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A Numerical Simulation Of Single Prey And Two Predators Model With Time Delay

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Abstract: **I**n this chapter, we propose a mathematical model comprising a single prey and two predators. In this model two predators (x_2, x_3) are preying on common prey (x_1) and are neutral to each other. All the three species have limited their own natural resources. A Distributed type of delay is included in the interaction prey (x_1) and second predator (x_3). The system is described by a system of integro differential equations. The co-existing state is identified and characterizes the local, global stability analysis at this state. The effect of Time delay on the dynamical behaviour of the system is studied using Numerical simulation**:**

Keywords: Prey, Predator, stability analysis, Numerical simulation.
Mathematics Subject Classification:34DXX

I.INTRODUCTION

The relation between prey-predator models is significant in biological relationships. Differential equations play a significant role to establish such relations. The application approach to study the dynamics of the models are by Braun [9] and Simon's [10]. Mathematical modelling approach using differential equation was initiated by Lokta [1] and Volterra [2]. Stability analysis of biological, ecological, epidemical models is briefly discussed by Kapur [3, 4]. May R M [5], Murry [6] and Freed man [7] explained the wide range of ecological models with detailed analysis.

Naturally any biological or ecological phenomenon not only depend on the current values, but also dependent on previous history. The concept of time delay is proposed and introduced to study system dynamics which depend on previous history. The time lags are classified as discrete, continuous, and distributed type. The appropriate time lags for ecological systems are distributed type and are dealt by authors [11-12]. The time delays are influence the dynamics of the system and tend to destabilize or stabilizes the system. The systems with delay arguments and the qualitative analysis are widely studied by the authors [13-15]. These lags will change the stable equilibrium to unstable or vice versa. The time lags are also significant in epidemiology and the instability tendencies in HIV and SIR epidemic models are dealt by Karuna [16] and Ranjith [17. Three species prey, predator and super predator models were dealt by Shiva Reddy [18]. The three species distributed type delay models with different iterations were extensively studied by Paparao [19-24]. These delay kernels play switchover behaviour from stable to unstable vice versa. Despite above models, we take a single prey and two predators' model for investigation. We studied the dynamics of the model at co-existing state and prove that the system is both locally and globally asymptotically stable.

2. Formation of Mathematical Model

The basic model is with single prey and two predators preying on the same prey species was dealt by Shiva Reddy [18] with exponential growth model. The system dynamics was studied at all possible equilibrium points and shown that the system is both locally and globally asymptotically stable. We proposed the mathematical model with logistic grow type of single prey and two predators. The two predators are generalist type. Paparao et al [25] studied the dynamics of the model and shown that the system is asymptotically stable globally. In spite of that we infuse a distributed time delay in prey-predator model (logistic) in the interaction prey and second predator. The model is characterized by the system of integro differential equations given by

$$\frac{dx_1}{dt} = a_1x_1\left[1-\frac{x_1}{L_1}\right] - \alpha x_1x_2 - \beta x_1\int_{-\infty}^{t} w_1(t-u)x_3(u)du$$

$$\frac{dx_2}{dt} = a_2x_2\left[1-\frac{x_2}{L_2}\right] + \delta x_1(t)x_2(t)$$

$$\frac{dx_3}{dt} = a_3x_3\left[1-\frac{x_3}{L_3}\right] + \varepsilon x_3\int_{-\infty}^{t} w_2(t-u)x_1(u)du \qquad (2.1)$$

With the following notations

$x_1(t)$ Prey population, $x_2(t)$ First predator population, $x_3(t)$ second predator population

$a_i(i=1,2,3)$: Growth rates of three populations

$\alpha,\beta,\delta,\varepsilon$: Mutual interference strengths of three species

L_i Carrying capacities of three species; $w_1(t-u)\&w_2(t-u)$ are weight kernels.

Let $t-u=z$ and substitute in equation (1.1) becomes.

$$\frac{dx_1}{dt} = a_1x_1\left[1-\frac{x_1}{L_1}\right] - \alpha x_1x_2 - \beta x_1\int_0^{\infty} w_1(z)x_3(t-z)dz$$

$$\frac{dx_2}{dt} = a_2x_2\left[1-\frac{x_2}{L_2}\right] + \delta x_1(t)x_2(t) \qquad (2.2)$$

$$\frac{dx_3}{dt} = a_3x_3\left[1-\frac{x_3}{L_3}\right] + \varepsilon x_3\int_0^{\infty} w_2(z)x_1(t-z)dz$$

Assume the solutions for the above model (2.2) as

$x_1 = A_1e^{\lambda t},\quad x_2 = A_2e^{\lambda t},\quad x_3 = A_3e^{\lambda t}$ we get.

$$\frac{dx_1}{dt} = a_1x_1\left[1-\frac{x_1}{L_1}\right] - \alpha x_1x_2 - \beta x_1x_3w_1(\lambda)$$

$$\frac{dx_2}{dt} = a_2x_2\left[1-\frac{x_2}{L_2}\right] + \delta x_2x_1 \qquad (2.3)$$

$$\frac{dx_3}{dt} = a_3x_3\left[1-\frac{x_3}{L_3}\right] + \varepsilon x_1x_3w_2(\lambda)$$

Where $w_1(\lambda) = \int_0^{\infty} w_1(z)e^{-\lambda z}dz$ is the Laplace Transform of $w_1(z)$

and $w_2(\lambda) = \int_0^{\infty} w_2(z)\ e^{-\lambda z}dz$ is the Laplace Transform of $w_2(z)$

All the constants are assumed to be positive.

From equation (2.3) we can write

$$\frac{1}{x_1}\frac{dx_1}{dt} = a_1\left[1-\frac{x_1}{L_1}\right] - \alpha x_2 - \beta x_3w_1(\lambda)$$

$$\frac{1}{x_2}\frac{dx_2}{dt} = a_2\left[1-\frac{x_2}{L_2}\right] + \delta x_1 \qquad (2.4)$$

$$\frac{1}{x_3}\frac{dx_3}{dt} = a_3\left[1-\frac{x_3}{L_3}\right] + \varepsilon x_1w_2(\lambda)$$

From the equation (2.4) **it** is evident that

$$x_1 = x_{10}e^{\int_0^t\left(a_1\left[1-\frac{x_1}{L_1}\right]-\alpha x_2-\beta x_3 w_1(\lambda)\right)dt} > 0$$
$$x_2 = x_{20}e^{\int_0^t\left(a_2\left[1-\frac{x_2}{L_2}\right]+\delta x_1\right)dt} > 0 \qquad (2.5)$$
$$x_3 = x_{30}e^{\int_0^t\left(a_3\left[1-\frac{x_3}{L_3}\right]+\varepsilon x_1 w_2(\lambda)\right)dt} > 0$$

From the above equation (2.5) the system (2.1) admits positive solutions in R_3^+ .

3. Existence of Equilibrium:

The co-existing state of the system (2.1) exist if the following conditions holds good.

(i) $a_1 > \alpha L_2 + \beta w_1(\lambda)L_3$ (ii) $\varepsilon w_2(\lambda)a_2 = \delta a_3$

The co-existing state $E(\overline{x_1}, \overline{x_2}, \overline{x_3})$ given by

$$\overline{x_1} = \frac{L_1 a_1 a_3(a_1 - \alpha L_2 - \beta w_1(\lambda)L_3)}{a_1 a_2 a_3 + a_3\delta\alpha L_1 L_2 + a_2\beta\varepsilon w_1(\lambda)w_2(\lambda)L_1 L_3}$$
$$\overline{x_2} = \frac{L_2[a_1 a_3 a_2 + a_1 a_3 \alpha L_1 + \beta w_1(\lambda)L_1 L_3(\varepsilon w_2(\lambda)a_2 - \delta a_3)}{a_1 a_2 a_3 + a_3\delta\alpha L_1 L_2 + a_2\beta\varepsilon K_1(\lambda)K_2(\lambda)L_1 L_3} \qquad (3.1)$$
$$\overline{x_3} = \frac{L_3[a_1 a_3 a_2 + a_1 a_2 \varepsilon w_2(\lambda)L_1 + \alpha L_1 L_2(\delta a_3 - \varepsilon w_2(\lambda)a_2)}{a_1 a_2 a_3 + a_3\delta\alpha L_1 L_2 + a_2\beta\varepsilon w_1(\lambda)w_2(\lambda)L_1 L_3}$$

4. Local Stability Analysis

Theorem 4.1: The system (2.3) is locally asymptotically stable at co-existing state.
Proof: Consider the Jacobean matrix for the system (2.3) is

$$J = \begin{bmatrix} -\frac{a_1 x_1}{L_1} & -\alpha x_1 & -\beta w_1(\lambda)x_1 \\ \delta x_2 & -\frac{a_2 x_2}{L_2} & 0 \\ \varepsilon w_2(\lambda)x_3 & 0 & -\frac{a_3 x_3}{L_3} \end{bmatrix} \qquad (4.1.1)$$

With the characteristic equation $\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0$
Where

$$b_1 = \left(\frac{a_1 x_1}{L_1} + \frac{a_2 x_2}{L_2} + \frac{a_3 x_3}{L_3}\right)$$
$$b_2 = \left(\frac{a_1 a_2 x_1 x_2}{L_1 L_2} + \frac{a_1 a_3 x_1 x_3}{L_1 L_3} + \frac{a_2 a_3 x_2 x_3}{L_2 L_3} + \alpha\delta x_1 x_2 + \beta\varepsilon w_1(\lambda)w_2(\lambda)x_1 x_2\right)$$
$$b_3 = x_1 x_2 x_3\left(\frac{a_1 a_2 a_3}{L_1 L_2 L_3} + \frac{a_3\delta\alpha}{L_3} + \frac{a_2\beta\varepsilon w_1(\lambda)w_2(\lambda)}{L_2}\right)$$

Calculate the following determinates.

$$D_1 = b_1 = \left(\frac{a_1 x_1}{L_1} + \frac{a_2 x_2}{L_2} + \frac{a_3 x_3}{L_3}\right) > 0$$

$$D_2 = b_1 b_2 - b_3 b_0$$

$$= \left(\frac{a_1 x_1}{L_1} + \frac{a_2 x_2}{L_2} + \frac{a_3 x_3}{L_3}\right)\left(\frac{a_1 a_2 x_1 x_2}{L_1 L_2} + \frac{a_1 a_3 x_1 x_3}{L_1 L_3} + \frac{a_2 a_3 x_2 x_3}{L_2 L_3} + \alpha\delta x_1 x_2 w_1(\lambda) + \beta\varepsilon w_1(\lambda)w_2(\lambda)x_1 x_2\right)$$
$$-x_1 x_2 x_3\left(\frac{a_1 a_2 a_3}{L_1 L_2 L_3} + \frac{a_3\delta\alpha}{L_3} + \frac{a_2\beta\varepsilon w_1(\lambda)w_2(\lambda)}{L_2}\right)$$

$$\left(\begin{array}{c}\frac{a_1^2a_2x_1^2x_2}{L_1^2L_2}+\frac{a_1^2a_3x_1^2x_3}{L_1^2L_3}+2\frac{a_1a_2a_3x_1x_2x_3}{L_1L_2L_3}+\frac{a_1a_2^2x_1x_2^2}{L_1L_2^2}+\frac{a_2^2a_3x_2^2x_3}{L_2^2L_3}+\frac{a_1a_3^2x_1x_3^2}{L_1L_3^2}+\frac{a_2a_3^2x_2x_3^2}{L_2L_3^2}\\+\frac{a_1\alpha\delta x_1^2x_2w_1(\lambda)}{L_1}+\frac{a_2\alpha\delta x_1x_2^2w_1(\lambda)}{L_2}+\frac{a_1\beta\varepsilon w_1(\lambda)w_2(\lambda)x_1^2x_2}{L_1}+\frac{a_2\beta\varepsilon w_1(\lambda)w_2(\lambda)x_1x_2^2}{L_2}\end{array}\right)$$

$D_2 > 0\ at\ (\overline{x_1},\overline{x_2},\overline{x_3})$

$D_3 = (b_1b_2 - b_3b_0)b_3 = b_3D_2 > 0$

Clearly $D_2 > 0$ & $b_3 > 0$ the product is also positive.

Hence $D_3 > 0\ at\ (\overline{x_1},\overline{x_2},\overline{x_3})$

Since all three determinates are positive, by Routh -Hurwitz criteria the co-existing state $E(\overline{x_1},\overline{x_2},\overline{x_3})$ is locally asymptotically stable

5. Global Stability

Theorem 5.1: The co-existing state $E(\overline{x_1},\overline{x_2},\overline{x_3})$ is globally asymptotically stable.
Proof: consider the suitable Lyapunov's function given below.

$$V(x,y,z) = \left[x_1 - \overline{x_1} - \overline{x_1}\,log\left(\frac{x_1}{\overline{x_1}}\right)\right] + m_1\left[x_2 - \overline{x_2} - \overline{x_2}\,log\left(\frac{x_2}{\overline{x_2}}\right)\right] + m_2\left[x_3 - \overline{x_3} - \overline{x_3}\,log\left(\frac{x_3}{\overline{x_3}}\right)\right] \quad (5.1.1)$$

Clearly $V(\overline{x_1},\overline{x_2},\overline{x_3}) = 0$&$V(x_1,x_2,x_3) > 0$
The time derivate of V along the solutions of equations (2.1) is.

$$\frac{dV}{dt} = \frac{dx}{dt}\left[1-\frac{\overline{x_1}}{x_1}\right] + m_1\frac{dy}{dt}\left[1-\frac{\overline{x_2}}{x_2}\right] + m_2\frac{dz}{dt}\left[1-\frac{\overline{x_3}}{x_3}\right] \quad (5.1.2)$$

$$= [x_1-\overline{x_1}]\left[a_1\left(1-\frac{x_1}{L_1}\right) - \alpha x_2 - \beta\int_{-\infty}^{t} w_1\,(t-u)x_3(u)du\right] + m_1[x_2-\overline{x_2}]\left[a_2\left(1-\frac{x_2}{L_2}\right)+\delta x_2\right]$$

$$+m_2[x_3-\overline{x_3}]\left[a_3\left(1-\frac{x_3}{L_1}\right)+\varepsilon\int_{-\infty}^{t} w_2\,(t-u)x_1(u)du\right] \quad (5.1.3)$$

Choose the proper set of values for $a_1, = \frac{a_1\overline{x}}{L_1} + \alpha\overline{x_2} + \beta\int_{-\infty}^{t} w_1\,(t-u)x_3(u)du, a_2 = \frac{a_2\overline{x_2}}{L_2} - \delta\overline{x_2}, a_3 = \frac{a_3\overline{x_3}}{L_3} - \varepsilon\int_{-\infty}^{t} w_2\,(t-u)x_1(u)du$ Then (5.1.3) becomes

$$\frac{dV}{dt} = -\frac{a_1}{L_1}(x_1-\overline{x_1})^2 - \frac{a_2}{L_2}(x_2-\overline{x_2})^2 - \frac{a_3}{L_3}(x_3-\overline{x_3})^2 + (x_1-\overline{x_1})(x_2-\overline{x_2})(\delta m_1 - \alpha)$$

choose $m_1 = \frac{\alpha}{\delta}$. $m_2 = 1$

$$\frac{dV}{dt} = -\frac{a_1}{L_1}(x_1-\overline{x_1})^2 - \frac{a_2}{L_2}(x_2-\overline{x_2})^2 - \frac{a_3}{L_3}(x_3-\overline{x_3})^2$$

$$\frac{dV}{dt} = -\left[\frac{a_1}{L_1}(x-\overline{x})^2 + \frac{a_2}{L_2}(y-\overline{y})^2 + \frac{a_3}{L_3}(z-\overline{z})^2\right]$$

Hence $\frac{dV}{dt} \leq 0$
Therefore, the co –existing state $E(\overline{x_1},\overline{x_2},\overline{x_3})$ is globally asymptotically stable.

Theorem 5.2: The system (1.3) cannot have any periodic orbits in the interior of the quadrant.

Proof: Using Bendixen -Dulac criterion we establish a dulac function $H(x_1,x_2,x_3) = \frac{1}{x_1x_2x_3}$

And define.

$$h_1(x_1,x_2,x_3) = a_1x_1\left[1-\frac{x_1}{L_1}\right] - \alpha x_1x_2 - \beta x_1x_3w_1(\lambda)$$
$$h_2(x_1,x_2,x_3) = a_2x_2\left[1-\frac{x_2}{L_2}\right] + \delta x_2x_1 \qquad (5.2.1)$$
$$h_3(x_1,x_2,x_3) = a_3x_3\left[1-\frac{x_3}{L_3}\right] + \varepsilon x_1x_3w_2(\lambda)$$

Clearly function $H(x_1,x_2,x_3) = \frac{1}{x_1x_2x_3}$ is a positive (since the population strengths x_1,x_2,x_3 are positive values) in the interior positive octant of $x_1x_2x_3$ space.

Calculate $\Delta(x_1,x_2,x_3)$ which is given by $\frac{\partial(H\,h_1)}{\partial x_1}+\frac{\partial(H\,h_2)}{\partial x_2}+\frac{\partial(H\,h_3)}{\partial x_3}$

$$\frac{\partial}{\partial x}\left(\frac{1}{x_1x_2x_3}\left\{a_1x_1\left[1-\frac{x_1}{L_1}\right]-\alpha x_1x_2-\beta x_1x_3w_1(\lambda)\right\}\right)+\frac{\partial}{\partial y}\left(\frac{1}{x_1x_2x_3}\left\{a_2x_2\left[1-\frac{x_2}{L_2}\right]+\delta x_2x_1\right\}\right)$$
$$+\frac{\partial}{\partial z}\left(\frac{1}{x_1x_2x_3}\left\{a_3x_3\left[1-\frac{x_3}{L_3}\right]+\varepsilon x_1x_3w_2(\lambda)\right\}\right)$$

$$=\frac{\partial}{\partial x_1}\left(\frac{1}{x_2x_3}\left\{a_1\left[1-\frac{x_1}{L_1}\right]-\alpha x_2-\beta x_3w_1(\lambda)\right\}\right)+\frac{\partial}{\partial x_2}\left(\frac{1}{x_1x_3}\left\{a_2\left[1-\frac{x_2}{L_2}\right]+\delta x_1\right\}\right)+\frac{\partial}{\partial x_3}\left(\frac{1}{x_1x_2}\left\{a_3\left[1-\frac{x_3}{L_3}\right]+\varepsilon x_1w_2(\lambda)\right\}\right)$$

$$=-\left(\frac{a_1}{x_2x_3}+\frac{a_2}{x_1x_3}+\frac{a_3}{x_1x_2}\right)<0$$

This shows that $\Delta(x_1,x_2,x_3)<0$

Therefore $\Delta(x_1,x_2,x_3)$ does not change the sign and identically zero in the positive quadrant of $x_1x_2x_3$ space. hence the system (1.3) does not produce any closed orbits and periodic oscillation.

6. Numerical Simulation:

Numerical simulation is performed using the parametric values given in example (5.1) exponential kernel described with $w_1(\lambda) = w_2(\lambda) = \frac{a}{\lambda+a}$ becomes

$$\frac{dx_1}{dt} = a_1x_1\left[1-\frac{x_1}{L_1}\right]-\alpha x_1x_2-\beta x_1x_3\frac{a}{\lambda+a}$$
$$\frac{dx_2}{dt} = a_2x_2\left[1-\frac{x_2}{L_2}\right]+\delta x_2x_1 \qquad (6.1)$$
$$\frac{dx_3}{dt} = a_3x_3\left[1-\frac{x_3}{L_3}\right]+\varepsilon x_1x_3\frac{a}{\lambda+a}$$

The figure(A) represents Time series plot and figure (B) represents phase portrait.

Example:5.1 $a_1 = 12, a_2 = 3, a_3 = 4, \alpha = 0.01, \beta = 0.1, \delta = 0.02, \varepsilon = 0.05, L_1 = 150, L_2 = 150, L_3 = 150, x_1 = 20, x_2 = 10, x_3 = 10$

with the above parametric values, the simulation is carried out for the system of equations (6.1) without impose delay arguments converging to fixed equilibrium point E (45,374,234) shown in the graphs 6.1(A) & 6.1 (B) respectively.

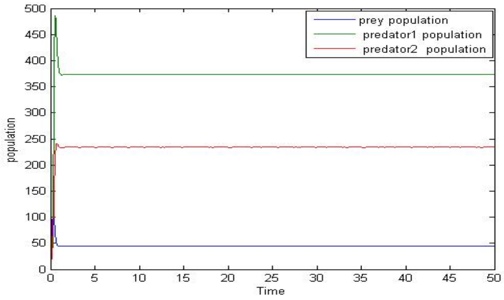

Fig. 6.1(A)

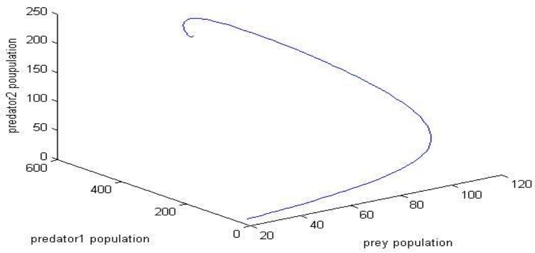

Fig. 6.1 (B)

Defined as follows with different kernel strengths as

Case (1) for a = 0.01, λ = 5.

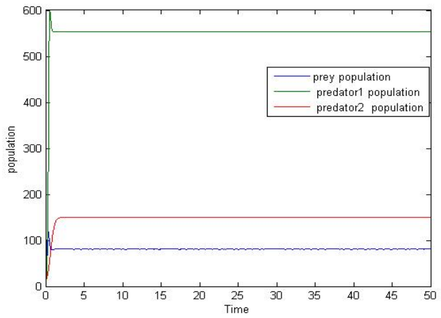

Fig. 6.1.1(A)

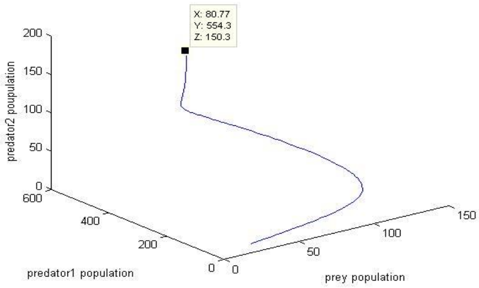

Fig. 6.1.2 (B)

Converging to fixed equilibrium point E (81,554,150)

Case (ii) a = 0.1, λ = 5 for this kernel strengths the system (2.3) converges to a fixed equilibrium point E (80,551,152)

Case (iii) a = 0.5, λ = 5 for this kernel strengths the system (2.3) converges to a fixed equilibrium point E (78,542,163)

Case (iv) a = 5, λ = 5 for this kernel strengths the system (2.3) converges to a fixed equilibrium point E (65,472,211)

Case(v) a = 50, λ = 50 for this kernel strengths the system (2.3) converges to a fixed equilibrium point E (65,472,211)

As on the weight kernel strength increase from $0.01 < a < 50$ and $0.5 < \lambda < 50$ the prey population & first predator population decreases, and second predator population increase when compared with the dynamics of the system without delay arguments.

7. Conclusion:

In this chapter we studied the delay dynamics of three species ecological model with a single prey and two predators. The system of integro-differential equations describes the system. The co-existing state is identified and studied the local and global dynamics of the model. The system does not admit any periodic oscillations shown using Dulac criteria. Hence the system is both locally and globally asymptotically stable.

Numerical simulation is performed with suitable parametric values and exponential type delay kernel and shown that the system is stable for different types of delay kernel strengths and the weight are significant in influencing the population dynamics.

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