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ANALYSIS OF BEAM ON ELASTIC FOUNDATION

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***Abstract*:** This paper deals with the analysis of beam on elastic foundation using finite element method. Winkler model is used as the foundation models, in which Winkler model is used because of its simplicity. Winkler assumes that the base of the beam consists of independently and closed spaced springs and stiffness of these springs explains the behavior of the elastic foundation. The efficient method for solving beams resting on elastic foundation is finite element method. It is easy to account for material non-linearity, non-linear soil effects, beam weight and boundary conditions. Numerous issues inside the building bundles related with soil-structure interaction can be displayed by utilizing a beam on elastic basis. Validation, convergence study and parametric study are carried out in this paper with a numerical example.

Key words: Finite element analysis, Beams on elastic foundation, Winkler model.

I. INTRODUCTION

The analysis of structures on elastic foundation has wide range of applications in the fields such as geotechnics, railroads, roads, bio-mechanics, foundation engineering etc. The soil structure interaction is not considered for conventional buildings. Normally the buildings such as single storey buildings and buildings in situated in hard soil are design without considering the soil structure interaction. For design of all buildings especially the multi storey buildings the soil structure interaction needs to be considered because it affects the buildings directly. Numerous issues inside the building bundles related with soil-structure interaction can be displayed by utilizing a beam on elastic basis. In spite of the way that couple of kind foundation model are exist, the other soil foundation models are Pasternak model, Filonenko-Borodich model, Hetenyi model and Vlasov-Leontiev model. In these model winkler model is the simplest model. The Winkler foundation model is significantly used by designers and analysts because of its simplicity. Winkler assumes that in elastic foundation the vertical displacement at a particular point is proportional to the pressure at that particular point and the vertical displacement at a particular point does not depend on the pressure of adjacent points of that particular point. Also assumes that the base consists of independently and closed spaced springs. The stiffness of these springs explains the behavior of the elastic foundation. The common numerical methods used for the analysis of beams resting on elastic foundation are finite element method, finite difference method and boundary element method. In this paper finite element approach is used to solve beam on elastic foundation. Using the software ANSYS the analysis is carried out. The objectives of this study of analysis of beam on elastic foundation are to develop a Finite Element model for beam resting on elastic foundation using Winkler method and to validate the same and perform a detailed parametric study to understand the variation of elastic properties of soil on the response of beam. Scope of this analysis is soil structure interaction analysis of beam will reveal the variation in response when compared to the results when the soil is considered as infinitely stiff.

II. REVIEW OF LITERATURE

2.1 GENERAL

The foundation models used for the analysis of beam on elastic foundation, methods used for the analysis of beam on elastic foundation and the characteristics of the of the winkler model and finite element method are discussed.

2.2 GENERAL ASPECTS OF BEAM ON ELASTIC FOUNDATION

Analysis of bending of beams on elastic foundation is developed based on the assumption that the reaction forces in the foundation are proportional to the deflection of beam on the particular points. Basic assumptions of beam theory are plane sections remain plane before deformation and after deformation, strains are very small can be neglected, slopes of beam are small, cross section of the beam is symmetrical with respect to both horizontal and vertical planes, material of beam is linearly elastic, homogeneous, continuous and isotropic (Zissimos et al., 1987). The efficient method for solving beams resting on elastic foundation is finite element method. It is easy to account for material non-linearity, non-linear soil effects, beam weight and boundary conditions. The non-linearity behavior of materials such as steel and concrete gives low deformations only. The non-linearity behavior of elastic foundation leads to higher deformation (Mithaq, 2006). Basically, two theories are used for the formation of flexural beam models. The thin beam model is based on Euler-Bernoulli theory and the deep beam model is based on Timoshenko's theory. ANSYS computer program are consistent and precise to forecast the behavior of nonlinear geometric material behavior and nonlinear material behavior of deep beams resting on linear foundation and nonlinear Winkler foundation. Because of the cumulative deflection with reduced modulus sub-grade reaction and load increments, the maximum deflection is high for beam on nonlinear Winkler model than linear Winkler model (Azzawi et al.,2010). In this journal for the analysis of beam on elastic foundation is carried out by three methods finite element method, finite difference method, and general method. The result using finite difference method is more accurate than result from finite element method and general method. The only limitation of finite difference method is it finds out only the displacements at the predetermined grid points, the grid size effects on the accuracy of the solution (Bogdan, 2010). The Winkler foundation model assumes that only the displacements appear at the loaded zones and outside the loaded zones the displacement is considered as zero. For solving the problems of beam resting on elastic foundation, the finite element method has many of advantages such as non-linear modeling of soil medium, presentation of jump discontinuities in beam stiffness and foundation etc. Analysis of beam on elastic foundation has applications in the fields of geotechnics, biomechanics, marine engineering, roads etc (Dinev, 2012). In this paper the analytical and finite element approach are used for solving beam resting on elastic foundation. Also, in this paper the response of beam on elastic foundation under static and dynamic load is explained based on numerical examples. Result from the paper shows the behavior of beam on elastic foundation subjected to load with temporal variation and spatial variation (Karmvir and Ramakrishna, 2014). The common numerical methods used for the analysis of beams resting on elastic foundation are finite element method, finite difference method and boundary element method. The Winkler foundation model is mostly used by the researchers and engineers due to its simplicity. The differential equation of beam resting on elastic foundation is,

$$EI\frac{d^4}{dx^4}v + kv = -q(x)$$

where, E is Young's modulus, I is the moment of inertia, k is the foundation modulus and $q(x)$ is the distributed lateral load (Bekir et al. ,2016).

III. FINITE ELEMENT ANALYSIS

3.1 GENERAL

Finite element analysis is a computational tool used for the analysis of engineering problems. It contains mesh generation techniques used for dividing a complex problem into smaller elements. The finite element analysis is the best choice for the analysis of complex problems.

3.2 ANALYTICAL FORMULATION

3.2.1 The Governing Equation

A restrictive case is the Winkler foundation is that the displacement $u(z)$ at a point defined by the coordinate z depends only on the local force per unit length $p(z)$.

$$u(z) = \frac{p(z)}{k} \tag{3.1}$$

Where, k is known as the stiffness or modulus of the foundation.

Figure 3.1(a) shows the beam on elastic foundation, the beam has flexural rigidity EI, beam is supported on the Winkler foundation and a distributed load w(z) per unit length subjected to the beam. In Figure 3.1(b) shows that equilibrium of beam element having length δz. The force $w(z)\delta z$ is corresponding to the distributed load is and the downward force $p(z)\delta z$ provides by the support opposes the upward displacement u(z).

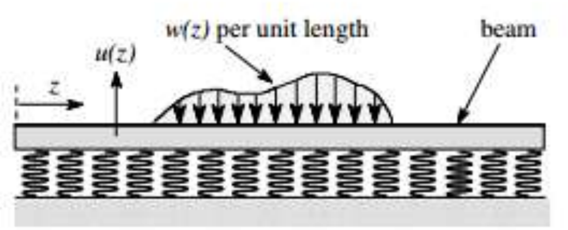
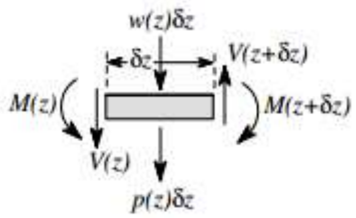


Figure 3.1 (a) Beam on a winkler foundation
(Barber,2000)



(b) Equilibrium of a beam element
(Barber,2000)

Shear force V satisfy the equation

$$V(z+\delta_z) - V(z) - W(z)\delta_z - P(z)\delta_z = 0 \qquad 3.2$$

$$\frac{V(z+\delta_z)-V(z)}{\delta_z} = W(z)\delta_z + P(z)\delta_z \qquad 3.3$$

Allowing $\delta_z \rightarrow 0$ and using equation 3.1 obtain that

$$\frac{dV}{dz} = W(z) + P(z) = W(z) + ku(z) \qquad 3.4$$

For moment equilibrium,

$$\frac{dM}{dz} = V \qquad 3.5$$

and the bending equation is

$$M = -EI\frac{d^2u}{dz^2} \qquad 3.6$$

To eliminate M and V, (equation3.4-equation3.6) we obtain that

$$EI\frac{d^4u}{dz^4} + ku = -w \qquad 3.7$$

The general solution of (equation 3.7) is the sum of a particular solution and the homogeneous solution.

$$EI\frac{d^4u}{dz^4} + ku = 0 \qquad 3.8$$

3.2.2 The Homogeneous Solution

If the beam has no distributed load w(z), equation 3.8 showing displacement becomes

$$u(z) = Ae^{bz} \qquad 3.9$$

In the above equation A and b are constants. Substitute these in equation 3.8

$$EIb^4Ae^{bz} + kAe^{bz} = 0 \qquad 3.10$$

if and only if the equation 3.9 will be a solution of equation 3.8 when

$$b^4 = -\frac{k}{EI} \qquad 3.11$$

The above equation has four complex roots and has no real roots, so it can be written

b = (±1 ± i) β,

Were,

$$\beta = \sqrt[4]{\frac{k}{4EI}} \qquad 3.12$$

The general solution of the homogeneous equation 3.8 can be written as

$$u(z) = A_1 e^{(1+i)\beta z} + A_2 e^{(1-i)\beta z} + A_3 e^{(-1+i)\beta z} + A_4 e^{(-1-i)\beta z} \qquad 3.13$$

In the above equation A_1, A_2, A_3, A_4 are independent complex constants. The displacement must be a real function, so we have $A_2 = \bar{A_1}$, $A_4 = \bar{A_3}$, where the complex conjugate is denoted by the over bar. After some algebraic manipulations, the general real function (equation 3.13) can be written as

$$u(z) = B_1 e^{\beta z} cos(\beta z) + B_2 e^{\beta z} sin(\beta z) + B_3 e^{-\beta z} cos(\beta z) + B_4 e^{-\beta z} sin(\beta z) \qquad 3.14$$

Another form which is sometimes more suitable for beams of finite length is

$$u(z) = C_1 cosh(\beta z) cosh(\beta z) + C_1 sinh(\beta z) sinh(\beta z) + C_1 cosh(\beta z) sinh(\beta z) + C_4 sinh(\beta z) cosh(\beta z) \qquad 3.15$$

The equations 3.14 and 3.15 are equal because of the identities

$$cosh(\beta z) = \frac{e^{\beta z}+e^{-\beta z}}{2} \quad ; \quad \sinh(\beta z) = \frac{e^{\beta z}-e^{-\beta z}}{2} \qquad 3.16$$

Equation 3.15 can be obtained from equation 3.14 by script

$$B_1 = \frac{C_1 - C_4}{2} \; ; \; B_2 = \frac{C_2 + C_3}{2} \; ; \; B_3 = \frac{C_1 + C_4}{2} \; ; \; B_4 = \frac{C_3 - C_2}{2} \qquad 3.17$$

3.2.3 Finite Beams

The finite beam also can treat as a semi-infinite beam due to the effect from one end may decayed before the other end reached. However, for decoupling requires that βL≫1, where beam length is L. If condition is not satisfied, wants to retain all the four constants in the homogeneous solution in equation 3.1 and need to determine using four simultaneous equations.

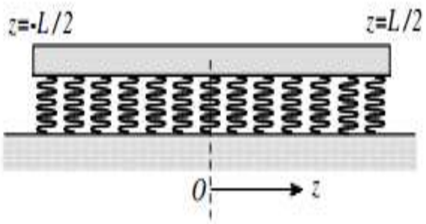


Figure 3.2 Coordinate system for the finite beam

The figure 3.2 shows that the origin moved to the center, and using the hyperbolic equation 3.15 of the homogeneous solution,

$$u(z) = C_1 g_1(\beta z) + C_2 g_2(\beta z) + C_3 g_3(\beta z) + C_4 g_4(\beta z) \qquad 3.18$$

Were,

$g_1(x) = cosh(x)\,cos(x)$; $g_2(x) = sinh(x)\,sin(x)$

$g_3(x) = cosh(x)\,sin(x)$; $g_4(x) = sinh(x)\,cos(x)$

3.19

In it g_1, g_2 are even functions of x and g_3, g_4 are odd functions, so only two of the four arbitrary constants want to be included for symmetric or anti symmetric problems. The derivatives of these functions satisfy the relations given below

$$\frac{dg_1}{dx} = g_4 - g_3 \;;\; \frac{dg_2}{dx} = g_4 + g_3 \;:\; \frac{dg_3}{dx} = g_1 + g_2 \;;\; \frac{dg_4}{dx} = g_1 - g_2 \qquad 3.20$$

And

$$\frac{d^2 g_1}{dx^2} = -2g_2 \;;\; \frac{d^2 g_2}{dx^2} = -2g_1 \;;\; \frac{d^2 g_3}{dx^2} = -2g_4 \;;\; \frac{d^2 g_4}{dx^2} = -2g_3 \qquad 3.21$$

The general equations for the slope, bending moment and shear force is given below

$$\theta(z) = C_1\beta[g_4(\beta z) - g_3(\beta z)] + C_2\beta[g_4(\beta z) + g_3(\beta z)] + C_3\beta[g_1(\beta z) + g_2(\beta z)] + C_4\beta[g_1(\beta z) - g_2(\beta z)] \qquad 3.22$$

$$\boldsymbol{M(z) = \frac{C_1 k}{2\beta^2} g_2(\beta z) - \frac{C_2 k}{2\beta^2} g_1(\beta z) - \frac{C_3 k}{2\beta^2} g_4(\beta z) + \frac{C_4 k}{2\beta^2} g_3(\beta z)} \qquad 3.23$$

$$V(z) = \frac{C_1 k}{2\beta^2}[g_4(\beta z) + g_3(\beta z)] - \frac{C_2 k}{2\beta^2}[g_4(\beta z) - g_3(\beta z)] - \frac{C_3 k}{2\beta^2}[g_1(\beta z) - g_2(\beta z)] + \frac{C_4 k}{2\beta^2}[g_1(\beta z) - g_2(\beta z)] \qquad 3.24$$

3.3 WINKLER MODEL

Winkler model is developed by Winkler. In it he assumes that in elastic foundation the vertical displacement at a particular point is proportional to the pressure at that particular point and the vertical displacement does not depends on the pressure adjacent to that point. The Winkler model is stated as a system of vertical springs which are not mutually dependent. The assumption in Winkler soil model is that only at the loaded zone displacements occurs. The other soil foundation models are Pasternak model, Filonenko-Borodich model, Hetenyi model and Vlasov-Leontiev model. In these model winkler model is the simplest model. The Winkler foundation model is significantly used by designers and analysts because of its simplicity.

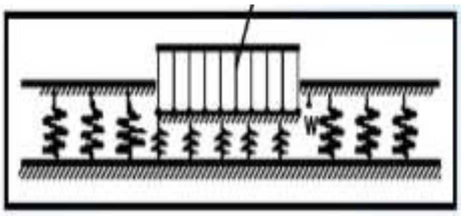


Figure 3.3 Winkler foundation model

3.4 FINITE ELEMENT MODEL

ANSYS software is used for the analysis of beam on elastic foundation. Beam of size 300mm x 200mm, has a length of 2m and E=200GPa is resting on the soil having safe bearing capacity 250 N/m^2 is considered for the analysis. The boundary conditions used were simply supported. Line pressure of 25kN/m is applied on the beam. Meshes are generated and springs are provided on each node in the bottom of the beam. The convergence study is carried out. From the convergence study obtained that with the increase of number of elements in the mesh increases the bending moment of the beam and mesh having eight number of elements is convergent than other meshes. Mesh having eight number of elements is used for parametric studies.

Table 3.1 Convergence study

Number of elements	Maximum bending Moment
2	12426
4	12443
8	12480
16	12487

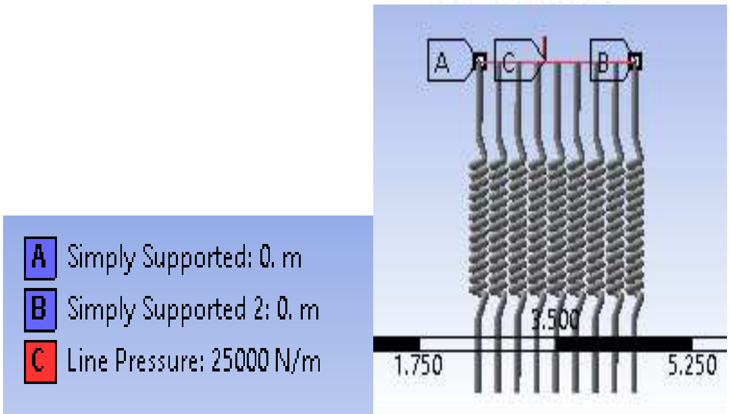


Figure 3.4 model of beam on elastic foundation

3.5 VALIDATION EXAMPLE

Consider a simply supported beam of size 300mm x 200mm, has a length of 2m and E=200GPa. A load P=25kN/m is uniformly distributed over the beam, is rest on soil having safe bearing capacity 250 kN/m^2.

when $\beta L \ll 1$

$$\text{Max Moment, M} = \frac{\text{WL}^2}{8} = \frac{25000 \times 2^2}{8} = 12500Nm \qquad 3.23$$

$$Deflection = \frac{WL^2}{24EI} = 0.1388mm \qquad 3.24$$

Maximum moment obtained from the analysis = 12480 Nm

Deflection obtained from analysis = 0.13166 mm

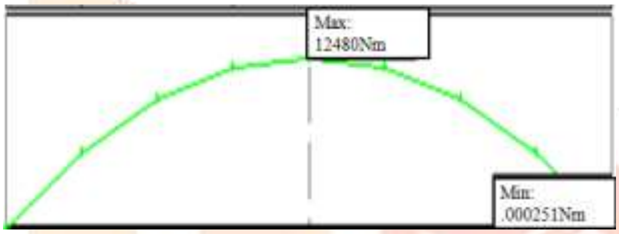


Figure 3.5 Bending moment diagram of beam on winkler foundation

The figure 3.5 shows the bending moment diagram of a beam on elastic foundation and from the figure we can see that maximum bending moment value is 12480 kN

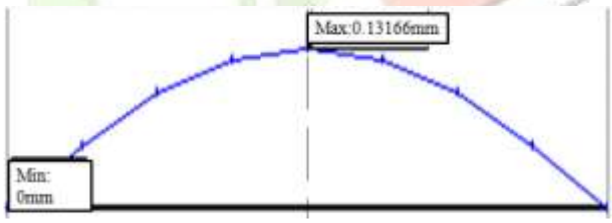


Figure 3.6 Deflection diagram of beam on winkler foundation

The figure 3.6 shows the deflection diagram of a beam on elastic foundation and from the figure we can see that maximum deflection in the beam is 0.13166 mm

3.6 PARAMERTIC STUDY

3.6.1 Safe Bearing Capacity

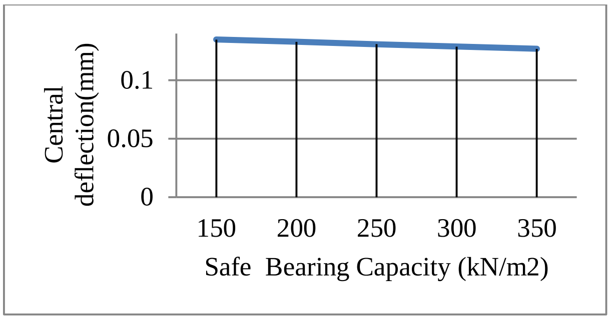


Figure 3.7 Variation in central deflection with respect to safe bearing capacity

From the figure 3.7 obtained that when the safe bearing capacity increases the deflection decreases. The safe bearing capacity of soil represents the strength of soil that means the deflection of the structure on high strength soil is low compared to the deflection of structure on other soil.

3.6.2 Width of Beam

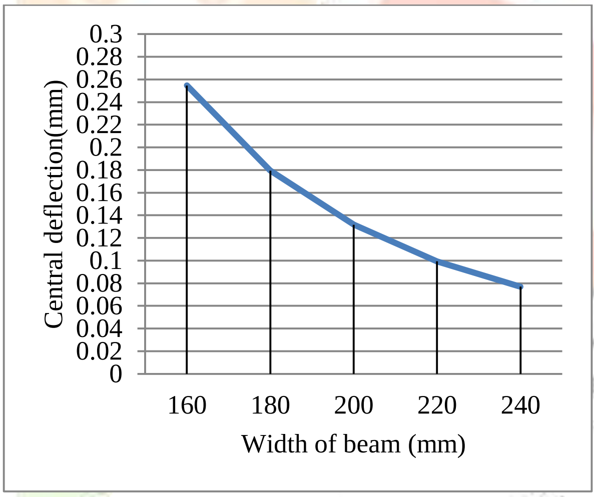


Figure 3.8 Variation in central deflection with respect to width of beam

From the figure 3.8 obtained that when the width of beam increases the deflection decreases. That means for structure having high width can transfer the load for a large area and also possess increased moment of resistance. When the loads acts on the structure transferred to large area and also due to moment of resistance will reduce the deflection of the structure where reduces.

3.6.3 Depth of Beam

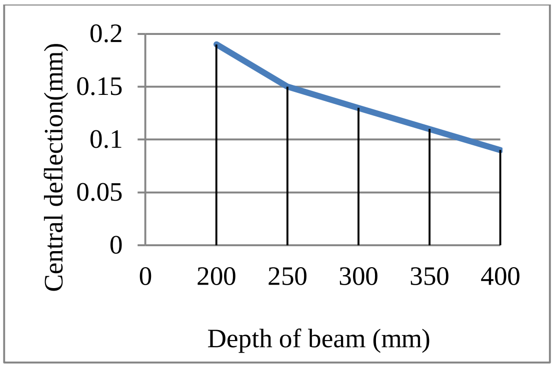


Figure 3.9 Variation in central deflection with respect to depth of beam

From the figure 3.9 obtained that when the depth of beam increases the deflection decreases. By the increase in depth moment of resistance and flexural stiffness are increases. Due to this increase of moment of resistance and flexural stiffness the deflection of the structure reduces.

3.6.4 Young's Modulus of beam

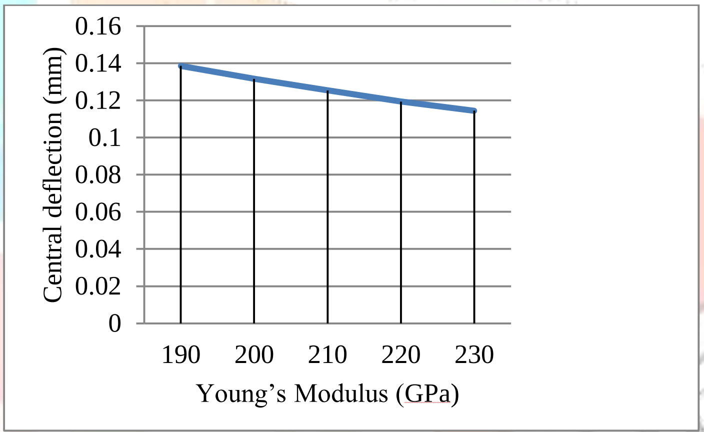


Figure 3.10 Variation in central deflection with respect to young's modulus of beam

From the figure 3.10 obtained that when the young's modulus increases the deflection decreases. As the young's modulus increases the strain will reduce. Due to this reduction in strain results in reduction of deflection.

IV. CONCLUSION

The analysis of beam on elastic foundation is completed and validation, convergence study and parametric study are done. From the parametric study it is observed that the parameters such as safe bearing capacity, width of beam, depth of beam and young's modulus of beam are directly related to the deflection of the beam.

- When the safe bearing capacity of soil increases the defection of the beam decreases.
- When the depth of beam increases the defection of the beam decreases.
- When the width of beam increases the defection of the beam decreases.
- When the young's modulus of beam increases the defection of the beam decreases.

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