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ON NANO REGULAR ^ GENERALIZED CLOSED SETS IN NANO TOPOLOGICAL SPACES

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Abstract: In this paper we introduce and investigate a new class of sets called Nano R^G- closed sets. Furthermore we introduce Nano R^G- open sets and investigate several properties of the new notions.

***Index Terms* - :** Nano R^G-closure and Nano R^G- interior **,** Nano R^G- closed set, Nano R^G- open set.

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1. INTRODUCTION

Topology can be formally defined as “The study of qualitative properties of certain objects called topological spaces". That is invariant under a certain kind of transformation called a continuous map. Topology also refers a structure imposed upon a set X, a structure that essentially characterizes' the set X as a topological space by taking proper care of. Properties such as convergence, connectedness and continuity upon transformation.

The theory of nano topology was introduced by Lellis Thivagar and Caramel Richard [15] which is defined in terms of approximations and boundary regions of a subset of a universe using an equivalence relation on it. The elements of topological spaces are called nano open sets. It originates from the Greek word 'Nanos' which means 'Dwarf' in its modern scientific sense, an order of magnitude - one billionth.

L. Thivagar [15] defined nano continuous functions, nano open mapping, nano closed mapping and nano homeomorphism. The concept of nano generalized continuous functions in nano topological space was introduced by K. Bhuvaneswari and K. Mithili Gnanapriya [8]. P. Sulochana Devi and Dr. K. Bhuvaneswari introduced the concept of nano regular generalized continuous functions. S.B Shalini and K.Indirani introduced nano generalized beta – continuous function . In 2016, Bhuvaneswari and Ezhilarasi [6] introduced irresolute maps and generalized irresolute map in nano topological maps. In 2013, Savithiri. D, Janaki. C [24] introduced R^G closed sets in topological spaces. In this article, Nano R^G- closed set is introduced in nano topological spaces.

2. PRELIMINARIES

All through this paper, X, Y, Z stand for nano topological spaces (X,τ), (Y,σ), (Z,η) with no separation axioms assumed. Let A ⊆ X, the Nano closure and Nano interior of A will be denoted by Ncl(A) and Nint(A) respectively.

Definition 2.1: Let U be a non-empty finite set of objects called the universe R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆ U.

(i) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by LR(X).
That is, LR(X)= {U{R(x) : R(x) ⊆ X}}; where R(x) denotes the equivalence class determined by x.

(ii) The Upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted byUR(X) = {∪{R(x) : R(x) ∩ X ≠∅}}

(iii) The Boundary region of X with respect to R is the set of all objects which canbe classified neither as X nor as not -X with respect to R and it is denoted by BR(X) = UR(X) − LR(X).

Definition 2.2: Let U be the universe, R be an equivalence relation on U and τR(X) = {U, ∅, LR(X), UR(X), BR(X)} where X ⊆ U and τR(X)satisfies the following axioms.

(i) U and ∅∈τR(X).

(ii) The union of the elements of any sub collection R(X) is in τ R(X).

(iii) The intersection of the elements of any finite sub collection of τ R(X) is in τ R(X).
That is, τ R(X) forms a topology U called as the nano topology on U with respect to X. We call (U, τ R(X)) as the nano topological space. The elements of τ R(X) are called as nano open sets. A set A is said to be nano closed if its complement is nano open.

Definition 2.3: A nano subset A of X is called nano r^g closed if Ngcl(A)⊆U whenever A⊂U and U is nano regular open in X.

Definition 2.4: A nano subset A of X is called nano g- closed set if Ncl(A)⊆U whenever A⊆U and U is nano open in X.

Definition 2.5: A nano subset A of X is called nano g* closed set if Ncl(A))⊆U whenever A⊆U and U is nano g open in X.

Definition 2.6: A nano subset A of X is called nano gw-closed set if Ncl(Nint(A))⊆U whenever A ⊆U and U is nano regular open in X.

Definition 2.7: A nano subset A of X is nano wg closed[18] set if Ncl(Nint(A))⊆U whenever A ⊆U and U is nano regular semi open in X.

Definition 2.8: A nano subset A of X is called a nano regular closed [18] set if A=Ncl(Nint(A)).

Definition 2.9: A nano subset A of X is called regular open in x if A=Nint(Ncl(A)).

Definition 2.10: A nano subset A of X is called a nano semi-open set if A ⊆Ncl(Nint(A));

Definition 2.11: A nano subset A of X is called nano semi closed set if Nint(Ncl(A)) ⊆A.

Definition 2.12: A nano subset A is called nano regular open[9] if A = Nint(cl(A)) and its complement is called nano regular closed. A nano subset A is called nano gclosed [9] if Ncl(A) ⊆ U whenever A ⊆ U and

U is nano open. The intersection of all nano gclosed sets is called nano gclosure of A and it is denoted by Ngcl(A).

3. NANO REGULAR ^ GENERALIZED CLOSED SETS

Definition 3.1 : A Nano subset A of (X,τ) is called a nano regular ^ generalized closed **(briefly Nr^g closed)** if Ngcl(A) $\subset$ U, whenever A $\subset$ U and U is nano regular open in X.

We denote the family of nano r^g closed sets in space X by NR^GC(X).

Theorem 3.2 : Every nano closed set of a nano topological space (X, τ) is nano r^g closed set.

Proof : Let A $\subset$ X be a nano closed set and A $\subset$ U where U be nano regular open. Since A is nano closed and every nano closed set is nano g closed, Ngcl(A) $\subset$ Ncl(A) = A $\subset$ U. Hence A is an nano r^g closed set.

Remark 3.3 : The converse of the above theorem need not be true as seen in the following example.

Example 3.4 : Let U= {a,b,c,d} with U/R= {{c},{d},{a,b}} and X = {a,c}. Then τ_R(X) = {Φ,{c},{a,b},{a,b,c},U} .Let A= {a,c} , then A is an nano r^g closed set but it is not a nano closed set.

Theorem 3.5 : Every nano gclosed set is nano r^g closed.

Proof : Let A be a nano gclosed set. Let A $\subset$ U where U is nano regular open. Since every nano regular open set is nano open and A is nano gclosed, Ncl(A) $\subset$ U. Every nano closed set is nano gclosed therefore Ngcl(A) $\subset$ Ncl(A) $\subset$ U. Hence A is nano r^g closed.

Remark 3.6 : The converse of the above theorem need not be true as seen in the following example.

Example 3.7 : Let U={a,b,c,d} with U/R={{c},{d},{a,b}} , X = {a,c}. Then τ_R(X)= {Φ,{c},{a,b},{a,b,c},U}. Let A = {a,d} , then A is a nano gclosed set but it is not a nano closed set.

Theorem 3.8 : Every nano regular generalized closed set is nano r^g closed.

Proof : Let A be nano regular generalized closed set. Let A $\subset$ U and U be nano regular open. Then Ncl(A) $\subset$ U, since A is nano rgclosed. Every nano closed set is nano g-closed therefore Ngcl(A) $\subset$ Ncl(A) $\subset$ U. Hence A is nano r^g closed.

Theorem 3.9 : Every nano g*closed set is nano r^g closed.

Proof : Let A be nano g*-closed in (X,τ). Let A $\subset$ U and U be nano regular open. Since every nano regular open set is nano g-open. And A is nano g*-closed, Ncl(A) $\subset$ U. Every nano closed set is nano gclosed, then Ngcl(A) $\subset$ Ncl(A) $\subset$ U. Hence A is nano r^g closed.

Remark 3.10 : The converse of the above theorem need not be true as seen in the following example.

Example 3.11 : Let U={a,b,c,d} with U/R={{c},{d},{a,b}} and X = {a,c}.Then τ_R(X)= {Φ,{c},{a,b},{a,b,c},U}. Let A = {a,c} , then A is an nano r^g closed set but it is not a nano g*-closed set.

Theorem 3.12 : Every nano r^g closed set is nano rwg closed.

Proof : Straight forward.

Remark 3.13 : The converse of the above theorem need not be true as seen in the following example.

Example 3.14 : Let U={a,b,c,d} with U/R={{c},{d},{a,b}} and X = {a,c}.Then τ_R(X) = {Φ,{c},{a,b},{a,b,c}, U}. Let A={b},then A is nano rwg closed but not nano r^g closed.

Theorem 3.15 : Every nano r^g closed set is nano rgw closed.

Proof : Straight forward.

Remark 3.16: The converse of the above theorem need not be true as seen in the following example

Example 3.17 : Let U={a,b,c,d} with U/R={{c},{d},{a,b}} and X = {a,c}. Then τ_R(X) = {Φ,{c},{a,b},{a,b,c},U}.Let A={a},then A is nano rgw closed but not nano r^g closed.

Remark 3.18 : Nano r^g closed sets and nano semi closed sets are independent to each other as seen from following example.

Example 3.19 :

- Let U={a,b,c,d}, with U/R={{c},{d},{a,b}} and X={a,c}. Then τ_R(X)={ Φ,{c}, {a,b},{a,b,c},U}.Let A={c} . Then A is nano semi closed but not nano r^g closed.
- Let U={a,b,c,d},with U/R={{c},{d},{a,b}} and X={a,c}. Then τ_R(X)={ Φ,{c}, {a,b},{a,b,c},U}.Let A={a,c} . Then A is nano r^g closed but not nano semiclosed.

Remark 3.20 : Nano r^g closed sets and nano pre closed sets are independent to each other as seen in the following example.

Example 3.21 :

- Let U={a,b,c,d} with U/R={{c},{d},{a,b}} and X={a,c}. Then τ_R(X)={ Φ,{c}, {a,b},{a,b,c},U}. Let A={a} .Then A is nano pre closed but not nano r^g closed.
- Let U={a,b,c,d} with U/R={{c},{d},{a,b}} and X={a,c}. Then τ_R(X)={ Φ,{c}, {a,b},{a,b,c},U}. Let A={a,c} . Then A is nano r^g closed but not nano preclosed.

Remark 3.22 : Nano r^g closed sets and nano semi- pre closed sets are independent to each other as seen in the following example.

Example 3.23 : Let U={a,b,c,d} with U/R={{a,c},{b},{d}} and X={a,d}. Then τ_R(X)={ Φ,{d}, {a,c},{a,c,d},U}. Let A={a}.Then A is nano semi pre-closed but not nano r^g closed.

- Let U={a,b,c,d} with U/R={{a,c},{b},{d}} and X={a,d}. Then τ_R(X)={ Φ,{d}, {a,c},{a,c,d},U}. Let A={a,c,d} . Then A is nano r^g closed but not nano semipre-closed

Remark 3.24 : Nano r^g closed sets and nano wg closed sets are independent to each other as seen in the following example.

Example 3.25 : Let U={a,b,c,d} with U/R={{c},{d},{a,b}} and X={a,c}. Then τ_R(X)={ Φ,{c}, {a,b},{a,b,c},U}. Let A = {a} . Then A is nano wg closed but not nano r^g closed.

- Let U={a,b,c,d} with U/R={{c},{d},{a,b}} and X={a,c}. Then τ_R(X) = { Φ,{c}, {a,b},{a,b,c},U}. Let A={a,c} . Then A is nano r^g closed but not nano wgclosed.

Remark 3.26 : The concept of nano r^g closed sets as nano gs closed sets are independent to each other as seen in the following example.

Example 3.27 :

- Let U={a,b,c,d} with U/R={{c},{d},{a,b}} and X={a,c}. Then τ_R(X)={ Φ,{c}, {a,b},{a,b,c},U}. Let A={b} . Then A is nano gs closed but not nano r^g closed .
- Let U={a,b,c,d} with U/R={{c},{d},{a,b}} and X={a,c}. Then τ_R(X)={ Φ,{c}, {a,b},{a,b,c},U}. Let A={b,c} . Then A is nano r^g closed but not nano gsclosed.

Remark 3.28 : The concept of nano r^g closed sets and nano sg closed sets are independent to each other as seen in the following example.

Example 3.29 : Let U={a,b,c,d} with U/R={{c},{d},{a,b}} and X={a,c}. Then τ_R(X)={ Φ,{c}, {a,b},{a,b,c},U}. Let A={c} . Then A is nano sg closed but not nano r^g closed.

- Let U={a,b,c,d} with U/R={{c},{d},{a,b}} and X={a,c}. Then τ_R(X)={ Φ,{c}, {a,b},{a,b,c},U}. Let A={b,c}. Then A is nano r^g closed but not nano sg closed.

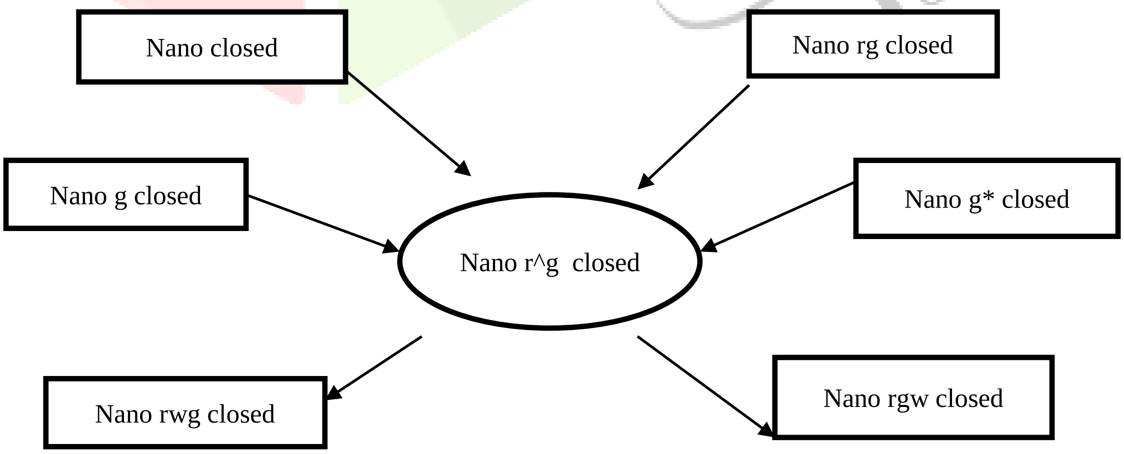
Remark 3.30 : Nano r^g closed sets and nano swg closed sets are independent to each other as seen in the following example.

Example 3.31 : Let U={a,b,c,d} with U/R={{a,c},{b},{d}} and X={a,d}. Then τ_R(X)={ Φ,{d}, {a,c},{a,c,d},U }. Let A = {a} . Then A is nano swg closed but not nano r^g closed.

- Let U={a,b,c,d} with U/R={{a,c},{b},{d}} and X={a,d}. Then τ_R(X)={ Φ,{d}, {a,c},{a,c,d},U}. Let A={a,c,d}. Then A is nano r^g closed but not nano swg closed.

Remark 3.32 :

The above discussions are diagrammatically represented as follows.



Theorem 3.33 : Let A be an nano r^g closed set in a nano topological space X. Then Ngcl(A) - A contains no non - empty nano regular closed set in X.

Proof : Let F be a nano regular closed set such that F⊂Ngcl (A)-A. Then F⊂X - A implies A⊂X-F. Since A is nano r^g closed and X-F is nano regular open, Then Ngcl(A)⊂X-F. That is F⊂X- Ngcl(A). Hence F⊂Ngcl(A) ∩ (X-gcl (A)) = Φ .Thus F= Φ, where Ngcl(A)-A does not contain non empty nano regular closed set.

Theorem 3.34 : The converse of the above theorem need not be true, that means if Ngcl (A)-A contains no non empty nano regular closed set, Then A need not to be a nano r^g closed as seen in the following example.

Example 3.35 : Let U= {a,b,c,d} with U/R= {{c},{d},{a,b}} and X={a,c}. Then τ_R(X)={ Φ,{c},{a,b},{a,b,c},U}. Let A={c}.Then Ngcl (A) - A ={d},it does not contain non -empty nano regular closed set in U. But A={c} is not an nano r^g closed set .

Theorem 3.36 : The finite Union of two nano r^g closed sets is nano r^g closed.

Proof : Assume that A and B are nano r^g closed sets in X. Let $A \cup B \subset U$ where U is nano regular open. Then $A \subset U$ and $B \subset U$. Since A and B are nano r^g closed, Ngcl(A) $\subset$ U and Ngcl(B) $\subset$ U. Then Ngcl($A \cup B$)=Ngcl(A) $\cup$ Ngcl(B) $\subset$ U. Hence $A \cup B$ is nano r^g closed.

Remark 3.37 : The intersection of two nano r^g closed set in X need not be a nano r^g closed set as seen in the following example.

Example 3.38 : Let U={a,b,c,d} with U/R={{a,c},{b},{d}} and X={a,d}. Then τ_R(X)={ Φ, {d},{a,c},{a,c,d}, U}. If A={a,b} and B={a,d}. Then A and B are nano r^g closed sets. But $A \cap B$={a} is not an nano r^g closed set .

Theorem 3.39: In a nano topological space X, if NRO(X) ={X, Φ}, then every nano subset of X is a nano r^g closed set.

Proof: Let X be a Nano topological space and NRO(X) ={X, Φ }. Let A be any arbitrary nano subset of X. Suppose A=Φ, then Φ is a nano r^g closed set in X. If A ≠ Φ, then X is the only set containing A and so Ngcl(A) $\subset$ X. Hence A is nano r^g closed. Thus every nano subset of X is nano r^g closed.

Remark 3.40: The converse of the above theorem need not be true as seen in the following example.

Example 3.41: Let U={a,b,c} with U/R={{a},{b,c}} and X={a,c}. Then τ_R(X) ={ Φ,{a}, {b,c}}. All the subsets of (X,τ_R(X)) are nano r^g closed sets, but NRO(X)={ Φ,{a},{b,c},U}.

Theorem 3.42 : Let A be an nano r^g closed set in the nano topological space (X, τ). Then A is nano g closed iff Ngcl(A)-A is nano regular closed.

Proof :

Necessity : Let A be nano gclosed then Ngcl(A)=A and so Ngcl(A)-A= Φ which is nano regular closed.

Sufficiency : Suppose Ngcl(A)-A is nano regular closed. Then Ngcl(A)-A= Φ by theorem 3.34 that is Ngcl(A)= A. Hence A is nano gclosed.

Theorem 3.43 : If A is an nano r^g closed subset of X such that $A \subset B \subset$ Ngcl(A) , then B is an nano r^g closed.

Proof : Let $B \subset U$ where U is nano regular open. Then $A \subset B$ implies $A \subset U$. Since A is nano r^g closed, Ngcl(A) $\subset$ U. By hypothesis Ngcl(B) $\subset$ Ngcl(Ngcl(A))=Ngcl(A) $\subset$ U. Hence B is nano r^g closed.

Remark 3.44: The converse of the above theorem need not be true as seen in the following example.

Example 3.45: Let U={a,b,c,d} with U/R={{c},{d},{a,b}} and X={a,c}. Then τ_R(X)={ Φ,{c},{a,b},{a,b,c}, U}, Let A={d} and B={a,d} . Then A and B are nano r^g closed sets. But A subset B is not a subset of Ngcl(A) .

Theorem 3.46 : Let (X,τ) be a nano topological space, then for x∈X, the set X \ {x} is either nano r^g closed set or nano regular open set.

Proof: If X \ {x} is a not a nano regular open set, then X is the only nano regular open set containing X \ {x} . This implies that Ngcl{X \ {x} }⊂X . Hence X \ {x} is nano r^g closed set.

4. NANO REGULAR ^ GENERALIZED OPEN SET

Definition 4.1 : A nano set A of a nano topological space X is called nano regular ^ generalized open **(briefly Nr^g open)** set if and only if its complement is nano regular ^ generalized closed. The collection of all nano r^g open set is denoted by NR^GO(X).

Remark 4.2 : Ngcl(X - A) = X - Ngint(A)

Theorem 4.3 : A nano subset X is nano r^g open iff F⊆Ngint(A), whenever F is nano regular closed and F⊂A

Proof : Necessity : Let A be nano r^g closed and F⊂A . Then X-A⊂X-F where X-F is nano regular open and nano r^g closedness of X-A implies Ngcl(X-A)⊂X-F. By remark 4.2 , X-Ngint(A)⊂X-F. Therefore F⊂Ngint(A).

Sufficiency : Suppose F is nano regular closed and F⊂A then F⊂Ngint(A). Let X-A⊂U, where U is nano regular open. Then X-U⊂A, where X-U is nano regular closed. By hypothesis , X-U⊂ Ngint(A).Then X-Ngint(A)⊂U. By remark 4.2, Ngcl(X-A)⊂U. Hence X-A is nano r^g closed. And A is nano r^g open.

Theorem 4.4 : If NgintA⊂B⊂A and if A is nano r^g open then B is nano r^g open.

Proof : Given NgintA⊂B⊂A , X-A⊂X-B⊂Ngcl(X-A). Since A is nano r^g open, X-A is nano r^g closed. This implies X-B is nano r^g closed. Hence B is nano r^g open.

Remark 4.5 : For any nano set A⊂X, Ngint(Ngcl(A)-A) = Φ.

Theorem 4.6 : If a nano subset A of a nano topological space X is nano r^g closed, then Ngcl(A)-A is nano r^g open.

Proof : Let A be an nano r^g closed and Let F be a nano regular closed set such that F⊂Ngcl(A)-A. Then by theorem 3.34 , F = Φ. So F⊂Ngint(Ngcl(A)-A). By theorem 4.3 Ngcl(A)-A is nano r^g open.

Remark 4.7 : The converse of the above theorem need not be true as shown below.

Example 4.8 : Let U={a,b,c,d} with U/R={{a},{b,c},{d}} and X={b,d}. Thenτ_R(X)={ Φ,{d},{b,c},{b,c,d},U}. Let A={d}, then Ngcl(A)-A={c} which is nano r^g open in X but A is not nano r^g closed in X.

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