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Foreign exchange market portfolio construction using Quantum harmonic oscillator

Atman Bhatt[1], Ravi Gor[2]
[1]*Research scholar, Department of Applied Mathematical Science,*
Actuarial Science and Analytics, Gujarat University
[2]*Department of Applied Mathematical Science,*
Actuarial Science and Analytics, Gujarat University

Abstract:
Many different types of risks like Leverage risk, Interest rate risk, Transaction risk, Counterparty risk and Country risks affects the foreign exchange markets because of its high liquidity and comparatively high trading volume. Therefore, to reduce risk, efficient portfolio for foreign exchange market is very much essential nowadays. This study proposes use of quantum harmonic oscillator model to generate portfolio return ratio and getting idea about portfolio construction. This approach will also help to establish relation between different probabilities of the eigenstates of currency market and its portfolio return.

Keywords: Quantum harmonic oscillator, Leverage risk, Interest rate risk, Transaction risk, Counterparty risk, Country risks, foreign exchange market, Eigenstate and Portfolio return.

Introduction:
Nowadays changes in the world's currency exchange rates have a big impact on investments' value. Investors need to understand how the foreign exchange market affects the assets they own and the extent of their currency exposure. The forex market is highly volatile in nature, so it is crucial to manage risk effectively to become a profitable trader. To manage internal and external risks, one needs to create portfolio of different foreign currency assets and should try to optimize it.

Any collection of financial assets, including stocks, bonds, and cash, is referred to as a "portfolio". Portfolios can be maintained by private investors, financial professionals, hedge funds, banks, and other financial institutions. The goal of portfolio optimization is typically to maximize expected return while to minimize expenses like financial risk.
There are various methods to optimize the portfolio, some of the important methods are mentioned below:

1. Linear programming.
2. Quadratic programming.
3. Nonlinear programming.
4. Mixed integer programming.
5. Meta-heuristic methods.
6. Stochastic programming.
7. Copula based methods.
8. Principal component-based methods.

This paper will use quantum harmonic oscillator approach to optimize the portfolio.

A brief about Quantum Harmonic Oscillator [4]:

- The most common application of eigenvalues is to identify the direction in which the data set has the greatest variance. The variance along an eigenvector's direction in a covariance matrix will increase as the eigenvalue increases (principal component). The stability or instability of a fixed point, commonly referred to as an equilibrium point, can be assessed using eigenvalues. The value of a measurable quantity connected to the wavefunction is referred to as an eigenvalue.
- One must use the Hamiltonian operator on the wavefunction to measure a particle's energy. The kinetic energy operator and the potential energy operator combine to form the Hamiltonian operator.

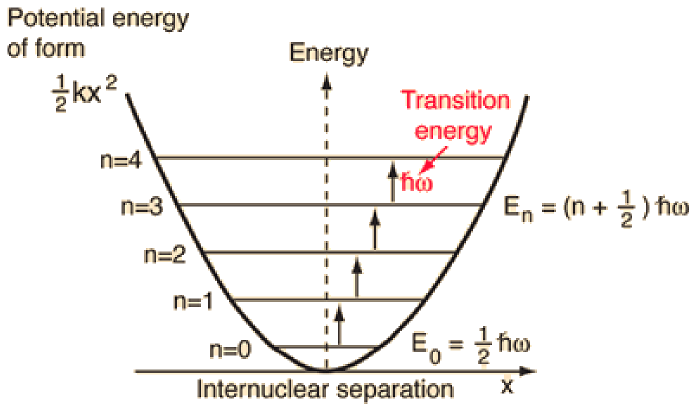


Figure 1

- Amplitude is the value of highest deviation of the item from equilibrium, in either the positive or negative x-direction. Moreover, Fundamental levels of Harmonic motion is always cyclic in nature.
- The time it takes for an object to complete one oscillation and return to its initial position is measured in terms of a period.
- In quantum physics, a body that is experiencing a restoring force proportionate to its displacement from its equilibrium location is referred to as a quantum harmonic oscillator, which is a counterpart to a classical one.
- The least energetic and most stable configuration is the ground state configuration in harmonic oscillator. A higher energy configuration is called an excited state configuration (it requires energy input to create an excited state).
- A diatomic molecule vibrates like two masses being supported by a spring, and its potential energy is inversely proportional to the square of the deviation from equilibrium. But the quantized energy levels have values that are evenly spaced apart.
- The so-called "zero-point vibration" of the $n = 0$ ground state is the most unexpected variation for the quantum scenario. This suggests that even at absolute zero temperatures, molecules are not entirely at rest. Beyond the simple diatomic molecule, the quantum harmonic oscillator has significant implications.

Literature Review:

Ahn et. al. [1] (2018) demonstrate that the quantum harmonic oscillator model outperforms traditional stochastic process models, e.g., Geometric Brownian Motion and the Heston model, with smaller fitting errors and better goodness of fit statistics. The solution of the Schrodinger equation for the quantum harmonic oscillator shows that stock returns follow a mixed distribution, which describes Gaussian and non-Gaussian features of the stock return distribution. In addition, they provide an economic rationale of the physics concepts such as the eigenstate, eigenenergy, and angular frequency, which sheds light on the relationship between Finance and Physics literature.

Jeknić-Dugić [4] (2018) pursued the quantum-mechanical challenge to the efficient market hypothesis for the stock market by employing the Quantum Brownian motion model. He also introduced the external harmonic field for the Brownian particle and use the quantum Caldeira-Leggett master equation as a potential phenomenological model for the stock market price fluctuations.

Lee [5] et al. (2020) examined the weak-form efficient market hypothesis of the crude palm oil market by adopting the quantum harmonic oscillator. This method permits Lee to analyse market efficiency by approximating one constraint: the probability of finding the market in a ground state where conclusion established that the crude palm oil market is more efficient than the West Texas Intermediate crude oil market.

Orrell [8] (2020) addressed issues regarding intrinsically uncertain demand by consuming a quantum context to model supply and demand as, not a cross, but a probabilistic wave, with an allied entropic force. The approach is used to derive from first principles a technique for modelling asset price changes using a quantum harmonic oscillator, that has been previously used and empirically tested in quantum finance. The method is established for a simple system, and claims in other areas of economics are discussed.

Ryu [9] (2021) looked at the weak-form efficient market theory because the log price series for REIT stocks for US REIT equities, contradicted the random walk theory as a model specification, the variance ratio test revealed that the general stock market and REIT markets were not efficient in the weak-form. Instead, he used the quantum harmonic oscillator to present definite evidence. The ground state solution for a random walk included in the quantum harmonic oscillator turned out to be a more effective way to test the efficient market hypothesis.

Bhatt and Gor [2] (2022) showcased an interesting structure of Risk Neutral system. They also examine single step and multistep quantum binomial option pricing model. This approach elaborates circuit proposed by A. Meyer.

Bhatt and Gor [3] (2022) review applications of quantum harmonic oscillator model in financial mathematics and discussed about different applications of quantum harmonic oscillator and its characteristics.

Zhang [10] (2022) enhances the IOAS algorithm's neighbourhood and out-of-bounds movement rules. It then suggests the DETS support vector regression algorithm, which is based on enhanced tabu search and differential evolution and uses error indicators to compare related algorithms. The findings demonstrate the algorithm's effectiveness and viability in exchange rate prediction.

Mba, [6] (2022) examined the typical mean-variance (MV) optimization model in this work by means of two modifications to the MV formulation. Additionally, these results demonstrate that equities with lower behavioural scores do better than counterpart portfolios with higher behavioural scores.

Xiangyi et al. [7] (2022) explores the behaviour of stocks in daily price-limited stock markets. In a quantum spatial-periodic harmonic oscillator potential well, the stock price is oscillating and damping. With the energy band structure of the model in question, a sophisticated non-linear relationship between the volatility and trading volume of a stock is statistically calculated, including inter-band positive correlation and intra-band negative correlation. The efficiency of price limits is re-examined, with this quantum model being used to investigate some observable aspects of price-limited stock markets in China. The Financial Times-Stock Exchange (FTSE) All Share Index's instantaneous return is modelled as a frictionless particle travelling in a one-dimensional square well with a non-trivial probability of tunnelling through the well's retaining walls.

Methodology and Data Collection:

In this paper, three different pairs of foreign currencies say, USDINR, GBPINR and CADINR; were taken for the holding period of one week between the period of 1st January 2021 to 31st December 2021 from YAHOO finance.
Descriptive analysis for all the three pairs of currencies are calculated with the help of MS Excel. The results are mentioned below:

Table 1: Descriptive Analysis

	USDINR	GBPINR	CADINR
Planck Constant	6.26E-34	6.26E-34	6.26E-34
Mean (average return)	0.000300106	0.000217701	0.000386629
Standard Deviation	0.006717247	0.009331408	0.009125824
Delta T (Holding Period in days)	7	7	7
Average Square Value	1.052626409	1.000363415	1.045386135
Average Value	1.025975833	1.000181691	1.022441262
Diffusion Coefficient (D)	0.526313205	0.500181708	0.522693067
Mass (m)	3.72E-67	3.92E-67	3.75E-67
Elastic Constant (k)*	0.0001	0.0001	0.0001
Angular Frequency (w)	1.64E+31	1.59773E+31	1.63329E+31
mw	6.10E-36	6.26E-36	6.12E-36
Gamma	0.00513	0.00500	0.00511
b (Business Evolutionary Pressure)	0.0001	0.0001	0.0001

(* fixed for all the three currencies to make comparison better)

Here Gamma measures how the oscillations of the system decay after an initial force is applied and it gives information about the damping of oscillator. Here proposed model is underdamped for all the three currencies as the value of gamma is less than one for all the three currencies. Value of gamma is calculated as:

$$\gamma = \omega\sqrt{\frac{mD}{2}}$$

As the Fick's First Law states, flux is proportional to the concentration gradient, and the proportionality constant D is the diffusion coefficient. Diffusion coefficient is the ratio of flux density to the negative of the concentration gradient in direction of diffusion. Quantum harmonic oscillator model calculate different parameters which are mentioned in the following tables:

Table 2: Parameters of Quantum Harmonic Oscillator (USDINR)

Quantum Harmonic Oscillator					
USD TO INR					
State	0	1	2	3	4
Energy	0.00513	0.01539	0.025649	0.035909	0.046169
Hermite Polynomial	1	0.202581	-1.958961	-1.207172	11.509216
Amplitude	0.236009	0.166883	0.083442	0.034065	0.012044
Eigen Function	0.234801	0.033634	-0.162623	-0.040912	0.137905
Eigen Value	0.013078	0.006472	0.001602	0.000264	0.000033
Probability	0.793441	0.194331	0.011899	0.000324	0.000005

Table 3: Parameters of Quantum Harmonic Oscillator (GBPINR)

Quantum Harmonic Oscillator					
GBP TO INR					
State	0	1	2	3	4
Energy	0.005001	0.015003	0.025005	0.035006	0.045008
Hermite Polynomial	1	0.200018	-1.959993	-1.192107	11.521513
Amplitude	0.237516	0.167949	0.083975	0.034282	0.012121
Eigen Function	0.236331	0.033425	-0.163769	-0.040665	0.138952
Eigen Value	0.013332	0.006600	0.001634	0.000270	0.000033
Probability	0.793351	0.194410	0.011910	0.000324	0.000005

Table 4: Parameters of Quantum Harmonic Oscillator (CADINR)

Quantum Harmonic Oscillator					
CAD TO INR					
State	0	1	2	3	4
Energy	0.005112	0.015337	0.025561	0.035785	0.046010
Hermite Polynomial	1	0.202232	-1.959102	-1.205119	11.510901
Amplitude	0.236213	0.167027	0.083514	0.034094	0.012054
Eigen Function	0.234981	0.033606	-0.162778	-0.040878	0.138047
Eigen Value	0.013331	0.006598	0.001633	0.000269	0.000033
Probability	0.793428	0.194342	0.011901	0.000324	0.000005

This model gives the value of eigenvalues and its respective probabilities. Now mean value for the quantum harmonic oscillator model is the average value of the summation of the product of eigenvalue and its respective probability.

Below table gives the value for the mean of quantum harmonic oscillator model for all the three currencies

Table 5: Mean value for QHO Model

	USDINR	GBPINR	CADINR
Quantum Harmonic Oscillator Mean (Q)	0.01165348	0.01187984	0.01187879

After calculating the values of mean for the model, this paper introduces concept of deciding weightages. Let w_1, w_2 and w_3 are the weightages decided to put for USDINR, GBPINR and CADINR respectively. Here it is known that $\sum_{i=1}^{3} w_i = 1$.

This model is giving expected portfolio return ratio which is shown below:

$$Expected\ Portfolio\ Return\ Ratio = \sum_{i=1}^{3} w_i Q_i$$

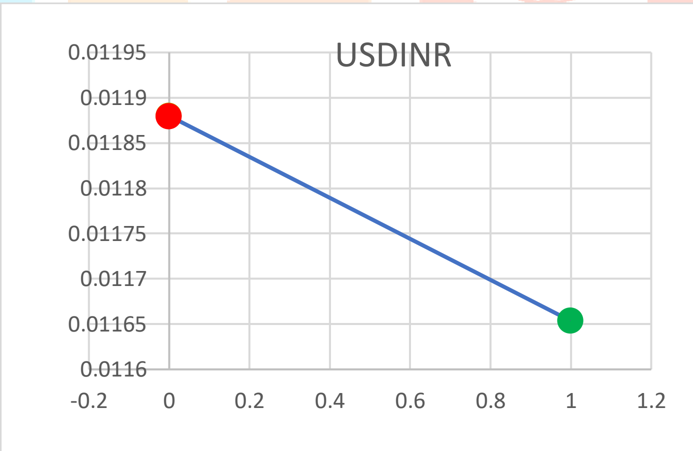
It is mandatory to mention here that the portfolio return ratio is not the actual return for the portfolio but the value which replicate the real value of the expected return. Table 6 showcases the different expected return ratios for the different weightages assigned.

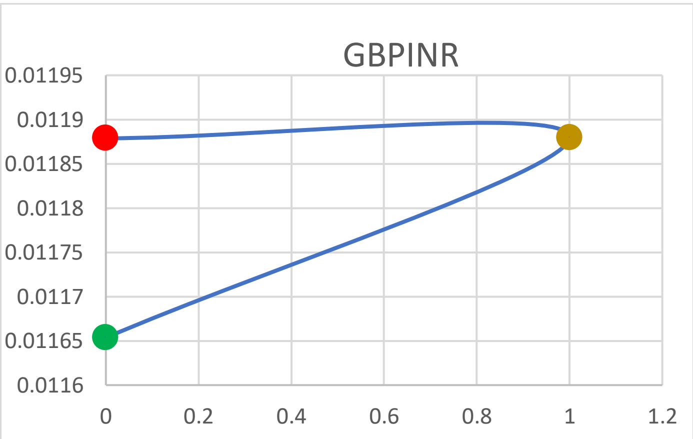
Table 6: Weightages and Return Ratio

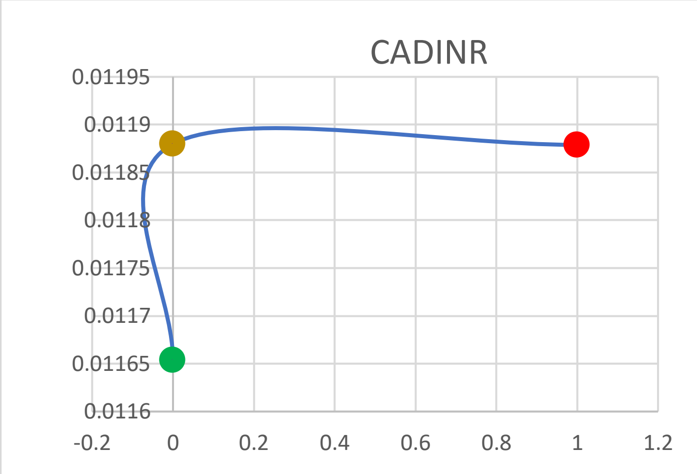
Weightages	USDINR	GBPINR	CADINR	Return Ratio
W_a	0	0	1	1.18788%
W_b	0	0.5	0.5	1.18793%
W_c	0.25	0.25	0.5	1.18227%
W_d	0.25	0.5	0.25	1.18230%
W_e	0.5	0.5	0	1.17667%
W_f	0	1	0	1.18798%
W_g	1	0	0	1.16535%

From the computed values of $W_a, W_b, W_c, W_d, W_e, W_f$ and W_g, it is observed that the weightage of GBPINR increases, the expected portfolio return ratio increases. Similarly, as the weightage of USDINR increases, the expected portfolio return ratio decreases.

Graphical Representation:







Conclusion:

This paper proposes quantum harmonic oscillator model to construct portfolio using various pairs of currency in foreign exchange markets. Here three different currency pairs were taken and using quantum harmonic oscillator model, it is calculated that the GBPINR and CADINR are positively correlated with the expected portfolio return ratio while USDINR is negatively correlated with the expected portfolio return ratio.

Future Scope:

- Sharpe ratio can be calculated by the variance of quantum harmonic oscillator is given by

$$\sigma_n^2 = (2n+1)\left(\frac{\hbar}{2m\omega}\right)$$

- Efficient frontier can be built by the proposed model to offer the highest expected return for a defined level of risk.
- With the help of quantum harmonic model and time series analysis, one can forecast the portfolio return for the desired period.
- This model can be compared with the other models of portfolio optimization to identify the better fitted model.

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