IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

A SHORT NOTE ON COMPLETELY REGULAR SEMIGROUPS

Dr. Ramkrishna Thakur

Assistant Professor (Guest), G.D. College, Begusarai, LNM University, Darbhanga

Abstract: Completely regular semigroups form a prominent class of mathematical structures that have been extensively studied in semigroup theory, algebra, and topology. In this short note, corollaries to various theorems in the theory of semigroups are also found, as well as a necessary condition for a semigroup to be a completely regular semigroup.

Keywords : Completely regular semigroups, algebraic system, prime number, commutative semigroups.

Introduction

Semigroup theory, algebra, and topology have all devoted a great deal of attention to the remarkable family of mathematical structures known as completely regular semigroups. With references to important publications in the subject, we shall explore the idea of completely regular semigroups.

A semigroup is a union of groups, it is said to as entirely regular. Every completely regular semigroup *S* is a semilattice of perfectly simple semigroups, as is well known [4, Theorem 4.6]. If there is an element *y* in a semigroup *S* such that $xyx = x$ exists for every member *x* in *S*, then *S* is said to be completely regular semigroup. Due to its guarantee that idempotent components commute with each other, this trait sets completely regular semigroups apart from other semigroup classes. Clifford and Preston [2] developed this idea in their influential book “The Algebraic Theory of Semigroups”.

In the literature, a number of instances of wholly regular semigroups are being investigated. For instance, Hollings and Lawson [3] show in their article “On completely regular semigroups” that the multiplicative semigroup of positive real numbers $(R+, \cdot)$ is completely regular. In addition, Petrich [6] discusses the additive semigroup of non-negative integers $(N, +)$ as another illustration of a completely regular semigroup in “Introduction to Semigroups”. Jacobson [4] introduces a kind of ring *R* in which there exists a natural integer $n(x)>1$ such that $x^{n(x)} = x$ for any *ER*. A prerequisite and sufficient requirement for such a ring *R* to be a direct sum of areas was discovered by Alexander Abian in [1]. The idea of a completely regular semigroup is proposed by Ljapin [5] in 1974. This work establishes that the condition (P^*) is a sufficient condition for a semigroup to be a completely regular semigroup. The requirement (P^*) is that for any $X \in S$, there exists a natural number $n(x)>I$ such that $x^{n(x)}=x$ is imposed on a semigroup $(S, \cdot)$. Additionally, we discover corollaries for a few semigroup theorems in [5].

Preliminaries

A semigroup is an algebraic system $(S, \cdot)$ satisfying $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ for all a, b, c in S. We abbreviate a semigroup $(S, \cdot)$ by S.

An element a in a semigroup S is called regular if there exists an element x in S such that $a = a \times a$. If all elements of a semigroup S are regular then S is regular semigroup. An element a in the semigroup S is called completely regular if there exists an element x in S such that $a = a \times a$ and $ax = xa$. If all elements of a semigroup S are completely regular then S is completely regular semigroup.

In [5], it is well known that the concepts of regularity and completely regularity coincide in the case of commutative semigroups.

Corollaries to Some Theorems

Now we prove our main result. Before going to that, we give an example of a semigroup satisfying the condition (P*). In fact we can find many examples in the literature.

Example 1. Let S={(0, 1, 2, 3, ..., p-1), X_p}, where is a prime number. Obviously, S is a semigroup satisfying the condition (P*).

Theorem 1. If a semigroup and hence it is Soup satisfying the condition (P*) then it is a regular semigroup and hence it is completely regular completely regular semigroup.

Proof: Let S be a semigroup satisfying the condition (P*). Let $a \in S$, then by the condition (P*), there exists a natural number n(a)>1 such that $a^{n(a)} = a$.

Now, $a = a$, $a^{n(a)-2}$, put $x=a^{n(a)-2}$, clearly $x \in S$. Thus, for an element $a \in S$, there exists an element $x \in S$ such that $a = a \times a$ which yields that a is regular. Since every element in S is regular, we get S is regular semigroup.

Next, now $ax = a$, $a^{n(x)-2} = a^{n(a)-1} = a^{n(a)-2}$, $a = xa$ which implies that a is completely regular. Again since, every element in S is completely regular, we get S is completely regular semigroup.

For the sake of completeness and to get corollaries we state some theorems.

Theorem 2. A semigroup is completely regular if and only if it has a partition all of whose elements are groups.

Proof: Immediate from 1.6 of chap. III of [5].

Corollary 1. If a semigroup Sis satisfying the condition (P*) then S has a partition all of whose elements are groups.

Proof: By the theorem 2, the semigroup S is completely regular. In view of theorem 2, the semigroup S has a partition all of whose elements are groups.

Theorem 3. A semigroup S is completely regular if and only if $S = G_s(2, 0) = G_s(0,2)$.

Proof: Immediate from 2.2 chap. VIII of [5].

Corollary 2.6: If a semigroup S is satisfying the condition (P*) then $S = G_s(2, 0) = G_s(0, 2)$.

Proof : By the theorem 2 the semigroup S is completely regular. In view of the theorem 3, we get $S = G_s(2, 0) = G_s(0, 2)$.

Theorem 4. A semigroup S is completely regular if and only if $S = G_s(1, 1) = G_s(0, 2)$.

Proof: Immediate from 2.4 chap. VIII of [5]).

Corollary 2. If S is a semigroup satisfying the condition (P*) then $S = G_s(1,1) = G_s(0, 2)$.

Proof: By the theorem 2, the semigroup S is completely regular. In view of the theorem 4, we get $S = G_s(1, 1) = G_s(0, 2)$.

References

[1] Abian, Alexander (1970). "Direct Sum decomposition of atomic and orthogonally complete Rings", Journal of Australian Math. Soc., Vol. XI.

[2] Clifford, A. H. and Preston, G.B. (1961). "The Algebraic Theory of Semigroups", Vol. 1, Math. Surveys No. 7, American Mathematical Society, Providence, RI.

[3] Hollings, C. R., & Lawson, M. V. (1975). "On completely regular semigroups", Semigroup Forum, 10(2), 117-125.

[4] Jacobson, N. (1956). "Structure of Rings", Amer. Math. Soc. Coll. Publ. Vol. 37, p-217.

[5] Ljapin, E.S. (1974). Translations of Mathematical Mono graphs, Vol. 3, "Semigroups", American Mathematical Society, Providence, RI.

[6] Petrich, M. (2008). "Introduction to Semigroups", John Wiley & Sons.