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A MATHEMATICAL MODEL FOR THE IMPACT OF PHYSICAL EXERCISES AND NUTRITION ON DIABETIC PATIENTS

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Abstract

The disease of diabetes has become devastating for many people and increasing dramatically throughout the world. Diabetes mellitus is a chronic metabolic disorder characterized by an increase in the blood-glucose level resulting from a relative insulin deficiency or insulin resistance or both. Its effects result in an increased risk of medical comorbidities like heart attack, stroke, recent weight loss etc and consequently reducing the Gross Domestic Product (GDP) of a nation.

The objective of this study is to develop a mathematical model for the Blood Glucose Regulatory System (BGRS) using Linear Non-homogeneous differential equations. Stability of the proposed model is analyzed and observed that complications can be controlled and managed in such a way that, the peak which is the time period for insulin to be most effective in decreasing blood sugar, through regular physical activity and nutrition. It is a matter of attention that after doing physical activity for some minutes, we find that glucose and insulin levels in the blood plasma of diabetic patient decreases back to their normal levels. Thus if a patient does physical activity regularly for a certain period of time and observes correct diet, he does not need any insulin supplements. Stability of the model has been approached from Jacobian matrix method and numerical simulations have been done using various parameter values.

Keywords: *Diabetes mellitus, physical activity, nutrition, stability*

1. Introduction

Diabetes is a chronic disease that occurs when the beta cells in the Langerhans islets of the pancreas does not produce enough insulin or when the body cannot effectively utilize the insulin, it produces. During digestion, carbohydrates are broken down mainly into a simple sugar called glucose. When glucose enters the bloodstream, the pancreas goes into red alert. It secretes insulin into the blood, then the liver and muscles immediately remove glucose from the blood, when it fails to do so then diabetes occurs. It is a growing public health problem and is considered as one of the main threats to human health in the twenty-first century. It imposes a significant burden on patients and society. It is one of the leading causes of complexity of the illness, an increased risk of medical comorbidities like fatigue, recent weight loss, severe restriction in mobility and strength and increased propensity to falls. Healthy diet, regular exercise and maintaining a normal body weight have been recommended to diabetic patients for a long time which does not require more money compared to insulin supplement.

Regular physical activity reduces the risk of occurrence of Non-insulin Dependent Diabetes Mellitus(NIDDM) (Wasserman et al.,1991;Sigal et al.,1994;Sigal et al.,1996).Li et al [9];modeling the glucose insulin regulatory system and ultradian insulin secretory oscillations with two time delays. Derouich and Boutayeb (2002)[11] have used a simple mathematical model to illustrate the role of physical activity in improving insulin sensitivity and regulating blood glucose concentrations. Denghmingliani et al.[3] proposed a mathematical model to study the impact of physical exercises on glycemic regulations.

In this chapter, taking into account the effect of physical exercise and nutrition on the dynamics of glucose and insulin, we propose a mathematical model following the model of Elizabeth Mala, Wilys O. Mukuna and Maurice Owino Oduor(2021) and study the stability of the dynamical system of glucose and insulin but considering the external source of glucose q(t) to be a constant.

2. Materials and Methods

Diabetic model state equations with physical activity and nutrition are as follows:

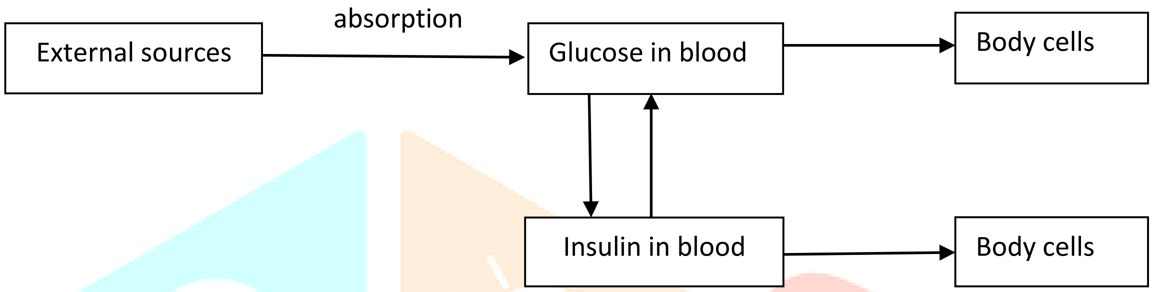

Figure 1: External source works in our body

$g'(t) = -(p_1 + p_3)g(t) + p_4 h(t) + q$ (1)
$h'(t) = p_3 g(t) - (p_2 + p_4)h(t)$ (2)
where
g (t): represents glucose available in blood
h (t): represents insulin available in blood
q: external source of glucose where $q = q_1 + q_2$
q_1: physical activity that fluctuate the glucose level
q_2: nutrition entering the blood
p_1: rate of glucose absorbed by blood cells
p_2: rate at which insulin is digested into body cells
p_3: rate at which excess glucose is converted to glycogen
p_4: rate at which glycogen is converted to glucose
The model given in equation (1) can be written in matrix form as follows:

$$X(t) = \begin{pmatrix} g \\ h \end{pmatrix} \quad (3)$$

where the coefficient matrix is given in the form

$$A = \begin{pmatrix} -(p_1 + p_3) & p_4 \\ p_3 & -(p_2 + p_4) \end{pmatrix} \quad (4)$$

$$B = q \quad (5)$$

Where $q = q_1 + q_2$
Now differentiating the equation (3) we have,

$$X'(t) = A.X(t) + B$$

$$X'(t) = \begin{pmatrix} -(p_1 + p_3) & p_4 \\ p_3 & -(p_2 + p_4) \end{pmatrix} X(t) + \begin{pmatrix} q \\ 0 \end{pmatrix}$$

Where matrix A in equation (4) is the Jacobian matrix and $B = \begin{pmatrix} q \\ 0 \end{pmatrix}$

We have the following equilibrium points,

$$-(p_1 + p_{3)} g(t) + p_4 h(t) + q = 0 \quad (6)$$

$$p_3 g(t) - (p_2 + p_4)h(t) = 0 \quad (7)$$

From equation (7) we have,

$$g(t) = \frac{p_2+p_4}{p_3} h(t) \quad (8)$$

Now substituting equation (8) into equation (6) we have,

$$q + p_4 h(t) - \frac{(p_1+p_3)(p_2+p_4)}{p_3} h(t) = 0 \quad (9)$$

Multiplying both sides by p_3 and regrouping h(t) we have,

$$h(t) = \frac{p_3 q}{(p_1+p_3)(p_2+p_4) - p_4 p_3} = \frac{p_3 q}{p_1p_2+p_1p_4+p_2p_3} \quad (10)$$

3. Stability Analysis

$$g(t) = \frac{(p_2+p_4)q}{p_1p_2+p_1p_4+p_2p_3},$$
$$h(t) = \frac{p_3 q}{p_1p_2+p_1p_4+p_2p_3} \quad (11)$$

Case I: For 3 × 3 matrix, we can check stability in following ways:

a) When $t \to \infty$, most of the solutions fall through and the real parts of some root is positive. Then the system of equation is unstable.

b) When $t \to \infty$, all solutions decompose to zero and the real parts of all roots are negative. Then system of equation is stable.

Case II: For 2×2 matrix, we can check stability as follows:

a) If $\det A < 0$,then we have the real roots with opposite sign and the system of equation is unstable.

b) If $\det A > 0$,then the roots are real but again in this situation we have another two cases:
Case i) If both real roots are positive then we have have to find out trace of the matrix. If $tr\ A > 0$, then the system of equation is unstable.
Case ii) If both real roots are negative and $tr\ A < 0$,then system of equation is stable.

c) If $\det A > 0$ and roots are imaginary then $Real\ \mu = \frac{trA}{2}$
Then again if tr A>0 then equation is unstable and if tr A<0 then equation is stable.

Now we use the characteristic polynomial in following form to find the eigenvalues of matrix A;

$$q(\mu) = \mu^2 - tr\ A + \det A = 0 \quad (12)$$

Where $tr\ A = -(p_1 + p_2 + p_3 + p_4) < 0$

And $\det A = (p_1 + p_3)(p_2 + p_{4)} - p_3 p_4$

$$= p_1p_2 + p_1p_4 + p_2p_3 > 0 \quad (13)$$

Therefore, the eigenvalues of the Jacobian matrix A are,

$$\mu_{1=} \frac{tr\ A + \sqrt{(tr\ A)^2 - 4\det A}}{2}$$

$$\mu_2 = \frac{tr\ A - \sqrt{(tr\ A)^2 - 4\det A}}{2}$$

Since,μ_1<0 and μ_2<0

Therefore,$tr\ A = \mu_1 + \mu_2 < 0$

And $\det A = \mu_1 \mu_2 - p_3 p_4 > 0$

Which implies that μ_1<0 and μ_2<0.Therefore the system of equations (1) is asymptotically stable.

The solution of equation (1) is,

$$X(t) = e^{tA} X_0 + \int_0^t e^{(t-y)A} B(y) dy \quad (13)$$

$e^{tA} = Q e^{tD} Q^{-1}$ where D is the diagonal matrix and Q is the matrix of eigenvectors.

4. Results and Discussion

Different parameter values are obtained from secondary data which is given in the following table. This is the data for glucose plasma levels during Intravenous Glucose Tolerance tests(IVGT) and it is assumed that there is no error in this data collection.

Table 1: Data for Glucose Plasma Levels during IVGT

Time(in minutes)	Glucose(mg/dl)	Insulin(μ/ml)	Time(in minutes)	Glucose(mg/dl)	Insulin(μ/ml)
0	170	10	31	254	48
3	236	12	41	234	35
6	251	85	51	227	27
9	314	53	61	203	25
12	211	45	71	192	22
15	207	41	81	188	19
18	192	90	91	192	18
22	298	124	101	177	19
24	294	150	121	170	8
28	278	160	141	82	12
30	268	114	161	85	12

Secondary values of glucose plasma levels and insulin during Intravenous Glucose Tolerance tests (IVGT) have been used to see the effects of external sources when incorporated so that, the time period for insulin to be most effective is in the acceptable therapeutic range[7].

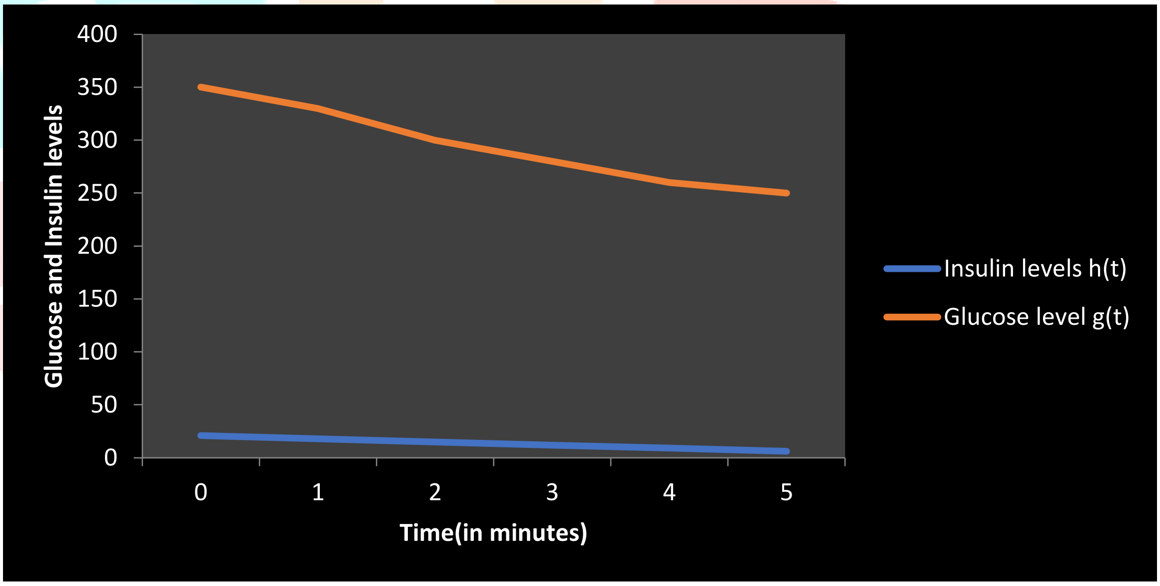

Figure 2: Effects of external rates on levels of glucose and insulin for T=5

From the graph, we can see that at initial level, glucose in blood plasma was almost 350 units and insulin level was also high as approx 50 units. But after doing physical activity for T=5 minutes, glucose level in blood is decreasing by almost 250 units and insulin level is also decreasing in the same way.

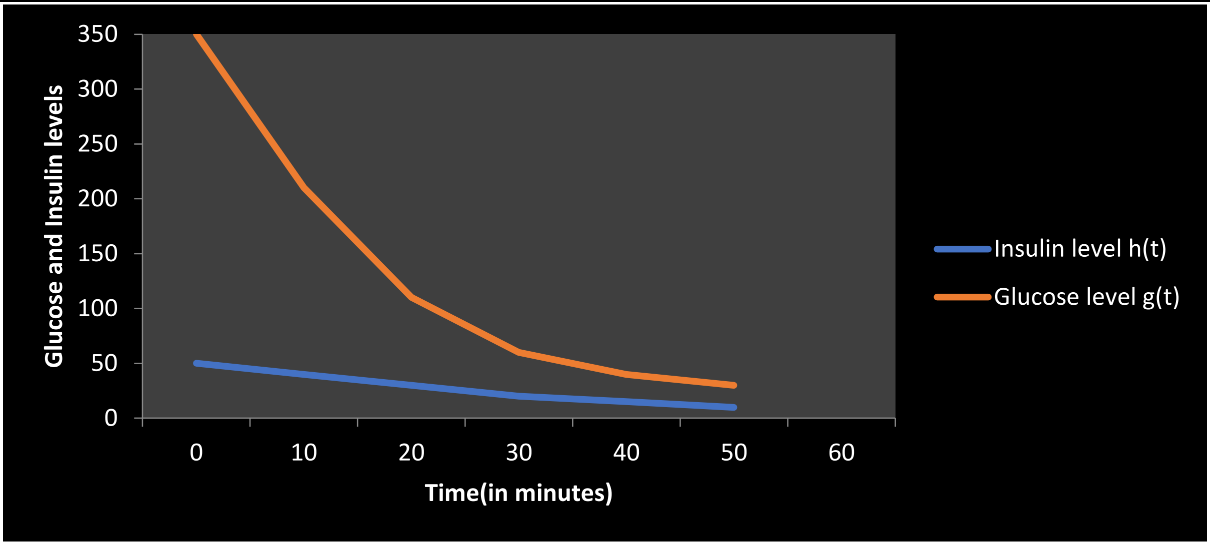

Figure 3: Effects of external rates on Glucose and Insulin levels at T=60 minutes

After doing physical activity for one hour i.e. when T=60 minutes glucose level in blood plasma reduced to 50 units i.e. at normal level and insulin level also decreasing to very low level which we can see in the above graph.

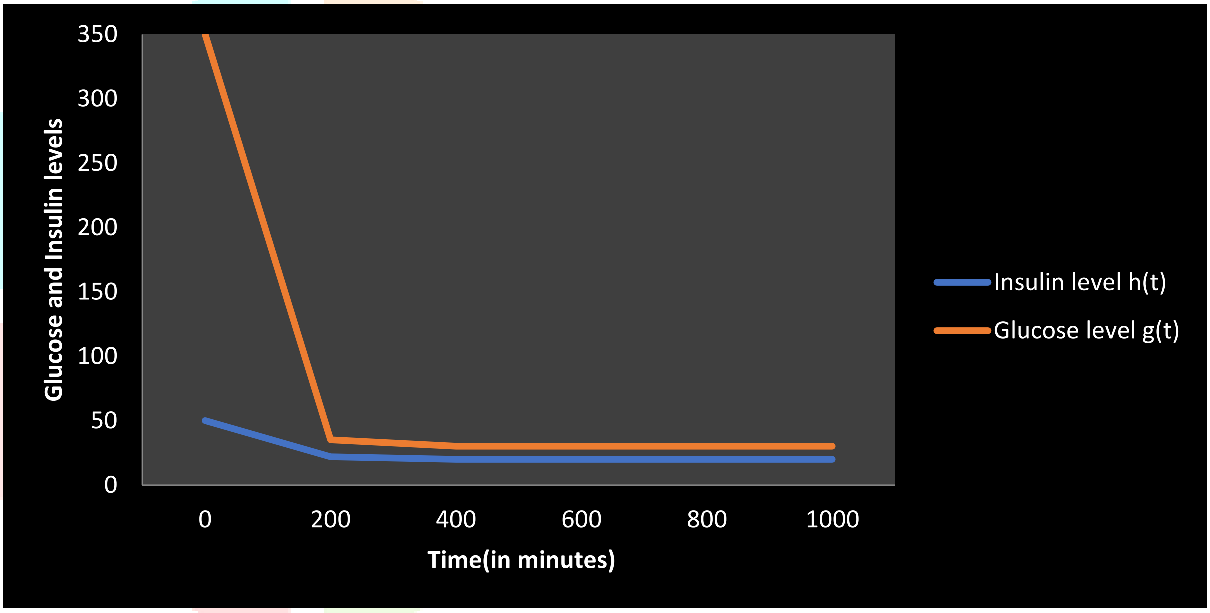

Figure 4: Effects of external rates on glucose and insulin levels for T=1000

Figure 4 shows that after doing physical exercises for t=1000 minutes, glucose and insulin level in blood of diabetic patient decreases back to their normal levels from 100th minute which remain constant. Hence we can say that if a diabetic patient does physical activity regularly and follows the right diet, there is no need of any medication (i.e. insulin supplement).Therefore physical activity and nutrition is an easier and cost effective method for a diabetic patient to control the disease.

5. Conclusion

The main objective of this study is to formulate a mathematical model based on blood glucose regulatory system which describes how external sources works in our body. The model is expressed in the form $X' = AX + B$ which is first order linear non-homogeneous differential equation. Stability of the model is analyzed analytically and numerically. Hence we can conclude that after doing physical exercises for a certain period of time and following the correct diet ,the glucose and insulin level in blood decreases to their normal levels. If a diabetic patient follow the same routine regularly, his/her body will produce enough amount of sugar in blood and insulin level in the body will be enough to control the normal glucose in blood. The body will be able to fight the disease without any expenses.

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