V , we get V ⊆ PFint(PFcl(A)) whenever V ⊆ A and V is a PFαCS in X.

**Theorem 4.11:** Let (X,τ) be a PFTS. Then for every A ∈ PF ∗ GαO(X) and for every B ∈ PFS(X), PFαint(A) ⊆ B ⊆ A implies B ∈ PF ∗ GαO(X).

Proof: By hypothesis, $A^C$ ⊆ $B^C$ ⊆ $(PF\alpha int(A))^C$. Let $B^C$ ⊆ U and U be a PFαOS. Since, $A^C$ ⊆ $B^C$,$A^C$ ⊆ U. But $A^C$ is a PFGαCS, PFαcl($A^C$) ⊆ U. Also, $B^C$ ⊆ $(PF\alpha int(A))^c$ = PFαcl($A^C$).
Therefore PFαcl($B^C$) ⊆ PFαcl($A^C$) ⊆ U. Hence, $B^C$ is a PF*GαCS, which implies B is a PF*GαO(X).

**Remark 4.12:** The intersection of any two PF*GαOSs is not a PF*GαOS in general.

**Example 4.13:** Let X = {a,b} and let τ = {0,U,1} be a PFTS on X, where U = {< a,0.6,0.4 >,< a,0.8,0.2 >}. Then the PFSs A = {< a,0.7,0.3 >,< a,0.6,0.4 >} and B = {< a,0.4,0.6 >,< a,0.3,0.7 >} are PF ∗ GαOSs but A ∩ B is not a PFαOS in X.

**Theorem 4.14:** A PFS A of a PFTS (X,τ), is a PF*GαOS if and only if F ⊆ PFαint(A) whenever F is a PFCS and F ⊆ A.

Proof: Necessity: Suppose A is a PF*GαOS in X. Let F be a PFCS and F ⊆ A. Then $F^C$ is a PFOS in X such that $A^C$ ⊆ $F^C$. Since $A^C$ is a PF*GαCS, we have PFαcl($A^C$) ⊆ $F^C$. Hence $(PF\alpha int(A))^C$ ⊆ $F^C$. Therefore, F⊆PFαint(A).

Sufficiency: Let A be a PFS of X and let F ⊆ PFαint(A) whenever F is a PFCS and F ⊆ A. Then $A^C$ ⊆ $F^C$ and $F^C$ is a PFOS. By hypothesis, $(PF\alpha int(A))^C$ ⊆ $F^C$ which implies PFαcl($A^C$) ⊆ $F^C$. Therefore $A^C$ is a PF*GαCS of X. Hence A is a PF*GαOS of X.

**Corollary 4.15:** A PFS A of a PFTS (X,τ) is a PF*GαOS if and only if F ⊆ PFint(PFcl(A)) whenever F is a PFCS and F ⊆ A.

Proof: Necessity: Suppose A is a PF*GαOS in X. Let F be a PFCS and F ⊆ A. Then $F^C$ is a PFOS in X such that $A^C$ ⊆ $F^C$. Since $A^C$ is a PF*GαCS, we have PFαcl($A^C$) ⊆ $F^C$. Therefore PFcl(PFint($A^C$)) ⊆ $F^C$. Hence $(PFint(PFcl(A)))^C$ ⊆ $F^C$. Therefore, F ⊆ PFint(PFcl(A)).

Sufficiency: Let A be a PFS of X and let F ⊆ PFint(PFcl(A)) whenever F is a PFCS and F ⊆ A. Then $A^C$ ⊆ $F^C$ and $F^C$ is a PFOS. By hypothesis, $(PFint(PFcl(A)))^C \subseteq F^C$. Hence $PFcl(PFint(A^C)) \subseteq F^C$ which implies, $PF\alpha cl(A^C) \subseteq F^C$. Hence A is a PF*GαOS of X.

**Theorem 4.16:** For a PFS A, A is a PFOS and a PF*GαCS in X if and only if A is a PFROS in X.

Proof: Necessity: Let A be a PFOS and a PF*GαCS in X. Then PFαcl(A) ⊆ A. This implies PFcl(PFint(A)) ⊆ A. Since, A is a PFOS, it is a PFαOS. Hence A ⊆ PFint(PFcl(A)). Therefore A = PFint(PFcl(A)) and hence, A is a PFROS in X.

Sufficiency: Let A be a PFROS in X. Therefore A = PFint(PFcl(A)). Let A ⊆ U and U is a PFαOS in X. This implies PFαcl(A) ⊆ A and hence A is a PF*GαCS in X.

## 5. APPLICATIONS OF PYTHAGOREAN FUZZY GENERALIZED ALPHA CLOSED SETS

**Definition 5.1:** A PFTS (X, τ ) is said to be a PF $_{\alpha}T_{1/2}$ space space if every PF*GαCS in X is a PFCS in X.

**Definition 5.2:** A PFTS (X, τ ) is said to be a $PF_{*g\alpha}T_{1/2}$ space ($PF_{*g\alpha}T_{1/2}$ space in short) if every PF*GαCS in X is a PFαCS in X.

**Theorem 5.3:** Every PF $_{\alpha}T_{1/2}$ space is a $PF_{*g\alpha}T_{1/2}$ space.

Proof: Let X be a $_{\alpha}T_{1/2}$ space and let A be a PF*GαCS in X. By hypothesis A is a PFCS in X. Since every PFCS set is a PFαCS, A is a PFαCS in X. Hence X is a $PF_{*g\alpha}T_{1/2}$ space.

But the converse need not be true which can be seen in the following example.

**Example 5.4:** Let X = {a,b} and let τ = {0,U,1} be a PFT on X, where U = {< a,0.8,0.2 >,< b,0.8,0.2 >}. Then (X,τ) is a $PF_{*g\alpha}T_{1/2}$ space. But it is not a PF $_{\alpha}T_{1/2}$ space since the PFS A = {< a,0.3,0.7 >,< b,0.6,0.4 >} is PF*GαCS but not a PFCS in X.

**Theorem 5.5:** Let (X,τ) be a Pythagorean Fuzzy topological space and X is a PF $_{\alpha}T_{1/2}$ space then

1. Any union of PF*GαCSs is a PF*GαCS.
2. Any intersection of PF*GαCSs is a PF*GαOS.

Proof: 1. Let $\{A_i\}_{i\in J}$ is a collection of PF*GαCSs in a PF $_{\alpha}T_{1/2}$ space (X,τ). Therefore every PF*GαCS is a PFCS.But the union of PFCS is a PFCS. Hence the union of PF*GαCSs is a PF*GαCS in X.

2. Let $\{A_i\}_{i\in J}$ is a collection of PF*GαOSs in a PF $_{\alpha}T_{1/2}$ space (X,τ). Therefore every PF*GαOS is an PFOS.But the intersection of PFOS is a PFOS. Hence the intersection of PF*GαOSs is a PF*GαOS in X .

**Theorem 5.6:** A PFTS X is a $PF_{*g\alpha}T_{1/2}$ space if and only if $PF*G\alpha O(X) = PF\alpha O(X)$.

Proof: Necessity: Let A be a PF*GαOS in X, then $A^C$ is a PF*GαCS in X. By hypothesis, $A^C$ is a PFαCSin X. Therefore A is a PFαOS in X. Hence $PF*G\alpha O(X) = PF\alpha O(X)$.

Sufficiency: Let A be a PF*GαCS in X. Then $A^C$ is a PF*GαOS in X. By hypothesis, $A^C$ is a PFαOS in X. Therefore, A is a PFαCS in X. Hence X is a $PF_{*g\alpha}T_{1/2}$ space.

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