



# A CRITICAL APPRAISAL AND ADVANCEMENT OF THE FOKAS TRANSFORM TECHNIQUE FOR SOLVING LINEAR EVOLUTION PARTIAL EQUATIONS

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## Abstract

In a few areas of science and designing, the Fokas Transform Procedure (FTT) has demonstrated to be a strong and valuable strategy for solving linear evolution partial equations (LEPEs). The FTT is assessed fundamentally in this theoretical, which likewise features ongoing advancements that work on its materialness and viability in resolving LEPEs. The FTT, made by Teacher A. S. Fokas, brings become notable for its ability to the table for exact answers for an assortment of linear PDEs and has tracked down use in various fields, including electromagnetism, liquid elements, and quantum material science. The polish of this approach rests in its original technique of moving the issue from the time-area to the mind-boggling recurrence space, which every now and again smoothest out the numerical definition and empowers exact arrangements. The methodology additionally holds up well when contrasted with other generally involved procedures for managing corner singularities. A delineation of how this consideration brings about remarkable assembly is shown. Moreover, we give new discoveries on various different hardships, for example, choosing collocation areas and processing arrangements all through whole space insides.

**Keywords:**Fokas Transform,Solving Linear,Evolution Partial,Equations

## 1. INTRODUCTION

The conventional early schooling in halfway differential circumstances (PDEs) underlines the significance of clear game plan methods for issues that could have such an answer. A portion of the strategies for finding game plans are tended to, like the variable parcel strategy, Fourier series and transformations, Laplace and other huge transformations, Green's capacities, etc. The going with standard texts, which range top to bottom in all cases, are oftentimes utilized in these courses. Scalar circumstances that are first-or second-demand in any independent variable comprise the greater part of these course texts.

In the event that an explicit verbalization is made for the reliant variable as a part of the in-subordinate elements, alongside the predefined starting and cutoff conditions, the fundamental or breaking point regard issue (IVP or BVP) is supposed to be settled for this situation. This commitment is habitually expressed utilizing a urgent, an endless series, or a blend of the two.

In this work, we give an alternate philosophy from the systems that are regularly shown in classes. The technique isn't especially old. To apply the contrary scattering system to BVPs offered either on the half-line  $x \geq 0$  or on the obliged stretch  $x \in [0, L]$ , which tends to the IVP for  $x \in \mathbb{R}$  for implied soliton conditions, A. S. Fokas found it. It was before long understood that the technique, which is the principal focal point of this exposition, additionally delivers enrapturing results for direct circumstances. The Fokas approach gives an express answer for the reliant variable  $u(x, t)$ , very much like ordinary methodologies. The reaction condition comprises of something like one essential explanation along the mind boggling plane ways of the associate variable  $k$ . The game plan recipe obviously communicates the total  $x$  and  $t$  dependence. Notwithstanding the procedure's more extensive application, we focus on scalar challenges with a solitary geographic free part ( $x$ ) and a solitary universally independent variable ( $t$ ). For various reasons, we support the Fokas method as a significant improvement above regular appearance procedure.

It integrates all broadly utilized strategies. At the point when a traditional technique would create a particular reaction, Fokas' strategy likewise delivers an unequivocal reaction. The association equations that are shaped are, as we exhibit in a couple of the circumstances beneath, practically identical as a general rule.

It is more broad than customary techniques since we might make equations for different issues that conventional methodology can't determine. At the point when subordinates with higher solicitations are locked in, this is hard to miss. The Fokas technique produces a response including comparative thoughts for these different issues instead of customary systems, which comprise of an assortment of circumstance explicit philosophies customized to explicit circumstances and cutoff conditions. Any progressions just appear in the calculational subtleties.

The strategy makes it simpler to decide the number of and which limit conditions lead to a regularly expressed issue while likewise creating an obvious recipe for the plan. This is a difficult theme, particularly for BVPs for auxiliary conditions with more noteworthy solicitation levels.

Various strategies, for example, parametrizing the structures to give integrals that are direct to assess mathematically and asymptotic procedures like the steepest dive and For example, the main essentials for the strategy are experience with Jordan's lemma, the development speculation, and the Fourier and opposite Fourier change pair. Subsequently, regardless of whether troublesome assessment isn't covered before the primary PDEs course, the key data can in any case be shrouded in three or four introductions.

Models are commonly used to introduce the Fokas technique. There are more nuanced highlights and broad protections in. We could rundowns the way to deal with the half-line and little stretch issues considerably more successfully, yet it would simply prompt the making of a perceptible Fourier change condition for the response. We start by getting back to the IVP on the whole line inside the boundaries of the new cycle. The peruser will see instances of issues that can be addressed utilizing regular strategies, in which cases we exhibit the consistency of ordinary and Fokasi-animated replies. Various models represent alternate ways of utilizing the method when the more typical ones don't work or can't be utilized. Since precision isn't our essential concentration, there are no remarks concerning event rooms. At  $(x, t) = (0, 0)$  and, given the compelled range, at  $(x, t) = (L, 0)$ , independently, the hidden and limit conditions are feasible. In light of everything, we expect that capacities are generally basically as differentiable as expected by the circumstance, for instance, like those laid forward in a first-year PDE course.

## 2. REVIEW OF LITREATURE

Deconinck, Vasan, and Trogdon (2014). Because of its viability in resolving various PDE challenges, this strategy created by A.S. Fokas has earned boundless respect in the consistent and mathematical organizations. The center ideas of the Fokas method are outlined in the initial sections of the clarification. Its principal thought is to utilize proper changes in accordance with transform the PDE issue into a progression of supporting ordinary differential circumstances (Recognitions), which can then be utilized to determine the underlying PDE. The Fokas method's imprint change stage offers a smart answer for taking care of troublesome direct PDEs. The review features the Fokas approach's versatility as one of its principal benefits. As per the columnists' examination, the methodology has been utilized in an extensive variety of specific and exploration spaces. There is no question that this article covers quantum mechanics, fluid components, heat conduction, and electromagnetics. The Fokas methodology's expansive materialness to an extensive variety of PDEs underscores this point.

David A. Smith (2012) makes critical progressions in both the investigation of frequently experienced beginning cutoff regard issues and the discipline of mathematical examination. Smith's exploration centers

around two-point starting cutoff regard issues, which are common in many areas of science and inside plan. One of Smith's fundamental exploration subjects is the requirement for a superior comprehension of the limit conditions that lead to profoundly introduced circumstances. A vital idea in mathematical examination is "well-posedness," which means that an issue has a solitary plan that reliably relies upon the basic information and cutoff conditions. In both made up math and sensible applications, the determination of fitting cutoff conditions for habitually experienced issues is urgent.

Bruno Deconinck, Thomas Trogdon, and Venkatakrishnan Vasani (2014) present a vigorous strategy for solving straight mostly differential circumstances (PDEs). Because of its flexibility in taking care of an assortment of PDEs, particularly those with non-minor breaking point conditions, this procedure created by A.C. Fokas has drawn in extensive consideration in the fields of mathematical material science and planning. The primary ideas and utilizations of the Fokas approach are completely framed in the article. The capacity of the Fokas strategy to deal with complex PDEs that happen in various genuine conditions, for example, power conduction, wave causing, and scattering processes, is quite possibly of its most essential component.

George D. Birkhoff's (1908) work, which zeroed in on worries with limit regard and arrangements, has extraordinarily helped the investigation of typical straight differential conditions. This revelation fundamentally impacted resulting progresses in the discipline and established the groundwork for a superior comprehension of these essential mathematical ideas. Birkhoff fostered a complete framework for researching and resolving various differential circumstances with limit conditions in his investigation of cutoff regard challenges. Mathematicians and specialists currently have a valuable and apt structure for taking care of differential circumstances in various applications because of the focal thought of breaking point regard troubles he framed.

Theodoros S. Papatheodorou and Andreas N. Kandili examine the new development and utilization of new numerical methods for solving starting cutoff esteem issues (IBVPs) that depend on the idea of Fokas changes in their 2009 distribution. IBVPs are fundamental mathematical models that are utilized across numerous legitimate and configuration disciplines to address dynamic cycles traveling through the real world. This work is especially appropriate on the grounds that witnessing real occasions requires exact and powerful IBVP designs. The designers of a unique numerical procedure utilize the Fokas change's responsibility.

### 3. THE FOKAS METHOD

#### 3.1 Conventions and Solution Type

We will momentarily go over specific standards and the sort of arrangement we are searching for before we audit the Fokas approach. Utilizing the standard that  $z_{n+1} = z_1$  and each corner  $z_j$  having an inward point  $j$  (0, 2), we list the polygon's corners in anticlockwise arrangement as  $z_1, \dots, z_n$ . We can't by and large expect smooth arrangements since our area isn't smooth. It is notable that the main request peculiarity for the Laplace's answer

for  $1/(2)/Z$  acts as follows if a polygon (or space with tapered places in two aspects) has a point among Neumann and Dirichlet edges that relates to  $= 0$  and  $= 0$  separately.

$$u \sim r^{1/(2\alpha)} \cos(\theta/(2\alpha))$$

near the corner. At the point when we consider blended limit conditions, we can see that in any event, for arched polygons, arrangements in  $H^{1+}()$  for smooth information are not generally suggested (this can be explained and shown with cut-off capabilities). For a few general discoveries on Lipschitz spaces, we direct the peruser to.

Let  $D$  be the association of the Dirichlet limit condition-pertinent edges, as well as the corner focuses between any two contiguous such sides. Characterize  $R$  along these lines for Robin limit conditions. Assuming  $f$  and  $g_i$  are adequately smooth, the accompanying states that our concern is very much presented

### 3.2 Integral Formulation

We will currently go through how the Fokas approach is commonly utilized. Green's subsequent character fills in as the starting point.

$$\int_{\partial\Omega} \left( v \frac{\partial u}{\partial n} - u \frac{\partial v}{\partial n} \right) ds = \int_{\Omega} f v dV,$$

where  $v$  is any formal adjoint condition arrangement.

$$v_{xx} + v_{yy} \pm k^2 v = 0 \text{ in } \Omega.$$

We utilize  $v = \exp(iz)$  for the Poisson condition, letting  $z = x + iy$  and  $\bar{z} = x - iy$ . Making  $z$  and  $\bar{z}$  free by applying the overall personality

$$\frac{\partial F}{\partial n} ds = -i \frac{\partial F}{\partial z} dz + i \frac{\partial F}{\partial \bar{z}} d\bar{z},$$

Bringing about the situation

$$\int_{\partial\Omega} \exp(-i\lambda z) \left( \frac{\partial u}{\partial n} + \lambda u \frac{dz}{ds} \right) ds = \int_{\Omega} \exp(-i\lambda z) f dV.$$

These equations, which rely upon the perplexing boundary, are referred to for each situation as the worldwide connection and really comprise of a perpetual number of equations. This is the Fokas technique's central trademark, and it is fundamental for the resulting mathematical executions. On the off chance that  $u$  is valid, we can utilize the Schwartz formation to get a second worldwide connection by taking the complicated form and supplanting with. Because of a solid connection with Fourier examination, which empowers one to check exact outcomes as well as portrayal equations (which call for mix in the complicated plane), a perplexing detailing with dramatic kind answers for  $v$  is used.

This strategy and the invalid field technique share the accompanying qualities in the particular instance of the Helmholtz condition: both depend on Green's (second) personalities, with one of the two capabilities equivalent to the arrangement of the BVP and the other capability equivalent to a group of answers for the adjoint condition (without limit conditions). Indeed, even in this particular occasion, there are significant contrasts: The Fokas approach is utilized for inside hardships, though the nullfield technique is specific to the external Helmholtz dissipating issue. Second, the adjoint capabilities in the previous strategy are the active wave capabilities found by isolating factors in polar directions, while the adjoint capabilities in the last technique are the remarkable capabilities found by isolating factors in Cartesian directions.

Third, and most essentially, the invalid field strategy grows the obscure limit values  $w_j$  on a "worldwide premise," implying that the premise capabilities are upheld on the whole limit Normal premise capabilities incorporate the active wave capabilities themselves or their typical subsidiaries. The Fokas strategy, conversely, grows the unidentified line values  $w_j$  on a "nearby premise," implying that the premise capabilities don't uphold the whole boundary.<sup>3</sup> Using a neighborhood premise offers significantly more adaptability, permitting one to, for instance, remember singularities of the answer for the base

### 3.3 Global Relation and Basis Choice Approximations

By integrating a reasonable premise into the obscure limit values  $w_j$ , laying out an approximative worldwide relation is conceivable. There are various choices for bases in Legendre polynomials appear to be the best premise choice, assuming that the limit values are in  $L^2(j)$ . For the assessment of the D2N map, a Fourier premise just gives quadratic intermingling to smooth boundary values. Alternately, the use of Chebyshev or Legendre polynomial developments brings about outstanding intermingling for appropriately smooth obscure limit information (no corner singularities). The primary advantage of Legendre polynomials is that we can unequivocally figure the important fundamental transforms in shut structure.

To begin with, in the Legendre polynomial premise on one or the other side, broaden the obscure limit values ( $w_j$ ) and the realized limit values ( $g_j$ ) prior to shortening to the  $N$  expression.

$$w_j(t) \approx \sum_{l=0}^{N-1} a_l^j P_l(t), \quad g_j(t) \approx \sum_{l=0}^{N-1} b_l^j P_l(t),$$

$P_m$  represents the  $m$ th Legendre polynomial, which has been standardized so  $P_m(1) = 1$ . This estimation holds in the  $L^2$  sense and is saved by the Fourier transform on the off chance that the boundary information is in  $L^2()$ . Then, at that point, we delivered

$$\hat{P}_l(\lambda) = \int_{-1}^1 \exp(-i\lambda t) P_m(t) dt,$$



Address Pl's Fourier transform. Note that the association empowers the calculation of this essential transform in shut structure.

$$\int_{-1}^1 \exp(\alpha t) P_l(t) dt = \frac{2^{l+1} \alpha^l l!}{(2l+1)!} {}_0F_1\left(l + \frac{3}{2}, \frac{\alpha^2}{4}\right) = \frac{\sqrt{2\pi\alpha}}{\alpha} I_{l+\frac{1}{2}}(\alpha),$$

where  $I_\nu$  represents the primary sort of request's altered Bessel capability. Since the important branch cuts along the negative genuine hub drop, we have selected to utilize  $2/\alpha$  rather than  $\alpha/2$  for this articulation, which is entire in.

#### 4. NON-CONVEX POLYGONS

The area quits being curved when  $x^4 = \text{Re}(z^4)$  passes  $1/3$ . The ran bends in Figure 2 make clearly when the space is presently not curved, there is a huge debasement.

We have shown the best Legendre coefficient calculation blunder for the test issue canvassed in Segment 2.5 (Laplace).<sup>4</sup> For the adjusted Helmholtz and Helmholtz equations, we noticed equivalent peculiarities.

The plane wave 'test capabilities'  $e^{Iz}$  in (2.5) (and their analogs for changed Helmholtz/Helmholtz) develop/rot dramatically in unambiguous headings of, giving a heuristic clarification to this evil molding. Each side of a curved polygon will frequently experience bigger test capabilities than different sides while utilizing an adequate number of complex - values that are disseminated every which way from the beginning, implying that qualities along this side will prevail the commitments from different sides. Conversely, no matter what the - esteem, limits (counting corners) in indented segments of a non-raised polygon will constantly be overruled by impacts from other limit parts

##### 4.1 Offering a statistically sound technique with virtual sides

The accompanying techniques might have been utilized to coordinate along the quadrilateral's four sides: Add the results from sides 1, 2, and 5 to sides 3 and 4 and sides 5 (in the other way). Then, the side 5 commitments as well as the qualities for  $u \geq 5$  and  $u \leq 5$  would drop. On the off chance that we assess at something very similar - values, the end turns out to be numerically comparable to just following sides 1, 2, 3 and 4.

Equations that are equivalent numerically need not, in any case, be comparable mathematically. For example, the arrangement of a linear framework is unaffected by the request in which the equations are put down.

Nonetheless, trades — otherwise called turning — are oftentimes utilized by mathematical calculations to give mathematical dependability. We end up in this situation at the present time. The mathematical molding decays while incorporating at the edges 1, 2, 3, and 4. Then again, consolidating two all-around adapted assignments includes following the sides of two triangles and afterward mathematically taking out the outcomes along the common edge.

The accompanying two perceptions and the previously mentioned heuristic contention filled in as the impetus for the ongoing review: The issue isn't brought about by the BVP itself or inquiries of well-pawedness since (I) limit necessary strategies experience no comparing issues when a space fails to be raised, and (ii) Gaussian disposal with proper turning is notable not to deteriorate the molding of a linear framework, so permitting it to deal with the converging of very much adapted undertakings ought to be protected. There are a wide assortment of procedures in the writing that partition the space into subdomains; for more data, see the presentation. However, no exploration on this decay has been finished corresponding to the Fokas transform.

## 4.2 Numerical implementation of the virtual sides approach

For the two-triangle methodology referenced over, the comparable to will take the accompanying structure:

$$\begin{bmatrix} RN^{(1)} & 0 & 0 & RN^{(4)} & RD^{(5)} & RN^{(5)} \\ SN^{(1)} & 0 & 0 & SN^{(4)} & SD^{(5)} & SN^{(5)} \\ 0 & RN^{(2)} & RD^{(3)} & 0 & -RD^{(5)} & -RN^{(5)} \\ 0 & SN^{(2)} & SD^{(3)} & 0 & -SD^{(5)} & -SN^{(5)} \end{bmatrix} \begin{bmatrix} u_1^N \\ u_2^N \\ u_3 \\ u_4^N \\ u_5 \\ u_5^N \end{bmatrix} = - \begin{bmatrix} RD^{(1)} & 0 & 0 & RD^{(4)} \\ SD^{(1)} & 0 & 0 & SD^{(4)} \\ 0 & RD^{(2)} & RN^{(3)} & 0 \\ 0 & SD^{(2)} & SN^{(3)} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3^N \\ u_4 \end{bmatrix}$$

The main network in which relates to side 5 being followed two times, this way and that, has furthest right blocks that are indistinguishable with the exception of the signs that are exchanged. This demonstrates that we are matching the arrangement's Cauchy information across the two subdomains on the virtual side. As was recently referenced, this characteristic makes it charming to just add the base portion of every situation to the top portion of the equations, wiping out this lattice obstructs completely and, alongside it, erasing the questions  $u_5$ ,  $u_5^N$  preceding utilizing a linear framework solver. Be that as it may, nothing is accomplished on the grounds that we return to the framework subsequent to playing out this. The strong bends in Figure 2 are the consequence of solving as it is displayed above, which empowers the linear solver to apply totally stable disposal strategies. At the point when the quadrilateral loses convexity, we never again notice any adverse results. Because of the likelihood that side 5 would be generally short, it may not be imaginable to exactly recognize the high request coefficients in the vectors  $u_5$  and  $u_5^N$ . However, the opposite side's coefficients are unaffected by this. Yet again the methodology is incredibly speedy, with common seasons of under 0.06s; the bigger framework is just minimal slower.

## 4.3 Test Case: L-shaped domain

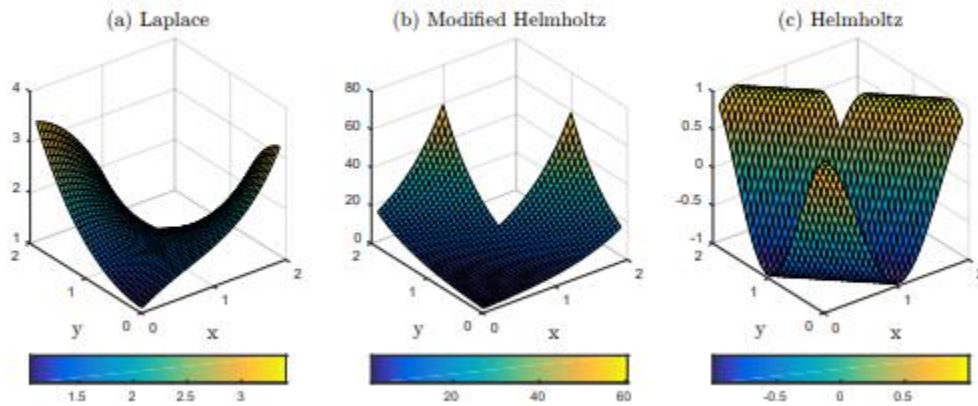
To settle the Laplace, Helmholtz, and changed Helmholtz equations in the space portrayed we currently utilize the idea of virtual sides. We will register within arrangement delivered involving the strategies in, as well as figuring the obscure boundary values. Each time, the limit information  $u + u^N$  (Robin limit conditions) are



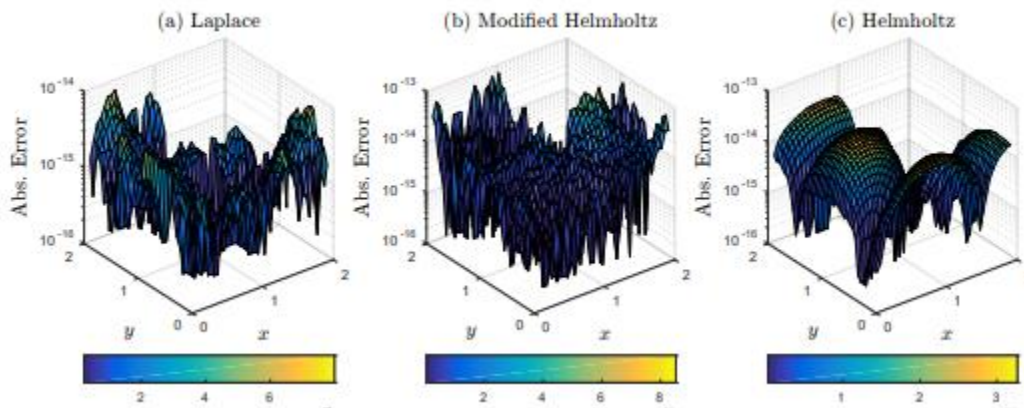
endorsed along sides 1 and 4, Neumann information are recommended along sides 2 and 5, and Dirichlet information are recommended along sides 3 and 6. Like this outcomes in the linear framework.

$$\begin{bmatrix} \frac{RN^{(1)}-RD^{(1)}}{2} & RD^{(2)} & RN^{(3)} & 0 & 0 & 0 & RD^{(7)} & RN^{(7)} \\ \frac{SN^{(1)}-SD^{(1)}}{2} & SD^{(2)} & SN^{(3)} & 0 & 0 & 0 & SD^{(7)} & SN^{(7)} \\ 0 & 0 & 0 & \frac{RN^{(4)}-RD^{(4)}}{2} & RD^{(5)} & RN^{(6)} & -RD^{(7)} & -RN^{(7)} \\ 0 & 0 & 0 & \frac{SN^{(4)}-SD^{(4)}}{2} & SD^{(5)} & SN^{(6)} & -SD^{(7)} & -SN^{(7)} \end{bmatrix} \begin{bmatrix} u_1^N - u_1 \\ u_2 \\ u_3^N \\ u_1^N - u_4 \\ u_5 \\ u_6^N \\ u_7 \\ u_7^N \end{bmatrix} \\
 = - \begin{bmatrix} \frac{RN^{(1)}+RD^{(1)}}{2} & RN^{(2)} & RD^{(3)} & 0 & 0 & 0 & 0 & 0 \\ \frac{SN^{(1)}+SD^{(1)}}{2} & SN^{(2)} & SD^{(3)} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{RN^{(4)}+RD^{(4)}}{2} & RN^{(5)} & RD^{(6)} & 0 & 0 \\ 0 & 0 & 0 & \frac{SN^{(4)}+SD^{(4)}}{2} & SN^{(5)} & SD^{(6)} & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1^N + u_1 \\ u_2^N \\ u_3 \\ u_4^N + u_1 \\ u_5^N \\ u_6 \end{bmatrix} .$$

Given the surmised Dirichlet and Neumann limit esteems, the state of the approximative arrangement inside a polygon was found in. It was exhibited that using a Chebyshev introduction alongside a speedy interpretation from Chebyshev to Legendre coefficients, it is feasible to figure the integrals accurately and proficiently.



**Figure 1:**The three test cases' analytical answers for the L-shaped domain



**Figure 2:**The interior absolute errors for each of the three test situations at N = 20

## 5. CONCLUSION

In conclusion, the careful evaluation and development of the Fokas transform method for resolving partial differential equations involving linear evolution mark an important turning point in the study of mathematical analysis and applied mathematics. This ground-breaking method has proven effective in addressing a variety of issues across numerous scientific and technical disciplines. Several important facts become clear as we consider the development in this area:

**Resilience and Versatility:** The Fokas transform method has demonstrated exceptional resilience and versatility when dealing with partial equations for linear evolution. It is an invaluable tool for both scholars and practitioners because of its capacity to tackle complex issues with variable boundary and initial circumstances.

**Interdisciplinary Applications:** The approach has applications outside of the realm of pure mathematics, including physics, engineering, and finance. Its ability to offer analytical solutions to issues that were previously difficult to solve has created new opportunities for research and invention.

**Despite the Fokas transform technique's:** primary concentration on analytical answers, its integration with contemporary computational techniques has further increased its utility. Based on this methodology, researchers have created numerical algorithms that effectively address a wider range of issues. The Fokas transform technique has been subject to a critical evaluation, which has advanced theory in addition to validating its efficacy. The mathematical foundations have become better understood by researchers, opening the door for more complex applications.

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