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STUDY OF HARMONIC ANALYSIS OF MULTILEVEL INVERTERS USING SELECTIVE HARMONIC ELIMINATION TECHNIQUE

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Abstract: Nowadays the utilization of high power Voltage Source Inverter is increased but the matter of harmonic and switching losses in inverter also are increased due to the utilization of power electronic semiconductor switches, which are used for better and efficient operation. In this paper, for eliminating the harmonics presented in full Bridge inverter during switching operation, Selective Harmonic Elimination Pulse Width Modulation (SHEPWM) Technique is employed. For full Bridge operation proper switching angles are calculated by solving nonlinear equation using Newton Raphson method. In this paper a single phase H-bridge inverter using MOSFETs with controlled output voltage with selective harmonic elimination, which reduces the cost of filter and thus improves the inverter voltage waveform. For a multilevel inverter, switching angles at fundamental are obtained by solving the selective harmonic elimination. Comparison of Fourier analysis of H bridge inverter with and without SHEPWM technique is completed.

Index Terms **- H-bridge Inverter, Selective Harmonic Elimination (SHE), Pulse width modulation (PWM), Voltage harmonics**

I. INTRODUCTION

The performance characteristics of inverter/rectifier conversion systems largely depend upon the selection of proper pulse width modulation (PWM) technique. PWM techniques can be broadly classified as carrier-based sinusoidal PWM (SPWM), space vector modulation (SVM) or selective harmonic elimination (SHE-PWM). Historically, SHE was proposed in the early 1960s, when it was found that low order harmonics could be suppressed by adding several switching angles in a square wave voltage. Switching angle transitions were calculated in such a way that the low-order harmonics are kept to zero while saving the fundamental value.

Since its introduction, SHE-PWM has drawn tremendous research interest and has also been developed for various applications, principally for high-voltage and high-power converters where switching losses are a major concern and their reduction is of prime importance. The concept of SHE-PWM techniques is based on decomposition of the PWM voltage/current waveform using Fourier theory. Different waveform formulations have been considered and analyzed in the technical literature, including: Bipolar, Unipolar and Stepped or PWM multilevel waveforms. Waveform properties such as symmetry and the number and amplitude of voltage levels are equally important factors in the analysis and play an essential role in determining the form and complexity of the solution space.

Finding the analytical solution of the SHE-PWM waveform is that the main challenge, and selection of an appropriate solving algorithm or method relies heavily on the formulation of the waveform. Numerous solving techniques, such as iterative approaches, optimization techniques and resultant theory have been discussed for obtaining the switching angles for different SHE-PWM waveforms.

SHE-PWM was initially studied for conventional two- and three-level converters. It has since been then extended to various multilevel and hybrid Multilevel converters for numerous applications. The number and sort of multilevel converters requires different implementation for every individual topology and may maximize the potential benefits that SHE-PWM offers to a specific converter. The aim of this paper is to provide an analytical review of progress in the field of SHE-PWM for multilevel converters and define the state of the art and outstanding issues with the SHE - PWM technique. Additionally, the paper aims to serve as a comprehensive resource on SHE-PWM and facilitate understanding of the features, benefits and limitations of this modulation technique. A thorough review of the well-established solving methods is also reported in this paper, with the aim of helping prospective researchers to identify appropriate algorithms for a given circuit topology and application. Special consideration is devoted to the implementation of SHE-PWM in the different multilevel converter topologies and their role in various industrial and utility applications.

II. FULL BRIDGE INVERTER

In H bridge inverter, four semiconductor switches are used. The main difference between half bridge and full bridge inverter is the maximum value of output voltage obtained. In half bridge inverter, peak voltage is half of the DC supply voltage. In full bridge inverter, peak voltage is same as the DC supply voltage. The circuit diagram of single phase H bridge inverter is as shown in below figure 1.

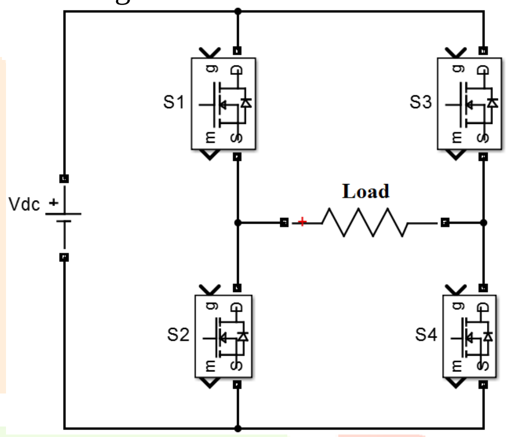

Figure 1: Basic H Bridge inverter

The SHE PWM technique is used to generate an output of a full-bridge inverter. In this paper, a three-level SHE PWM generated by a full-bridge inverter is considered. A full-bridge or H-bridge voltage source inverter comprises four switches and one dc source. Fig. 1 shows the basic H- Bridge inverter with four switches operated complementary. This particular inverter gives 3 levels in the output by proper switching of these four switches.

The gate pulse for S1 and S4 are same. Similarly, S2 and S3 have same gate pulses and operating at same time. But, S1 and S2 (vertical arm) never operate at same time. If this happens, then DC voltage source will be short circuited.

Three levels of an output waveform such as positive, negative, and zero, can be obtained with proper PWM arrangement. Fig 5 shows a generalized three-level output voltage waveform, which is synthesized by using the inverter circuit shown in Fig 1.

2.1For upper half cycle (Mode 1)

S1 and S4 get triggered and current will flow as shown in figure 2. In this part of time period, the current flow from positive to negative direction of load.

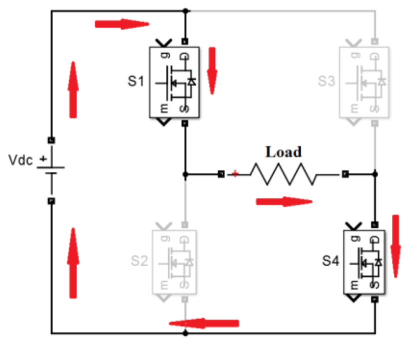

Figure 2: Mode 1 Operation

2.2Lower half cycle (Mode 2)

S2 and S3 get triggered and current will flow as shown in figure 3. In this time period, the current flow from negative to positive direction of load. The peak load voltage is same as DC supply voltage V_{dc} in both cases.

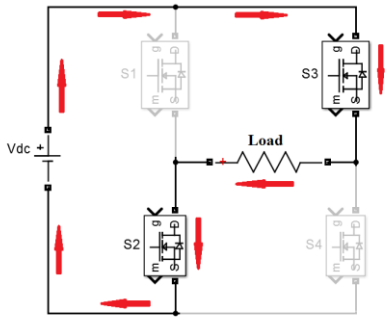

Figure 3: Mode 2 Operation

2.3 Zero level (Mode 3)

Switches S2 and S4 or switches S1 and S3 are triggered shown in figure 4. By this connection short circuit appear across the load resistance, makes zero output voltage.

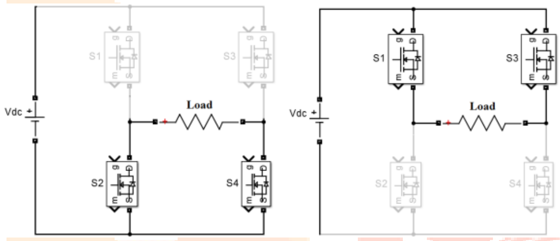

Figure 4: Mode 3 Operation

Figure 5 shows the basic 3 level output voltage of the H bridge inverter with maximum level of input voltage V_{dc}.

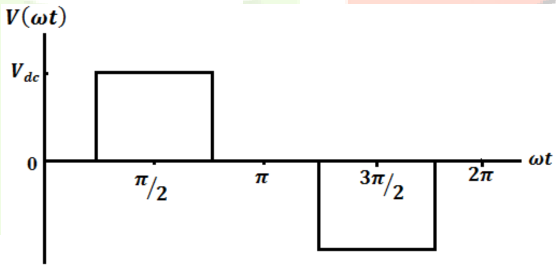

Figure 5: 3 Level Output

2.4Switching Angle Calculation

For generating the output from an inverter one simplest way is to find out the switching instants at which each inverter switches must be triggered. In order to calculate the switching angles one basic equation is given below

$$\theta_j = \sin^{-1}\left(\frac{2j-1}{m-1}\right)$$

Where m is the number of output levels

j = 0, 1, $\left(\frac{m-1}{2}\right)$

For the above 3 level (m=3) output, the equation generates two switching angles $\theta_1$and θ_2 for j=0 and j=1 respectively. This equation helps to find the switching angles for a staircase waveform. But we cannot reduce the %THD significantly

III. SELECTIVE HARMONIC ELIMINATION

3.1 Sinusoidal pulse-width modulation (SPWM)

The SPWM technique is one among the primitive techniques; it's wont to suppress harmonics present within the quasi-square wave. In SPWM, a carrier is compared with a reference wave. The reference wave corresponds to the specified fundamental at the output and therefore the triangular wave determines the frequency with which values are switched. By changing the frequency of the carrier, higher switching frequencies are often obtained. So as to avoid the elimination of the even harmonics, the carrier frequency should be an odd multiple of three.

3.2 Selective Harmonic Elimination Pulse Width Modulation (SHE PWM)

Selective Harmonic Elimination Pulse Width Modulation (SHE PWM) is a low switching frequency pulse generation technique by eliminating harmonics of specified order or eliminating harmonics in a band of specified frequencies. Compared to other conventional PWM technique, SHE is pre calculated non-carrier based PWM.

If K is the number of switching at each level, the switching frequency of the SHE PWM will be, K time's fundamental frequency f_0.

$$\mathrm{K}{*}f_0$$

Number of harmonic components that can be eliminated from the output voltage is evaluated by,

$$KP-1$$

Where, P is the number of actual switching or number of levels in quarter cycle.

SHE PWM method is able to eliminate the non-triple harmonics up to (3KP-2) when KP is odd and (3KP-1) when KP is even. To create the pulse pattern, different angles are calculated to form a staircase multilevel waveform having lowest possible Total Harmonic Distortion (THD). Switching angle calculation is performed by offline calculation and switching instant calculation is by desired fundamental and the harmonic components to be eliminated. SHE PWM is implemented based on optimisation technique and desired harmonic order can be eliminated. SHE technique is performed by Fourier series decomposition of the periodic PWM voltage waveform and generally uses a periodic waveform with quarter wave symmetry. Because of the odd quarter wave symmetry property of this waveform, the Fourier series are going to be simplified and therefore the study will be limited only to the quarter cycle.

Fourier series expansion;

$$v(\omega t)=a_0+\sum_{n=1}^{\infty}a_n\cos(n\omega t)+\sum_{n=1}^{\infty}b_n\sin(n\omega t)$$

Where $f_0$is fundamental frequency, n is the order of harmonics and $\omega=2\pi f_0$

$$v(\omega t)=a_0+\sum_{n=1}^{\infty}a_n\cos(2\pi f_0 nt)+\sum_{n=1}^{\infty}b_n\sin(2\pi f_0 nt)$$

$$a_0=\frac{1}{2\pi}\int_0^{2\pi}V_{dc}\,d\omega t$$

$$a_n=\frac{1}{\pi}\int_0^{2\pi}V_{dc}\cos(n\omega t)\;d\omega t$$

$$b_n=\frac{1}{\pi}\int_0^{2\pi}V_{dc}\sin(n\omega t)\;d\omega t$$

For a periodic PWM voltage waveform three types of symmetries are exist: Three phase symmetry, Half-wave symmetry and Quarter wave symmetry.

3.2.1Three phase symmetry

For a balanced operation of load, it is required that the converter output possesses three phase symmetry and all harmonics are balanced.

3.2.2 Odd symmetry

Condition for odd symmetry is

$$v(\omega t) = -v(\omega t)$$

Under this condition Fourier series parameters becomes;

$$a_0 = 0$$
$$a_n = 0$$
$$b_n = \frac{4}{\pi}\int_0^{\pi/2} V_{dc}\sin(n\omega t)\ d\omega t$$

3.2.3 Half wave symmetry

Condition for Half wave symmetry is

$$v(\omega t) = -v(\omega t + \pi)$$

Under this condition it eliminates all the even harmonics ($a_n = 0\ and\ b_n = 0\ for\ all\ even\ n$) and the Fourier series parameters become;

$$a_0 = 0$$
$$a_n = \frac{4}{\pi}\int_0^{\pi/2} V_{dc}\cos(n\omega t)\ d\omega t \quad for\ all\ odd\ n$$
$$b_n = \frac{4}{\pi}\int_0^{\pi/2} V_{dc}\sin(n\omega t)\ d\omega t \quad for\ all\ odd\ n$$

3.2.4 Quarter wave symmetry

Quarter wave symmetry exists if the waveform has symmetry around the midpoints of positive and negative half cycles.

$$a_0 = 0$$
$$a_n = 0 \quad for\ all\ n$$
$$b_n = 0 \quad for\ all\ even\ n$$
$$b_n = \frac{4}{\pi}\int_0^{\pi/2} V_{dc}\sin(n\omega t)\ d\omega t \quad for\ all\ odd\ n$$

The staircase voltage waveform is considered in SHE method in multilevel inverters. In this case for a (2N+1) level inverter, the number of switching in a quarter cycle is limited to N. So the number of harmonics can be eliminated from the output voltage is (N-1).

SHE PWM method is aimed to choose the set of switching angles $\theta_1, \theta_2, \theta_3, \dots \dots, \theta_N$ such that the identified lower order harmonics are suppressed and at the same time the amplitude of fundamental component become equal to the desired value.

SHE PWM is a novel method that provides more number of the Degree of Freedom (DoF) and makes available to eliminate more harmonic components with no need to change the hardware of the inverter. In order to increase the DoF and eliminating more harmonics than the case of fundamental frequency switching scheme without any manipulation on inverter hardware, the SHE PWM technique is proposed which is denominated Virtual Stage PWM. This technique is one of the powerful theories that apply to multilevel inverters in order to generate high quality voltage waveform with less switching frequency in comparison with other PWM methods.

3.3 Fundamental Frequency Switching Scheme

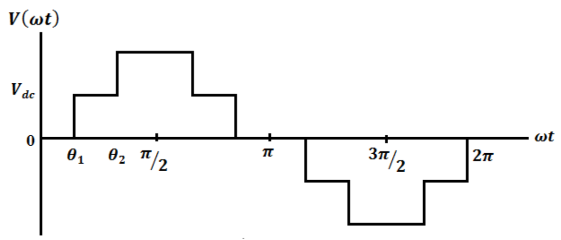

Figure 6: Fundamental Frequency Switching

Fundamental Frequency Switching is the basic staircase waveform of multilevel inverter with each level of specified step voltage (V_{dc}) at particular angle shown in figure 6.

$$b_n = \frac{4V_{dc}}{n\pi}[\cos(n\theta_1) + \cos(n\theta_2) + \cos(n\theta_3) + \dots \dots \cos(n\theta_N)]$$

$$b_n = \frac{4V_{dc}}{n\pi}\sum_{i=1}^{N}\cos(n\theta_i)$$

3.4 Virtual Stage PWM

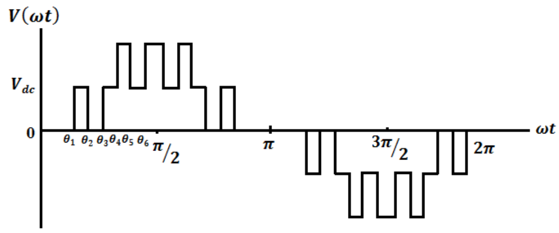

Figure 7: Virtual Stage PWM

In virtual stage SHE PWM scheme, number of switching per quarter cycle is more compared to fundamental frequency switching SHE PWM scheme in figure 7. Negative sign in the Fourier expression of the voltage indicate the falling edge of the staircase waveform and vice versa.

$$b_n = \frac{4V_{dc}}{n\pi}[\cos(n\theta_1) - \cos(n\theta_2) + \cos(n\theta_3) - \dots \dots \cos(n\theta_N)]$$

$$b_n = \frac{4V_{dc}}{n\pi}\sum_{i=1}^{N}P_i\cos(n\theta_i)$$

Where, $P_i = \begin{cases} 1\ ;\ \theta_i\ for\ rising\ edge \\ -1\ ;\ \theta_i\ for\ falling\ edge \end{cases}$

The generalized equation of the multilevel SHE PWM is

$$\frac{4}{\pi}\sum_{i=1}^{N}P_i\cos(\theta_i) = \left(\frac{L-1}{2}\right) M$$

Where, N is the number of switching angles in a quarter cycles, L is the number of levels of output phase voltage and M is the Modulation Index.

$$\sum_{i=1}^{N}P_i\cos(n\theta_i) = 0\ for\ n = 5,7,11,13 \dots \dots \dots$$

$$n = 5,7,11, \dots \dots \dots \dots (3N-2)\quad when\ N\ is\ odd$$

$$n = 5,7,11, \dots \dots \dots \dots (3N-1)\quad when\ N\ is\ even$$

3.5 Types of SHE PWM technique

Two types of SHE PWM are commonly used: bipolar SHE and unipolar SHE. For each half cycle period, both $+V_{dc}$ and $-V_{dc}$ voltage pulses are used, which is called a bipolar SHE. Similarly, for each half cycle period, only one of $+V_{dc}$ or $-V_{dc}$ is used, which is called unipolar SHE. Because of the quarter wave mirror symmetry and half wave odd symmetry, the pole voltage has only odd harmonics and has only sinusoidal components in the Fourier expansion.

$$v(\omega t) = \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

$$b_n = \frac{4}{\pi}\int_0^{\pi/2} V_{dc}\sin(n\omega t)\ d\omega t \qquad for\ all\ odd\ n$$

For SHE PWM, the switching instants are determined by solving a set of nonlinear equations. Due to nonlinear equations and transcendental characteristics, such equations can be solved numerically.

3.5.1 Unipolar SHE PWM

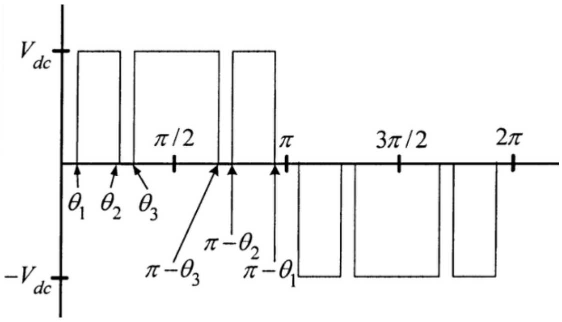

Figure 8: Unipolar SHE PWM

$$b_n = \frac{4V_{dc}}{n\pi}[\cos(n\theta_1) - \cos(n\theta_2) + \cos(n\theta_3) - \dots\dots \cos(n\theta_N)] \quad for\ all\ odd\ n$$

$$0 < \theta_1 < \theta_2 < \theta_3 \dots\dots\dots .\ \theta_N < {}^{\pi}/_{2}$$

$$b_n = \frac{4V_{dc}}{n\pi}\sum_{i=1}^{N}(-1)^{i+1}\cos(n\theta_i) \quad for\ all\ odd\ n$$

3.5.2 Bipolar SHE PWM

Case 1

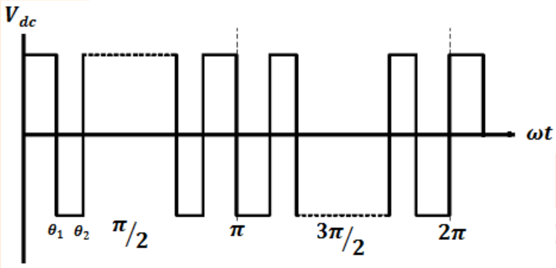

Figure 9: Bipolar SHE PWM

$$b_n = \frac{4V_{dc}}{n\pi}[1 - 2\cos(n\theta_1) + 2\cos(n\theta_2) - 2\cos(n\theta_3) - \dots\dots 2\cos(n\theta_N)] \quad for\ all\ odd\ n$$

$$0 < \theta_1 < \theta_2 < \theta_3 \dots\dots\dots .\ \theta_N < {}^{\pi}/_{2}$$

$$b_n = \frac{4V_{dc}}{n\pi}\left[1 - 2\sum_{i=1}^{N}(-1)^{i+1}\cos(n\theta_i)\right] \quad for\ all\ odd\ n$$

-ve sign indicates the falling edge of the voltage.

Case 2

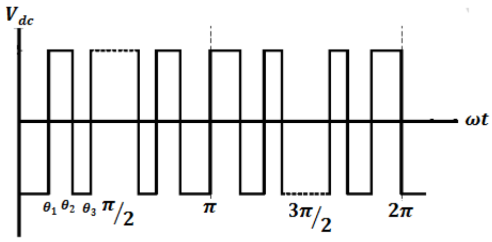

Figure 10: Bipolar SHE PWM

$$b_n = \frac{4V_{dc}}{n\pi}[-1 + 2\cos(n\theta_1) - 2\cos(n\theta_2) + 2\cos(n\theta_3) - \dots\dots 2\cos(n\theta_N)] \quad for\ all\ odd\ n$$

$$0 < \theta_1 < \theta_2 < \theta_3 \dots\dots\dots .\ \theta_N < {}^{\pi}/_{2}$$

$$b_n = \frac{4V_{dc}}{n\pi}\left[-1 - 2\sum_{i=1}^{N}(-1)^{i}\cos(n\theta_i)\right] \quad for\ all\ odd\ n$$

-ve sign indicates the falling edge of the voltage.

Only the fundamental frequency component in the output voltage is of interest and all other harmonic voltages are undesirable. As such one would really like to eliminate as many low order harmonics as possible.

Accordingly the elemental voltage magnitude could also be set at the specified value and therefore the magnitudes of 5^{th}, 7^{th} and other harmonics could also be set to zero.

To eliminate many more unwanted harmonic frequencies from the load voltage waveform but this will require introduction of more angles per quarter cycle. If there are N notch angles per quarter cycle, N number of equations may be written each of which determine the magnitude of a particular harmonic voltage.

3.6 Analysis of Unipolar SHE PWM

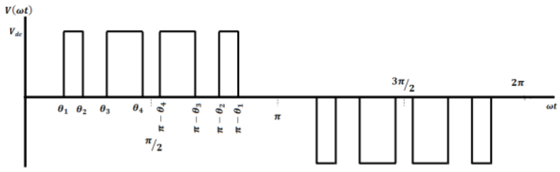

Figure 11: Unipolar SHE PWM with 4 angles

Figure 11 shows the unipolar SHE output with four angles per quarter. Fourier series corresponding to this output waveform is

$$b_n = \frac{4V_{dc}}{n\pi}[\cos(n\theta_1) - \cos(n\theta_2) + \cos(n\theta_3) - \cos(n\theta_4)]$$

Fundamental and Harmonic components are

$$b_1 = \frac{4V_{dc}}{\pi}[\cos(\theta_1) - \cos(\theta_2) + \cos(\theta_3) - \cos(n\theta_4)]$$
$$b_3 = \frac{4V_{dc}}{3\pi}[\cos(3\theta_1) - \cos(3\theta_2) + \cos(3\theta_3) - \cos(3\theta_4)]$$
$$b_5 = \frac{4V_{dc}}{5\pi}[\cos(5\theta_1) - \cos(5\theta_2) + \cos(5\theta_3) - \cos(5\theta_4)]$$
$$b_7 = \frac{4V_{dc}}{7\pi}[\cos(7\theta_1) - \cos(7\theta_2) + \cos(7\theta_3) - \cos(7\theta_4)]$$

If M is the modulation index,

$$M = \frac{b_1}{V_{dc}}$$

If the output contains N angles per quarter, N equations can be formed and $(N-1)$ harmonics can be eliminated from the output waveform by setting equations to zero. Lowest odd harmonics components to be eliminated from a single phase system and in a three phase system, the lowest non triplen harmonic components are eliminated.

$$\cos(\theta_1) - \cos(\theta_2) + \cos(\theta_3) - \cos(\theta_4) = \frac{M\pi}{4}$$
$$\cos(3\theta_1) - \cos(3\theta_2) + \cos(3\theta_3) - \cos(3\theta_4) = 0$$
$$\cos(5\theta_1) - \cos(5\theta_2) + \cos(5\theta_3) - \cos(5\theta_4) = 0$$
$$\cos(7\theta_1) - \cos(7\theta_2) + \cos(7\theta_3) - \cos(7\theta_4) = 0$$

3.7 Solution by Numerical Methods

To solve the SHE equations, Numerical methods are normally used. Two possible methods are Seidel's method and Newton's method. Seidel method can be applied if all the harmonic equations are functions of the trigger angle. Seidel's method is a method to assume the trigonometric function as a simple variable,

$$A = \cos a$$
$$B = \cos b$$

But this method is not suitable for the set of equations from the harmonic analysis equations. Because, the first group of equations is the cosine functions of the triggering angle. The second and third group of equations are cosine functions of 3^{rd} and 5^{th} multiples of the angles in the first equations. So Newton's method is suitable for obtaining the solution for the equation.

3.7.1 Newton's Method

Traditional method for solving SHE PWM problem using Newton- Raphson algorithm deeply depends on the selection of initial guess θ_0, taking suitable initial guess which must close enough to the exact solution in order to ensure the convergence of Newton's algorithm. Critical factor which can be heavily impact the calculation convergence is initial values. The creation of bad initial values leads to a much longer calculation time and create wrong solutions. Irrespective of the numerical methods used, the process of guessing initial value is quite tedious and deep comprehension of the switching pattern to be solved. Especially when the solution trajectories of the switching angles do not smoothly increase or decrease, it is more difficult to set appropriate initial values for the whole modulation index range. To conquer the problem for the selective harmonic elimination method, evolutionary algorithms are used to find all the solutions to the harmonic equations and the calculated solutions or the switching angles which are used to construct the switching pattern for the voltage source inverter to eliminate the lower order harmonics in the whole range of the modulation index to satisfy the application requirements.

The convergence of Newton's method and its performance solely depends on the accuracy of initial guess. If the initial guess is too different from the final solution, then the Newton's method may not even converge to the final value. A very good initial guess eventually reduces the number of iterations and hence improves the performance speed of the method. So the initial angles are generated by converting the equation into a Cauchy Problem and then plotting a set of trajectories of triggering angles as a function of the preset level of the fundamental or modulation index.

3.7.2 Newton Raphson Iteration Method

Algorithm

1. Propose a set of initial values for θ

$$\theta^j = \left[\theta_1{}^j \quad \theta_2{}^j \quad \theta_3{}^j \quad \cdots\cdots\cdots \quad \theta_N{}^j\right]^T \qquad with\ j = 0$$

2. Calculate the value of

$$F(\theta^j) = F^j$$

Where, F is the condenser vector format of the non-linear equation system

3. Linearize equation 2 about θ^j

$$F^j + \left(\frac{\partial f}{\partial \theta}\right)^j \partial\theta^j = T$$

Where, T is the amplitude of the fundamental and harmonic components

$$T = \left[\frac{M\pi}{4} \quad 0 \quad 0 \quad \cdots\cdots\cdots \quad 0\right]^T$$

F is the function connecting harmonics with switching angle

$$\left(\frac{\partial f}{\partial \theta}\right)^j = \begin{bmatrix} \frac{\partial f_1}{\partial \theta_1} & \frac{\partial f_1}{\partial \theta_2} & \frac{\partial f_1}{\partial \theta_3} & & \frac{\partial f_1}{\partial \theta_N} \\ \frac{\partial f_2}{\partial \theta_1} & \frac{\partial f_2}{\partial \theta_2} & \frac{\partial f_2}{\partial \theta_3} & \cdots\cdots\cdots & \frac{\partial f_2}{\partial \theta_N} \\ \vdots & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial f_N}{\partial \theta_1} & \frac{\partial f_N}{\partial \theta_2} & \frac{\partial f_N}{\partial \theta_3} & & \frac{\partial f_N}{\partial \theta_N} \end{bmatrix}$$

$$\partial\theta^j = \left[\partial\theta_1{}^j \quad \partial\theta_2{}^j \quad \partial\theta_3{}^j \quad \cdots\cdots\cdots \quad \partial\theta_N{}^j\right]^T$$

4. Solve $d\theta^j$ from equation 3

$$\partial\theta^j = INV\left[\frac{\partial f}{\partial \theta}\right]^j (T - F^j)$$

5. Change the initial values of each step by

$$\theta^{j+1} = \theta^j + \partial\theta^j$$

6. Repeat the process until $d\theta^j$ satisfied the desired degree of accuracy

Since the major problem of Newton's method is the knowledge of starting points of switching angles. For Unipolar SHE PWM, starting points are decided by following equations.

3.7.2.1 The Single Phase System

$$E_{ang} = \frac{\pi}{N+1} \qquad for\ N = 2, 3, 4 \ldots \ldots$$

Where, E_{ang} is the angular gap and N is the number of switching angles.

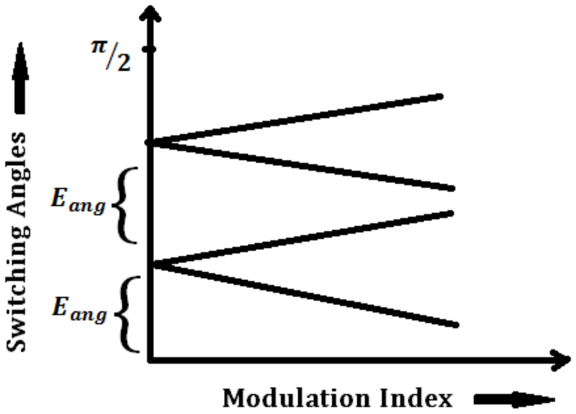

Figure 12: Trace of Starting points for Unipolar SHE PWM for single phase system

3.7.2.2 Three Phase System

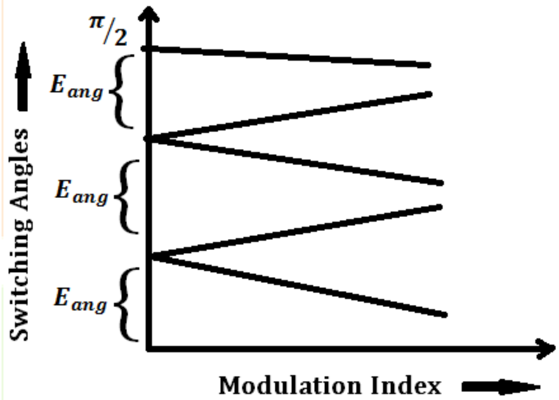

Figure 13: Trace of Starting points for Unipolar SHE PWM for three phase system

$$E_{ang} = \frac{2\pi}{3(N+1)} \qquad for\ N = 3, 5, 7 \ldots \ldots \ldots$$

Where, E_{ang} is the angular gap and N is the number of switching angles.

3.8 Cauchy Problem Formulation

Initial value for bipolar SHE PWM is decided by Cauchy Problem formulation method.
Initial values,

$$\theta_K = \frac{180 \times K}{2N+1}$$

Where, N is the number of angles and K= 1, 2, 3...........N
Before proceeding to the Cauchy problem, the given equations converted into matrix form;

$$f(\theta) = \begin{bmatrix} -1 + 2\cos\theta_1 - 2\cos\theta_2 + 2\cos\theta_3 - 2\cos\theta_4 - {}^{\pi M}/_{4} \\ -1 + 2\cos 3\theta_1 - 2\cos 3\theta_2 + 2\cos 3\theta_3 - 2\cos 3\theta_4 \\ -1 + 2\cos 5\theta_1 - 2\cos 5\theta_2 + 2\cos 5\theta_3 - 2\cos 5\theta_4 \\ -1 + 2\cos 7\theta_1 - 2\cos 7\theta_2 + 2\cos 7\theta_3 - 2\cos 7\theta_4 \end{bmatrix}$$

Jacobian matrix can be formed by differentiating the equations partially with respect to θ.

$$J(\theta)=\begin{bmatrix}-2\sin\theta_1 & 2\sin\theta_2 & -2\sin\theta_3 & 2\sin\theta_4\\ -6\sin 3\theta_1 & 6\sin 3\theta_2 & -6\sin 3\theta_3 & 6\sin 3\theta_4\\ -10\sin 5\theta_1 & 10\sin 5\theta_2 & -10\sin 5\theta_3 & 10\sin 5\theta_4\\ -14\sin 7\theta_1 & 14\sin 7\theta_2 & -14\sin 7\theta_3 & 14\sin 7\theta_4\end{bmatrix}$$

In this case,

$$J(\theta)\left[\frac{d\theta}{dM}\right]=b[1\quad 0\quad 0\quad 0\quad \cdots\cdots\cdots 0\quad 0]^T$$

When $\theta_{M=M_0} = \theta_0$ and $b = {}^{M\pi}/_{4}$

$$\begin{bmatrix}-\sin\theta_1 & \sin\theta_2 & -\sin\theta_3 & \sin\theta_4\\ -3\sin 3\theta_1 & 3\sin 3\theta_2 & -3\sin 3\theta_3 & 3\sin 3\theta_4\\ -5\sin 5\theta_1 & 5\sin 5\theta_2 & -5\sin 5\theta_3 & 5\sin 5\theta_4\\ -7\sin 7\theta_1 & 7\sin 7\theta_2 & -7\sin 7\theta_3 & 7\sin 7\theta_4\end{bmatrix}\begin{bmatrix}{}^{d\theta_1}/_{dM}\\ {}^{d\theta_2}/_{dM}\\ {}^{d\theta_3}/_{dM}\\ {}^{d\theta_4}/_{dM}\end{bmatrix}=\begin{bmatrix}{}^{M\pi}/_{8}\\ 0\\ 0\\ 0\end{bmatrix}$$

For this value of θ_1, θ_2, θ_3 and θ_4, it can be proved that the fundamental, third and fifth harmonics have zero amplitude.

3.9 Steps to get the trajectories of θ versus M

If the value of increment of M is small enough then the method proves to be pretty effective. So the increment for M is chosen as 0.001.

1. Set $M = M_0 = 0$ and

$$\theta_K = \frac{180\times K}{2N+1}\ \ for\ K = 1, 2, 3 \ldots \ldots . N$$

2. Solve the matrix equation to get the next points of θ
3. Increment the value of M by $\Delta M = 0.001$ so that $M_{k+1} = M_k + \Delta M$. The next value is $\theta_{k+1} = \theta_k + \Delta\theta$.
4. Repeat step 2 until $\theta_k > 0$

The above procedure in the form of M- file program. The program accepts the number of harmonics equations as input (N) and generates a set of trajectories of $\theta_1, \theta_2, \theta_3 \ldots \ldots \ldots . . \theta_N$ as a function of M. It was found that the trajectories tend to zero as M reaches the value 1.068. The limiting value varies according to the number of harmonics to be eliminated, 1.068 is the value of M for the case of N=3.

IV. SIMULINK MODEL AND RESULTS

Table 1 describes the simulation parameters used for the simulation. Matlab/Simulink platform is used for software simulation.

Table 1: Simulation Parameters

Simulation Parameter	Specification
DC Source	230Volt
Load Resistance	100 Ohm
MOSFET Switches	4nos.

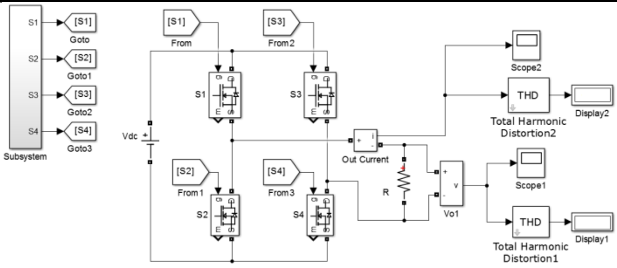

Figure 14: Simulink model of Basic H bridge inverter

Figure 14 shows the Simulink model of H bridge inverter triggered by SHE PWM.

4.1 Unipolar SHE PWM

Figure 15 shows the graphical representation of the initial values used for Unipolar SHE PWM. With these final values Simulink is run and the corresponding output voltage shown below.

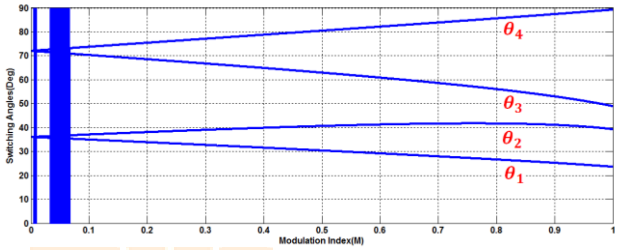

Figure 15: Trace of Starting points of Unipolar SHE PWM

Figure16 shows the Unipolar 3 level inverter with 4 switching angles in one quarter.

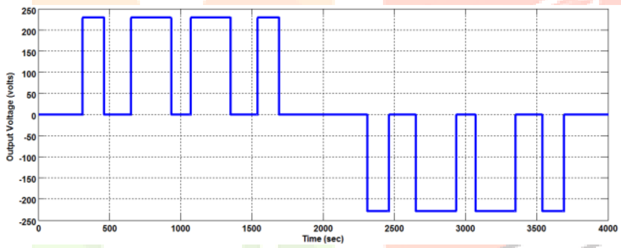

Figure 16: Inverter output using Unipolar SHE PWM

4.2 Bipolar SHE PWM

4.2.1 Case 1

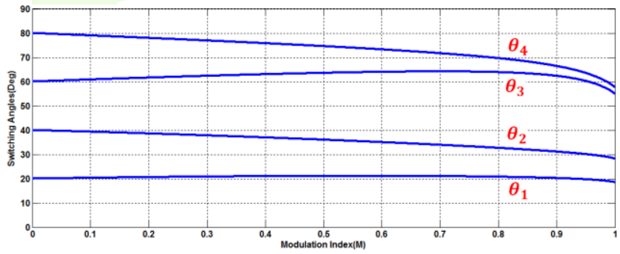

Figure 17: Trace of Starting points of Bipolar SHE PWM

Figure 17 shows the locus of the starting values of Bipolar SHE Case 1 using Cauchy Problem formulation technique explained earlier.

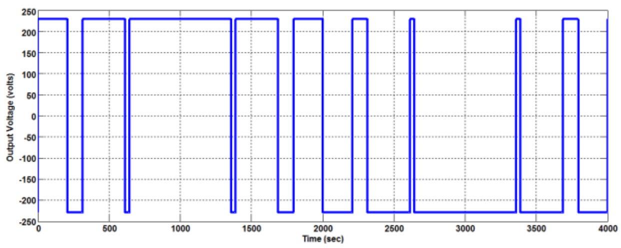

Figure 18: Inverter output using Bipolar SHE PWM

Figure 18 shows the corresponding output voltage of the basic H Bridge inverter using Bipolar SHE method and output voltage contains 4 switching in each quarter.

4.2.2 Case 2

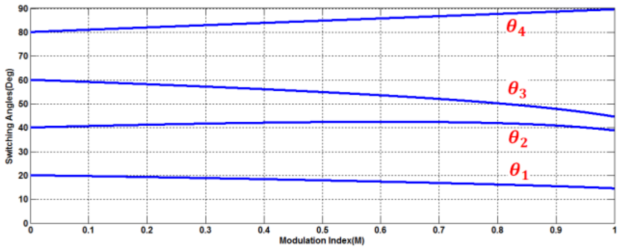

Figure 19: Trace of Starting points of Bipolar SHE PWM

Figure 19 shows the locus of the starting values of Bipolar SHE Case 1 using Cauchy Problem formulation technique explained earlier.

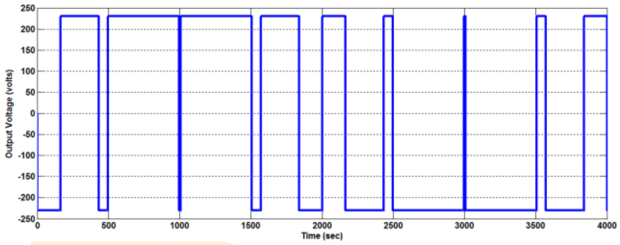

Figure 20: Inverter output using Bipolar SHE PWM

Figure 20 shows the corresponding output voltage of the basic H Bridge inverter using Bipolar SHE method and output voltage contains 4 switching in each quarter.

V. RESULTS AND DISCUSSION

A. Harmonic Analysis of Inverter Circuit with Square Gate Pulse

The switching waveforms, output voltage waveform and harmonic spectrum of the output voltage of the simulation model are shown in Fig. 21, Fig. 22and Fig. 23 respectively.

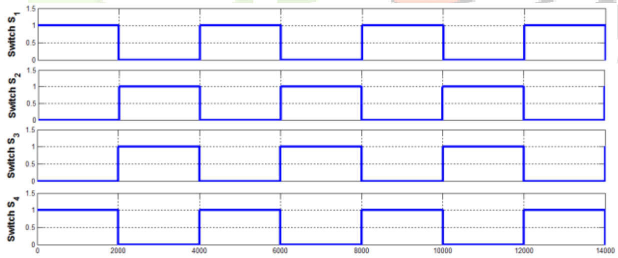

Figure 21: Switching Pulse of Square pulse Inverter

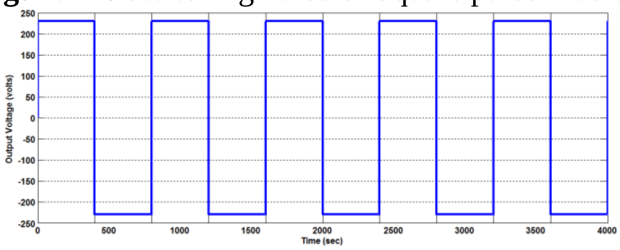

Figure 22: Square Pulse Inverter output

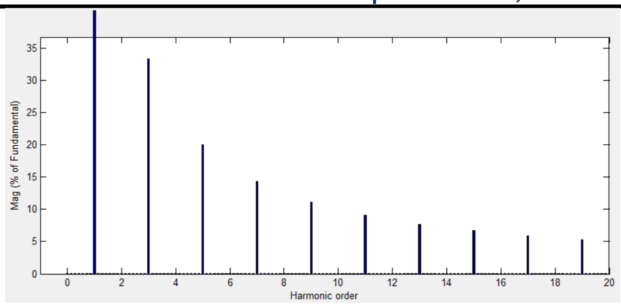

Figure 23: FFT Spectrum of Square pulse Inverter

Table 2 shows the harmonic content expressed as a percentage of fundamental for each order of the harmonic.

Table 2: Comparative study of Harmonic orders with percentage magnitude

Harmonic Order	**Magnitude (% of fundamental)**
h1	100.00%
h3	33.33%
h5	20.00%
h7	14.29%
h9	11.11%
h11	9.09%
h13	7.69%
h15	6.67%
h17	5.88%
h19	5.26%

Thus from the Table 2, it is observed that as the order of the harmonic increases the magnitude of the harmonic content expressed as a percentage of the fundamental decreases. Also from the spectrum analysis it is observed that the magnitudes harmonic order of 3^{rd}, 5^{th}, 7^{th}, 9^{th}, 11^{th} etc., are higher and they are detrimental to the operation of critical equipment's. Thus if these order of harmonics can be eliminated, it is possible to operate the critical equipment's with better performance.

5.2 Elimination of the First Three Odd Harmonics Using SHE PWM

Table 3 shows the percentage value of 3^{rd}, 5^{th} and 7^{th} harmonic component in the inverter output using SHE PWM technique. By considering 4 switching angles 3 harmonic components can be eliminated. In this paper 3^{rd}, 5^{th} and 7^{th} harmonic components are successfully eliminated for the single phase system. To eliminate the first three odd lower order harmonics the four notching angles are obtained from the developed program. The values of the notching angles are: $\theta_1 = 23.5598°, \theta_2 = 39.2596°, \theta_3 = 48.960°$ and $\theta_4 = 89.2240°$. These notching angles are converted into seconds and directly given to the gate drive circuitry of the inverter. Figure 24 shows the harmonic spectrum of the H bridge inverter output voltage using Unipolar SHE PWM.

Table 3: Comparative study of Harmonic orders with different PWM schemes

Harmonic Order	Unipolar SHE	Bipolar SHE Case 1	Bipolar SHE Case 2
3rd Harmonic	0.04%	0.12%	0.42%
5th Harmonic	0.26%	0.17%	0.19%
7th Harmonic	0.04%	0.11%	0.21%

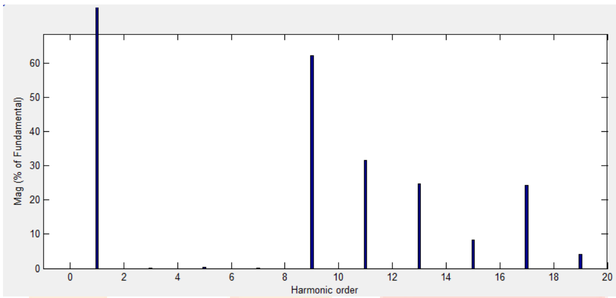

Figure 24: FFT Spectrum of Unipolar SHE PWM Inverter

From the harmonic spectrum it is found that the lower order harmonics 3rd, 5th and 7th are eliminated and their magnitudes are very close to zero.

5.3 Harmonic Analysis of Inverter Circuit Using SHE PWM

Table 4: Comparative study of %THD with different PWM methods

	SHE PWM	SPWM	By Calculation
% THD	33.29%	26.8%	17.58%

In Table 4, it compares the %Total Harmonic Distortion of the inverter output using different PWM techniques. Sinusoidal PWM uses the Triangular carrier wave and sinusoidal modulating wave for generating corresponding switching instants.

Table 5: Comparative study of 3rd Harmonic component with different PWM methods

	SHE PWM	SPWM	By Calculation
3rd Harmonic	0.04%	0.07%	1.98%

Percentage value of 3rd harmonic component present in the inverter output using different PWM method describes in Table 5. By SHE PWM method, 3rd harmonic component eliminates effectively compared to other two methods. So with SHE PWM any harmonic component can be cancel out by selecting appropriate switching angles.

VI. CONCLUSIONS

In this paper a simple and effective, minimization technique to solve the selective harmonic elimination using computed PWM control method for single phase voltage source inverters has been discussed. By solving the harmonic equations, the values of notch angles are obtained, which control the switching instants of the PWM wave. The PWM wave generated by the computed PWM technique takes less time when compared to the sine triangular comparison method. They do not involve any 'trial and error' procedures and the PWM pulse can triggered accurately at the desired values of the notch angles. The comparison between two techniques shows that the proposed method successfully eliminates 3rd, 5th, and 7th harmonic components along with dc and even harmonics.

VII. REFERENCES

[1]. Muhammad H. Rashid, "POWER ELECTRONICS HAND BOOK DEVICES, CIRCUITS, AND APPLICATIONS", Third Edition, ISBN 978-0-12-382036-5, Copyright C 2011, Elsevier Inc.

[2]. H. S. Patel And R. G. Hoft "Generalized Harmonic Elimination And Voltage Control In Thyristor Inverters. Part1. Harmonic Elimination," IEEE Trans. Ind. App., 9 (May/June) (1973) Pp. 310-317.

[3]. Prasad N. Enjeti, Phoivos D. Ziogas, James F. LINDSAY, "Programmed PWM Techniques To Eliminate Harmonics: A Critical Evaluation", IEEE TRANSACTIONS ON INDUSTRY APPLICATIONS, VOL. 26, No. 2, Marchiapril 1990.

[4]. Y. Sahali, M. K. FELLAH,"Selective Harmonic Eliminated Pulse-Width Modulation Technique (SHE PWM) Applied To Three-Level Inverter / Converter 2003", IEEE International Symposium on Industrial Electronics (Cat. No.03TH8692) Volume: 2 Pages: 1112 - 1117

[5]. Jagdish Kumar, Biswarup Das, and Pramod Agarwal, "Selective Harmonic Elimination Technique for A Multilevel Inverter", Fifteenth National Power Systems Conference (NPSC), IIT Bombay, December 2008.

[6]. L. M. Tolbert, J. N. Chiasson, K. McKenzie, and Z. Du, "Elimination of harmonics in a multilevel converter with non equal DC sources, " in *Proc. IEEE Appl. Power Electron. Con*!,Miami, FL, Feb. 9-13, 2003, pp. 589-595.

[7]. Kh. El-Naggar and T. H. Abdelhamid, "Selective harmonic elimination of new family of multilevel inverters using genetic algorithms", *Elsevier Journal of Energy Conversion and Management*, vol. 49, issue 1, pp. 89-95, Jan. 2008.

[8]. J. Rodriguez, J. Lai and FZ. Peng, "Multilevel converters: a survey of topologies, controls, and applications", *IEEE Trans. Industrial Application*, vol. 49(4), pp. 724–738, Aug. 2002.

[9]. P. Lezana, J. Rodriguez and D. A. Oyarzun, "Cascaded multilevel inverter with regeneration capability and reduced number of switches", *IEEE Trans. on Industrial Electronics*, vol. 55, no. 3, pp. 1059-1066, Mar. 2008.

[10]. Ovaska SJ, Bose T and Vainio O., "Genetic algorithmassisted design of adaptive predictive filters for 50/60 Hz power systems instrumentation", *IEEE Trans. Instrument Measurement*, vol. 54(5), pp. 2041-2048, 2005.

[11]. D. G. Holmes and T. A. Lipo, *Pulse Width Modulation for Power Converters*. Piscataway, NJ: IEEE Press, 2003.

[12]. S. Kouro, J. Rebolledo, and J. Rodriguez, "Reduced switching frequency modulation algorithm for high-power multilevel inverters," *IEEE Trans.Ind. Electron.*, vol. 54, no. 5, pp. 2894–2901, Oct. 2007.

[13]. W. Fei, X. Du, and B. Wu, "A generalized half-wave symmetry SHEPWM formulation for multilevel voltage inverters," *IEEE Trans. Ind. Electron.*, vol. 57, no. 9, pp. 3030–3038, Sep. 2010.

[14]. Y. Liu, H. Hong, and A. Q. Huang, "Real-time algorithm for minimizing THD in multilevel inverters with unequal or varying voltage steps under staircase modulation," *IEEE Trans. Ind. Electron.*, vol. 56, no. 6, pp. 2249–2258, Jun. 2009.

[15]. B. Ozpineci, L. M. Tolbert, and J. N. Chiasson, "Harmonic optimization of multilevel converters using genetic algorithms," *IEEE Power Electron. Lett.*, vol. 3, no. 3, pp. 92–95, Sep. 2005.

[16]. H. Taghizadeh and M. Tarafdar Hagh, "Harmonic elimination of multilevel inverters using particle swarm optimization," in *Proc. IEEE Int. Symp. Ind. Electron.*, 2008, pp. 393–396.

[17]. A. I. Maswood, Shen Wei and M. A. Rahman, *"A Flexible Way to Generate PWM-SHE Switching Patterns Using Genetic Algorithm,'' Conference Proceedings of IEEE (APEC)*, 2001, pp. 1130-1134.

[18]. R. Salehi, N. Farokhia, M. Abedi, and S.H. Fathi, *"Elimination of Low Order Harmonics in Multilevel Inverters Using Genetic Algorithm,''* Journal of Power Electronics, volume 11, no. 2, Mar. 2011, pp.132-139.