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TWO GENERALIZED CLASSES OF ESTIMATORSQUASI-MIMIMAX AND MOCK-MINIMAX IN LARGE SAMPLE ASYMPTOTIC APPROACH

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ABSTRACT

In this paper concerns with proposing two generalızed classes of estimators utilizing both types of information, apriori and sample, for the estimation of coefficient vector inclassical linear regression model. Bias vectors, mean square matrices and weighted quadratic risks are found employing large asymptotic theories. Some better estimators in the sense of having lesser risk than those of the estimators already in the literature are investigated. In generalized classes of estimators in regression models with apron information deals with developing generalized classes ofestimators. There two generalized classes of estimator **QUASI-MIMIMAX and MOCK-MINIMAX** estimators utilizing sample in large sample asymptotic approach due to **KADANE (1871),** we find the approximation bias, risk associated with b^* and b^* and compare from Ordinary least Square estimator b.

THE MODEL AND THE ESTIMATORS

Let the linear regression model be

$$y = x\beta + u$$

Where y is $T \times 1$ vector of observations on the variable to be explained, X isa non - stochastic TXP full Column rank matrix of observations on p- explanatory variable, β is a $p \times 1$ vector of regression coefficient and u is $T \times 1$ vector of disturbances. They found the minima linear estimator of β to

$$\hat{\beta}_1 = [x'x + \sigma^2 H^{-1} x'y]$$

$$\hat{\beta}_2 = [1 - \frac{\sigma^2 + rA(x'x)^{-1}}{C_2(AH^{-1}) + \sigma^2 + rA(xx)^{-1}}]$$

He gave the adaptive or operational version of estimators $\hat{\beta}_1$ and $\hat{\beta}_2$ as

$$b_1 = \{x'x + \hat{\sigma}^2 H\}^{-1} x'y$$

$$b_2 = \{1 - \frac{\hat{\sigma}^2 \mathrm{tr} A(x'X)^{-1}}{C(AH^{-1}) + \hat{\sigma}^2 \mathrm{tr}\, A(x'X)^{-1}}\} b$$

and call there estimators as the quasi minimax estimators and the mock-minimax estimator respectively

Replacing σ^2 involved in the quasi minimax estimator b_1 and the mock- minimax estimator b_2 by the generalized estimator σ_g^2 of σ^2, we get the following proposed estimators

$$b_1^* = \{x'x + \hat{\sigma}_g^2 H\}^{-1} x'y$$

and

$$\} b \frac{\hat{\sigma}^2 \mathrm{tr}\, A(x'X)^{-1}}{2} \qquad b^*_g = \{1 - \frac{g}{k(AH^{-1}) + \hat{\sigma}^2 \mathrm{tr}\, A(X'X)^{-1}}$$

and we call these estimators as the generalized quasi minimax estimator and the generalized mock-minimax estimator respectively.

We have

$$b^*_1 = \{x'x + a_g^2 H\}^{-1} x'y$$

$$= \{I + \hat{a}^2_g (x'x)^{-1} H^{-1} (x'x)^{-1} x'y.$$

Expanding $g(u)$ in $\sigma^2_g = s^2 g(u)$ in second order Taylor's series about the point $u = 1$ and noting that $g(u = 1) = 1$,
we get

$$\hat{\sigma}^2_g = s^2 \left[g(1) + (u^* - 1)g'(1) + \frac{(u^* - 1)^2}{2!} g''(1) + \cdots\right]$$

$$= s^2 \left[1 + (u^* - 1)g'(1) + \frac{(u^* - 1)^2}{?!} g''(1) + \cdots \ldots\right]$$

$$= s^2 \left\{1 + \left(\frac{\sigma_0^2}{s^2} - 1\right) g'(1) + \frac{\left(\frac{\sigma_0^2}{s^2} - 1\right)^2}{2} g''(1) + \cdots \ldots \ldots\right\}$$

$$= s^2 + s^2\left(\frac{\sigma_0^2 - s^2}{s^2}\right) g'(1) + \frac{s^2(u^* - 1)^2}{2} g''(1) + \cdots \ldots$$

$$= (\sigma^2 + \epsilon) + (\sigma_0^2 - \sigma^2 - \epsilon) g'(1) + \frac{s^2(u^* - 1)^2}{2} g''(1) + \cdots$$

where $s^2 - \sigma^2 = \epsilon$. Substituting σ^2_g,
we observe that

$$b_1^* = [x'x + \sigma_g^2 H]^{-1} x'y$$

$$\begin{aligned}\mathrm{MSE}(b_1^*) = \; & \sigma^2(x'x)^{-1} - 2a^2\{o^2 g'(1) + o_0^2(1 - g'(1))\} \\ & \cdot (x'x)^{-1}H(x'x)^{-1} + \{o^2 g_0'(1) \\ & + \sigma^2(1 - g'(1))\}^2 (x'X)^{-1}H\beta\beta'H(x'x)^{-1}\end{aligned}$$

$\mathrm{Var}(b_1^*)$ up to order $O(T^{-2})$ is

$$\begin{aligned}\mathrm{Var}(b_1^*) = \; & \mathrm{MSE}(b_1^*) - \{\text{ Bas } (b_1^*)\}^2 \\ = \; & \sigma^2(x'x)^{-1} - 2\sigma^2\{\sigma^2 g_0'(1) + \sigma^2(1 - g'(1))\}(x'x)^{-1} \\ & \cdot H(x'x)^{-1} + \{\sigma^2 g_0'(1) + \sigma^2(1 - g'(1))\}^2 \\ & \cdot (x'x)^{-1}H\beta\beta'H(x'x)^{-1} \\ & -\{\sigma_0^2 g'(1) + \sigma^2(1 - g'(1))\}^2(x'x)^{-1}H\beta\beta'H(x'x)^{-1} \\ = \; & \sigma^2(x'x)^{-1} - 2\sigma^2\{\sigma^2 g_0'(1) + \sigma^2(1 - g'(1))\} \\ & \cdot (x'x)^{-1}H(x'x)^{-1}\end{aligned}$$

Weighted quadratic risk upto order o(T^{-2})

$$\begin{aligned}R(b_1^*, A) = \; & E(b_1^* - \beta)A(b_1^* - \beta) \\ = \; & \sigma^2 A(x'x)^{-1} - 2\sigma^2\{\sigma^2 g'(1) + \sigma^2(1 -_0 g'(1))\}^2 \\ & \cdot \mathrm{trA}(x'x)^{-1}H(x'x)^{-1} + \{\sigma^2 g'(1) +_0 \sigma^2(1 - g'(1))\}^2 \\ & \beta'H(x'x)^{-1}A(x'x)^{-1}H\beta\end{aligned}$$

Minimax risk up to order O(T^{-2})

$$\begin{aligned}\rho(b_1^*, A) = \; & \sup_{B^F H \le 1} R(b_1^*, A) \\ = \; & \sigma^2 \mathrm{trA}(X'X)^{-1} - 2\sigma^2\{\sigma^2 g_0'(1) + \sigma^2(1 - g'(1))\} \\ & - \mathrm{trA}(X'X)^{-1}H(X'X)^{-1} + \{\sigma^2 g_0'(1) \\ & + \sigma^2(1 - g'(1))\}^2 \text{的}.\ H(X'X)-^1 A(X'X)-^1\end{aligned}$$

Proceedings on the some lines as for b_1^*, the bias up to order $O(T^{-1})$, $\mathrm{MSE}(b_2^*)$ and the risk $\mathrm{R}(b_2^*\ A_4)$ up to order O(T^{-2}) for the estimator b_2^* are found to be

$$\text{Bias } (D^*) = -\{0^2 g'(1) + o^2(1 - g'(1))\} \cdot \frac{\text{tEA}(x'x)^{-1}}{\text{Ch}(A^{-1})} \cdot \beta$$

2 0

$$\text{MSE}(b^*) = \sigma^2(x'x)^{-1} - 2\sigma^2\{g'(1)\sigma^2 + (1 - g'(1))\sigma^2\}$$

2 0

$$\cdot \frac{\text{trAH}(x'x)^{-1}}{\text{Ch}(\text{AH}^{-1})} \cdot (x'x)^{-1}$$

$$+\{g'(1)\sigma^2 + (1 - g'(1))\sigma^2\}^2$$

0 2

$$\cdot \left(\frac{\text{trAH}(x'x)^{-1}}{\text{Ch}(\text{AH}^{-1})}\right) \beta\beta'$$

$$\text{Var}(b^*) = \text{MSE}(b^*) - (\text{Bias } b^*)^2$$

2 2 2

$$= \sigma^2 I(x'x)^{-1} - 2\sigma^2\{g'(1)\sigma^2$$

0

$$+(1 - g'(1))\sigma^2\}(x'x)^{-1}H(x'x)^{-1}$$

Weighted risk for b* upto $O(\text{ T}^{-2})$

2

$$R(B^*, A) = E(B^* - \beta) \cdot A(B^* - \theta)$$

2 2 2

$$= \sigma^2 \text{trA}(x'x)^{-1} - 2a^2\{\sigma^2 g'(1)$$

0

$$+(1 - g'(1))\sigma^2\} \frac{\text{trA}(x'x)^{-1} 2}{\text{Ch}(\text{AH}^{-1})}$$

$$+\{g'(1)\sigma^2 + (1 - g'(1))^2\}^2$$

0 2

$$\cdot \left(\frac{\text{trA}(X'X)^{-1}}{\text{Ch}(\text{AH}^{-1})}\right) (\beta' AB)$$

Minimax risk up to order $O(\text{ T}^{-1})$

$$\rho(b^*, A) = \sup_{\beta^F H\beta \leq 1} R(b^*, A)$$

$$= \sigma^2 \mathrm{tr} A(x'x)^{-1} + \{g'(1)\sigma^2_0 + (1-g'(1))\sigma^2\} \cdot \{g'(1)\sigma^2_0 - (1-g'(1))\sigma^2\} \frac{(\mathrm{tr}\, A(x'x)^{-1})^2}{\mathrm{Ch}(AH^{-1})}$$

COMPARISON OF ESTIMATORS

(a) From the quadratic risk of the generalized quasi minimax estimator b^* given by under large asymptotic theory, we see that b^* will be superior to ordinary least squares estimator b in the sense of having smaller risk if andonly if

$$\beta' H(X'X)^{-1}A(X'X)^{-1}H\beta < \frac{2\sigma^2}{g'(1)\sigma^2_0 + (1-g'(1))\sigma^2} - \mathrm{tr}\, A(X'X)^{-1}H(X'X)^{-1}$$

A sufficient condition for *to hold true is*

$$\frac{Ch^{-}A(X'X)^{-1}H(X'X)^{-1}}{\mathrm{tr}\, A(X'X)^{-1}H(X'X)^{-1}} < \frac{2o^2}{g'(1)o^2_0 + (1-g'(1))\sigma^2}$$

which will be satisfied if

$$\frac{2o^2}{g'(1)0^2_0 + (1-g'(1))0^2} > 1$$

or if

$$\sigma^2 > \frac{\sigma^2}{1 + g'(1)} \qquad \frac{g'(1)}{0} \quad \rule{2cm}{0.4pt}$$

Further, from the generalized Mock minimax estimator $b^\star$ is superior to the least squares estimator b in the sense of having smaller risk if and only if 2

A sufficient condition for to hold true is

$$\left(\frac{\sigma^2}{\sigma_0^2}\right) > (\quad , \quad) \quad - \quad \frac{g'(1)}{1 + g\ (1)}$$

which is the same as obtained for the superiority of the generalized quasiminimax estimator b^* over the (ols) ordinary least square estimator b_1.

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