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M-Polynomial And Some Topological Indices Of Some Windmill Graphs.

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Abstract: The study of degree based topological indices plays very important role in QSAR/QSPR. The different degree based topological indices are used to characterize the chemical compound. In the this paper we compute the M-polynomial for different windmill graph, and also from the obtained M-polynomial, we derive different degree based indices.

Kea words: M-polynomial, degree based topological index, wind mill graphs, Dutch wind mill graph, French wind mill graphs, Kulli path and cycle wind mill graphs.

1. Introduction

The graph considered here is a finite undirected and simple graphs. Let $G\{V,E\}$ is a graph, where V is the vertex set and E is the edge set. If the graph G $\{V,E\}$ is connected if there exists edge between every pair of vertices in G. If $|V(G)|=n$ and $|E(G)|=m$, then the graph is also called (n,m) graph. The degree of vertex v is denoted by $d_G(v)$ is the number of vertices adjacent to v. The edge connecting the vertices u and v is denoted by $e=uv$. The degree of an edge $e=uv$ is denoted by $d_e(e)$ such that $d_e(e)=d_G(u)+d_G(v)-2$. The concept of degree in graph theory is closely related to the concept of valance in chemistry. For details on basics of graph theory we refer the standard text

[1][2][3].

A molecular graph is a graph in which the vertices corresponds to the atoms and edges corresponds to the chemical bonds of molecules. A single number that can be computed from the molecular graph and used to characterize some property of the underlying molecule is said to be a topological index or molecular structure descriptor. Several such descriptors have been considered in theoretical chemistry and have some application especially in QSAR/QSPR.

Numerous algebraic polynomials have useful applications in chemistry such as Hosaya Polynomial

(Wiener Polynomial) which plays a very important role in determining distance based topological indices.

Similarly among several polynomial plays very impartant role in determining closed form of many degree based topological indices [4,5] . The major purpose of M- polynomial is the it gives large information about the degree based indices.

The M-polynomial of a graph was introduce by **SANDI KLAVZAR .** Using M-polynomial the degree based topological indices can be routinely calculated. The wind mill graph $W(k,n)$ is an undirected graph constructed for $k \geq 2$ and $n \geq 2$ by joining n copies of the complete graph K_k at a shared universal vertex. The different types of windmill graph are Dutch windmill graph, French windmill graph, Kullli path windmill graph, Kuli cycle windmill graph etc.

Basic definitions and results:

The M-Polynomial of a Graph G is denoted and defined as $M(G,x,y) = \sum_{\delta \leq i \leq j \leq \Delta} m_{ij}(G) x^i y^j$.

Where $\delta =$ Min$\{d_v / v \in V(G)\}$, $\Delta =$ Max$\{d_v / v \in V(G)\}$ and $m_{ij}(G)$ is the edge $vu \in E(G)$ such that $\{d_v, d_u\} = \{i, j\}$.

The first Zagreb index was introduced by Guttmann and Trinajstic [9]. It is an important descriptor and has been closely correlated with many chemical properties. The first Zagreb index of G is defined as

$$M_1(G) = \sum_{u \in V(G)} d_G(u)^2 \text{ or } M_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)].$$

The second Zagreb indices were denoted and defined by $M_2(G) = \sum_{uv \in E(G)} [d_G(u) d_G(v)]$.

introduced to take account of the contribution of pair of vertices.

In 1975 Randic Introduced the best known degree based index is known as the Randic index denoted and defined by

$$R(G) = \sum_{uv \in E(G)} (\deg_G(u) \deg_G(v))^{-\frac{1}{2}}$$

More generally the general Randic index denoted and defined by

$$R_\alpha(G) = \sum_{uv \in E(G)} (\deg_G(u) \deg_G(v))^{\alpha}$$

This is also known as branching index or connectivity index.

The second modified Zagreb index is denoted and defined as ${}^m M_2(G) = \sum_{uv \in E(G)} \frac{1}{d_G(u) d_G(v)}$.

The symmetric division degree index is defined as

$$SSD(G) = \sum_{uv \in E(G)} \left\{ \frac{\min(d(u), d(v))}{\max(d(u), d(v))} + \frac{\max(d(u), d(v))}{\min(d(u), d(v))} \right\}.$$

The Harmonic index is defined as $H(G) = \sum_{uv \in E(G)} \frac{2}{d(u) + d(v)}$.

The inverse sum index is defined as $I(G) = \sum_{uv \in E(G)} \frac{d(u) d(v)}{d(u) + d(v)}$.

The augmented Zagreb index is defined as

$$A(G) = \sum_{uv \in E(G)} \left\{ \frac{d(u)d(v)}{d(u)+d(v)-2} \right\}^3$$

The table 1 given below gives the relation between some degree based topological indices and

M-polynomial of corresponding windmill graphs.

Topological index	Formula from of $M(G;x,y)$
First Zagreb index $M_1(G)$	$(D_x + D_y)(M(G;x,y))_{x=y=1}$
Second Zagreb index $M_2(G)$	$(D_x D_y)(M(G;x,y))_{x=y=1}$
Randic index $R_\alpha(G)$	$(D_x^\alpha D_y^\alpha)(M(G;x,y))_{x=y=1}$
Second modified Zagreb index ${}^m M_2(G)$	$(S_x S_y)(M(G;x,y))_{x=y=1}$
Inverse Randic index $RR_\alpha(G)$	$(S_x^\alpha S_y^\alpha)(M(G;x,y))_{x=y=1}$
Symmetric division index $SSD(G)$	$(D_x S_y + S_x D_y)(M(G;x,y))_{x=y=1}$
Harmonic index $H(G)$	$2S_x J(M(G;x,y))_{x=y=1}$
Inverse sum index $I(G)$	$(S_x J(D_x D_y))(M(G;x,y))_{x=y=1}$
Augmented Zagreb index $A(G)$	$S_x^3 Q_{-2} J\,(D_x^3 D_y^3)(M(G;x,y))_{x=y=1}$

Table 1

Where $D_x = x.\frac{\partial(f(x,y))}{\partial x}$, $D_y = y.\frac{\partial(f(x,y))}{\partial y}$, $S_x = \int_0^x \frac{f(t,y)}{t} dt$, $S_y = \int_0^y \frac{f(x,t)}{t} dt$

$J(f(x,y)) = f(x,x)$ and $Q_\alpha(f(x,y)) = x^\alpha f(x,y)$.

2. Main results.

Theorem 2.1: The M-polynomial of Dutch windmill graph is given by

$$M\left[D_n^m(x,y)\right]=m(n-2)x^2y^2+2mx^2y^{2m}$$

Proof:

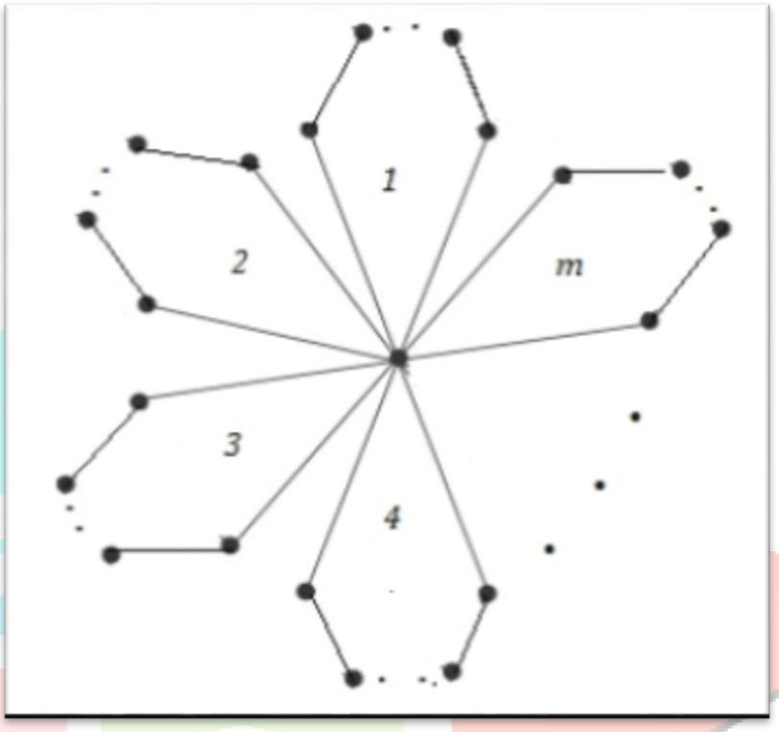

Figure 1: Dutch wind mill graph

The Dutch windmill graph $D_n^{(m)}$ is the graph obtained by taking $m \geq 1$copies of the cycle $C_n : n \geq 3$ with a vertex in common. The Dutch windmill graph has the vertex set $V(G)$ and edge set $E(G)$ where the cardinality of $V(G)$ is $|V(G)| = m(n-1)+1$ and the cardinality of $E(G)$ is $|E(G)| = mn$ .From the diagram of Dutch windmill graph have two partitions of vertex set and two partitions of edge set.

The partition of vertex set is as follows.

$V_2 = \{v \in V(G) : d_G(v) = 2\}$ such that $|V_2| = m(n-1)$ and

$V_{2m} = \{v \in V(G) : d_G(v) = 2m\}$ such that $|V_2| = 1$

The partition of edge set is as follows.

$E_4 = \{uv \in E(G) : d_G(u) = 2, d_G(v) = 2\}$ such that $|E_4| = m(n-2)$ and

$E_{2m+2} = \{uv \in E(G) : d_G(u) = 2, d_G(v) = 2m\}$ such that $|E_{2m+2}| = 2m$

Now the M-polynomial of Dutch windmill graph is defined as

$$M\left[D_n^{(m)}(x,y)\right]=\sum_{\delta\prec i\le j\le\Delta} m_{ij}x^i y^j$$

$$M\left[D_n^{(m)}(x,y)\right]=\sum_{2\le 2} m_{22}x^2y^2+\sum_{2\le 2m} m_{2(2m)}x^2y^{2m}$$

$$M\left[D_n^{(m)}(x,y)\right]=\sum_{2\le 2}|E_4|x^2y^2+\sum_{2\le 2m}|E_{2m+2}|x^2y^{2m}$$

$M\left[D_n^{(m)}(x,y)\right]=m(n-2)x^2y^2+2mx^2y^{2m}$ is the M-polynomial of Dutch windmill graph.

Theorem 2.2: Let $D_n^{(m)}$ be the Dutch windmill graph, then

1. $M_1\left(D_n^{(m)}\right)=4m(n-2)+4(m+m^2)$.
2. $M_2\left(D_n^{(m)}\right)=4m(n-2)+8m^2$.
3. $R_\alpha\left(D_n^{(m)}\right)=2^{2\alpha}m(n-2)+2^{2\alpha+1}m^{\alpha+1}$.
4. ${}^mM_2\left(D_n^{(m)}\right)=\dfrac{m(n-2)}{4}+\dfrac{1}{2}$.
5. $RR_\alpha\left(D_n^{(m)}\right)=\dfrac{m(n-2)}{2^{2\alpha}}+\dfrac{2m}{2^\alpha(2m)^\alpha}$.
6. $SSD\left(D_n^{(m)}\right)=2m(m+n-1)$.
7. $H\left(D_n^{(m)}\right)=\dfrac{m(n-2)}{2}+\dfrac{2m}{m+1}$.
8. $I\left(D_n^{(m)}\right)=m(n-2)+\dfrac{4m^2}{m+1}$.
9. $A\left(D_n^{(m)}\right)=8mn$.

Proof: From theorem (1) we have M-polynomial of Dutch Windmill graph is

$M\left[D_n^{(m)}(x,y)\right]=m(n-2)x^2y^2+2mx^2y^{2m}$, Then we have

$$D_x\left[M\left(D_n^{(m)}(x,y)\right)\right]=2m(n-2)x^2y^2+4mx^2y^{2m}.$$

$$D_y\left[M\left(D_n^{(m)}(x,y)\right)\right]=2m(n-2)x^2y^2+4m^2x^2y^{2m}.$$

Consider $\left[(D_x+D_y)\left(M\left(D_n^m(x,y)\right)\right)\right]=4m(n-2)x^2y^2+4(m+m^2)x^2y^{2m}$

At $x=1\ \&\ y=1$

The First Zagreb index of wind mill graph is $M_1\left(D_n^{(m)}\right)=4m(n-2)+4(m+m^2)$.

We have $D_xD_y\left[M\left(D_n^{(m)}(x,y)\right)\right]=4m(n-2)x^2y^2+8m^2x^2y^{2m}$.

At $x=1\ \&\ y=1$

The Second Zagreb index of wind mill graph is $M_2\left(D_n^m\right)=4m(n-2)+8m^2$.

We have $D_x^{\alpha} D_y^{\alpha}\left[M\left(D_n^{(m)}(x,y)\right)\right] = 2^{2\alpha} m(n-2)x^2 y^2 + 2^{2\alpha+1} m^{\alpha+1} x^2 y^{2m}$.

At $x = 1 \,\&\, y = 1$

The Randic index of Wind mill graph is $R_{\alpha}\left(D_n^{(m)}\right) = 2^{2\alpha} m(n-2) + 2^{2\alpha+1} m^{\alpha+1}$

We have $S_x S_y\left[M\left(D_n^{(m)}(x,y)\right)\right] = \frac{m(n-2)}{4} x^2 y^2 + \frac{x^2 y^{2m}}{2}$.

At $x = 1 \,\&\, y = 1$

The second modified Zagreb index of windmill graph is

$${}^2M_2\left(D_n^{(m)}\right) = \frac{m(n-2)}{4} + \frac{1}{2}.$$

$$S_x^{\alpha} S_y^{\alpha}\left[M\left(D_n^{(m)}(x,y)\right)\right] = \frac{m(n-2)}{2^{2\alpha}} x^2 y^2 + \frac{2m x^2 y^{2m}}{2^{2\alpha} m^{\alpha}}.$$

At $x = 1 \,\&\, y = 1$

Inverse Randic index of Windmill graph is $RR_{\alpha}\left(D_n^{(m)}\right) = \frac{m(n-2)}{2^{2\alpha}} + \frac{2m}{2^{\alpha}(2m)^{\alpha}}$.

$$[S_x D_y + S_y D_x]\left[M\left(D_n^{(m)}(x,y)\right)\right] = 2m(n-2)x^2 y^2 + 2\left(m + m^2\right)x^2 y^{2m}.$$

At $x = 1 \,\&\, y = 1$

Symmetric division index of windmill graph $SSD\left(D_n^{(m)}\right) = 2m(m+n-1)$.

$$S_x J\left[M\left(D_n^{(m)}(x,y)\right)\right] = \frac{m(n-2)x^4}{4} + \frac{2m x^{2+2m}}{2+2m}.$$

At $x = 1$

Harmonic index of Dutch windmill graph $H\left(D_n^{(m)}\right) = \frac{m(n-2)}{2} + \frac{2m}{m+1}$.

$$S_x\left\{J\left[D_x D_y M\left(D_n^{(m)}(x,y)\right)\right]\right\} = m(n-2)x^4 + \frac{4m^2}{(m+1)} x^{2m+2}.$$

At $x = 1$

Inverse sum index of Dutch wind mill graph $I\left(D_n^{(m)}\right) = m(n-2) + \frac{4m^2}{m+1}$.

$$S_x^3\left\{Q_{-2} J\left[D_x^3 D_y^3 M\left(D_n^{(m)}(x,y)\right)\right]\right\} = 8m(n-2)x^2 + 16m x^{2m}.$$

At $x = 1$

Augmented Zagreb index $A\left(D_n^{(m)}\right) = 8mn$.

Corollary: For a windmill graph having 5copies of cycle with 5 vertices we have

1. $M_1\left(D_n^{(m)}\right)=180.$
2. $M_2\left(D_n^{(m)}\right)=260.$
3. $R_\alpha\left(D_n^{(m)}\right)=15.2^{2\alpha}+2^{2\alpha+1}5^{\alpha+1}=15.2^{2\alpha}+10.2^{2\alpha}.5^{\alpha}.$
4. ${}^mM_2\left(D_n^{(m)}\right)=\frac{17}{4}.$
5. $RR_\alpha\left(D_n^{(m)}\right)=\frac{15}{2^{2\alpha}}+\frac{10}{(20)^\alpha}.$
6. $SSD\left(D_n^{(m)}\right)=90.$
7. $H\left(D_n^{(m)}\right)=\frac{55}{6}.$
8. $I\left(D_n^{(m)}\right)=\frac{95}{3}.$
9. $A\left(D_n^{(m)}\right)=200.$

Theorem 2.3: The M-polynomial of French windmill graph is given by

$$M\left[F_n^m(x,y)\right]=\frac{m\left(n^2-3n+2\right)}{2}x^{n-1}y^{n-1}+m(n-1)x^{n-1}y^{m(n-1)}.$$

Proof:

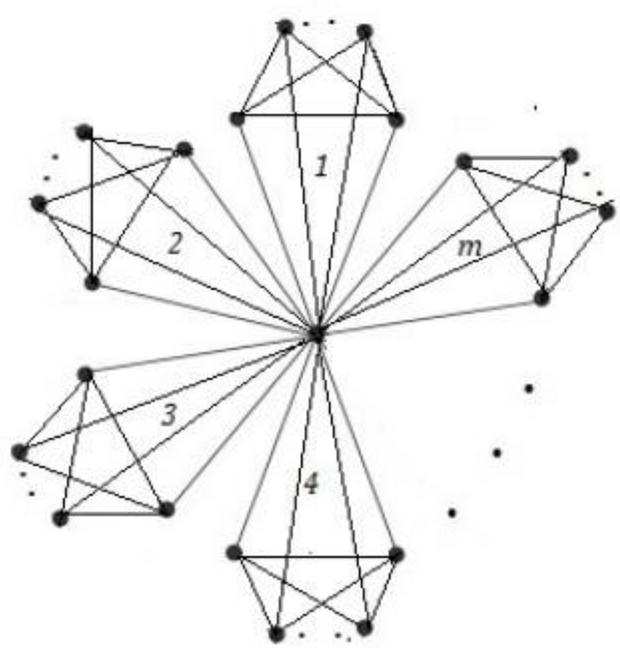

Figure 2: French wind mill graph

The French windmill graph $F_n^{(m)}$ is the graph obtained by taking $m \geq 2$ copies of the complete graph $K_n : n \geq 2$ with a vertex in common. The French windmill graph has the vertex set $V(G)$ and edge set $E(G)$ where the cardinality of $V(G)$ is $|V(G)| = m(n-1)+1$ and the cardinality of $E(G)$ is $|E(G)| = \frac{mn(n-1)}{2}$. From the diagram of French windmill graph we the two partition of vertex set and two partition of edge set.

The partition of vertex set is as follows.

$V_{n-1} = \{v \in V(G) : d_G(v) = n-1\}$ such that $|V_{n-1}| = m(n-1)$ and

$V_{(n-1)m} = \{v \in V(G) : d_G(v) = (n-1)m\}$ such that $|V_{(n-1)m}| = 1$

The partition of edge set is as follows.

$E_{2(n-1)} = \{uv \in E(G) : d_G(u) = n-1, d_G(v) = n-1\}$ such that $|E_{2(n-1)}| = \frac{m(n^2-3n+2)}{2}$ and

$E_{2(n-1)m} = \{uv \in E(G) : d_G(u) = n-1, d_G(v) = m(n-1)\}$ such that $|E_{2(n-1)m}| = m(n-1)$

Now the M-polynomial of French windmill graph is defined as

$$M[F_n^{(m)}(x,y)] = \sum_{\delta \prec i \leq j \leq \Delta} m_{ij} x^i y^j$$

$$M[F_n^{(m)}(x,y)] = \sum_{(n-1)\leq(n-1)} m_{(n-1)(n-1)} x^{n-1} y^{n-1} + \sum_{(n-1)\leq m(n-1)} m_{(n-1)\leq(m(n-1)} x^{n-1} y^{m(n-1)}$$

$$M[F_n^{(m)}(x,y)] = |E_{2(n-1)}| x^{n-1} y^{n-1} + |E_{2(n-1)m}| x^{n-1} y^{m(n-1)}$$

$M[F_n^{(m)}(x,y)] = \frac{m(n^2-3n+2)}{2} x^2 y^2 + m(n-1) x^{n-1} y^{m(n-1)}$ is the expression for M-polynomial of French windmill graph.

Theorem 2.4: Let $F_n^{(m)}$ be the French windmill graph, then

1. $M_1(F_n^{(m)}) = m(n-1)(n^2-3n+2) + m(m+1)(n-1)^2$
2. $M_2^{(m)} = \frac{m(n-1)^3(n-2)}{2} + m^2(n-1)^3 = \frac{(n-1)^3(m(n-2)+2m^2)}{2}$
3. $R_\alpha(F_n^{(m)}) = \frac{m(n-1)^{2\alpha+1}(n-2)}{2} + (n-1)^{2\alpha+1} m^{\alpha+1} = (n-1)^{2\alpha+1}\left[\frac{m(n-2)+2m^{\alpha+1}}{2}\right]$.
4. ${}^m M_2(F_n^{(m)}) = \frac{m(n-2)+2}{2(n-1)}$.
5. $RR_\alpha(F_n^{(m)}) = \frac{m^\alpha(n-2)+2}{2m^{\alpha-1}(n-1)^{2\alpha-1}}$.
6. $SSD(F_n^{(m)}) = (n-1)(m^2+mn-2m+1)$.

7. $H\left(F_n^{(m)}\right)=\frac{m(n-2)}{2}+\frac{2m}{m+1}$.
8. $I\left(F_n^{(m)}\right)=\frac{m(n-1)^2(n-2)}{4}+\frac{m^2(n-1)^2}{m+1}$.
9. $A\left(F_n^{(m)}\right)=\frac{m(n-1)^7}{16(n-2)^2}+\frac{m^4(n-1)^7}{(mn-m+n-3)^3}$.

Proof: From theorem (1) we have M-polynomial of French Windmill graph is

$M\left[F_n^{(m)}(x,y)\right]=\frac{m(n-1)^2(n-2)}{2}x^{n-1}y^{n-1}+m(n-1)x^{n-1}y^{m(n-1)}$, Then we have

$$D_x\left[M\left(F_n^{(m)}(x,y)\right)\right]=\frac{m(n-1)^2(n-2)}{2}x^{n-1}y^{n-1}+m(n-1)^2x^{n-1}y^{m(n-1)}.$$

$$D_y\left[M\left(F_n^{(m)}(x,y)\right)\right]=\frac{m(n-1)^2(n-2)}{2}x^{n-1}y^{n-1}+m^2(n-1)^2x^{n-1}y^{m(n-1)}.$$

Consider

$$\left[(D_x+D_y)\left(M\left(D_n^m(x,y)\right)\right)\right]=m(n-1)\left(n^2-3n+2\right)x^{n-1}y^{n-1}+\left(m(n-1)^2+m^2(n-1)^2\right)x^{n-1}y^{m(n-1)}$$

At $x=1\,\&\,y=1$

The First Zagreb index of wind mill graph is $M_1\left(F_n^{(m)}\right)=(n-1)^2\left(m^2+mn-m\right)$.

$$D_xD_y\left[M\left(F_n^{(m)}(x,y)\right)\right]=\frac{m(n-1)^3(n-2)}{2}x^{n-1}y^{n-1}+m^2(n-1)^3x^{n-1}y^{m(n-1)}.$$

At $x=1\,\&\,y=1$

The Second Zagreb index of wind mill graph is $M_2\left(F_n^m\right)=\frac{(n-1)^3\left(2m^2-2m+mn\right)}{2}$.

$$D_x^\alpha D_y^\alpha\left[M\left(F_n^{(m)}(x,y)\right)\right]=\frac{m(n-1)^{2\alpha+1}(n-2)}{2}x^{n-1}y^{n-1}+(n-1)^{2\alpha+1}m^{\alpha+1}x^{n-1}y^{m(n-1)}.$$

At $x=1\,\&\,y=1$

The Randic index of Wind mill graph is $R_\alpha\left(F_n^{(m)}\right)=\frac{(n-1)^{2\alpha+1}\left[2m^{\alpha+1}-2m+mn\right]}{2}$

$$S_xS_y\left[M\left(F_n^{(m)}(x,y)\right)\right]=\frac{m(n-2)}{2(n-1)}x^{n-1}y^{n-1}+\frac{x^{n-1}y^{m(n-1)}}{(n-1)}.$$

At $x=1\,\&\,y=1$

The second modified Zagreb index of windmill graph is ${}^{2}M_{2}\left(F_{n}^{(m)}\right)=\frac{m(n-2)+2}{n-1}$.

$$S_{x}^{\alpha}S_{y}^{\alpha}\left[M\left(F_{n}^{(m)}(x,y)\right)\right]=\frac{m(n-1)(n-2)}{2(n-1)^{2\alpha}}x^{n-1}y^{n-1}+\frac{m(n-1)x^{n-1}y^{m(n-1)}}{(n-1)^{2\alpha}m^{\alpha}}.$$

At $x=1\,\&\,y=1$

Inverse Randic index of Windmill graph is $RR_{\alpha}\left(F_{n}^{(m)}\right)=\frac{m^{\alpha}(n-2)+2}{2m^{\alpha-1}(n-1)^{2\alpha-1}}$.

$$[S_{x}D_{y}+S_{y}D_{x}]\left[M\left(F_{n}^{(m)}(x,y)\right)\right]=m(n-1)(n-2)x^{n-1}y^{n-1}+(n-1)(1+m^{2})x^{n-1}y^{m(n-1)}.$$

At $x=1\,\&\,y=1$

Symmetric division index of windmill graph $SSD\left(F_{n}^{(m)}\right)=(n-1)(m^{2}+mn-2m+1)$.

$$S_{x}J\left[M\left(F_{n}^{(m)}(x,y)\right)\right]=\frac{m(n-2)x^{2n-2}}{4}+\frac{mx^{(1+m)(n-1)}}{m+1}.$$

At $x=1$

Harmonic index of French windmill graph $H\left(F_{n}^{(m)}\right)=\frac{m(n-2)}{2}+\frac{2m}{m+1}$.

$$S_{x}\left\{J\left[D_{x}D_{y}M\left(F_{n}^{(m)}(x,y)\right)\right]\right\}=\frac{m(n-1)^{2}(n-2)}{4}x^{2n-2}+\frac{m^{2}(n-1)^{2}}{(m+1)}x^{(n-1)(m+1)}.$$

At $x=1$

Inverse sum index of French wind mill graph $I\left(F_{n}^{(m)}\right)=\frac{m(n-1)^{2}(n-2)}{4}+\frac{m^{2}(n-1)^{2}}{m+1}$.

$$S_{x}^{3}\left\{Q_{-2}J\left[D_{x}^{3}D_{y}^{3}M\left(D_{n}^{(m)}(x,y)\right)\right]\right\}=\frac{m(n-1)^{7}}{4(2n-4)^{2}}x^{2n-4}+\frac{m^{4}(n-1)^{4}}{(mn-m+n-3)^{3}}x^{(mn-m+n-3)}.$$

At $x=1$

Augmented Zagreb index $A\left(F_{n}^{(m)}\right)=\frac{m(n-1)^{7}}{4(2n-4)^{2}}+\frac{m^{4}(n-1)^{7}}{(mn-m+n-3)^{3}}$.

Corollary : For a French windmill graph having 5copies of cycle with 5 vertices we have

1. $M_{1}\left(F_{n}^{m}\right)=720$
2. $M_{2}\left(F_{n}^{(m)}\right)=2080$
3. $R_{\alpha}\left(F_{n}^{m}\right)=\frac{4^{2\alpha+1}\left(2.5^{\alpha+1}+15\right)}{2}=2.2^{2\varepsilon}\left(2.5^{\alpha+1}+15\right)$

4. ${}^{m}M_2\left(F_n^{(m)}\right)=\frac{17}{4}$.
5. $RR_\alpha\left(F_n^{(m)}\right)=\frac{5^\alpha.3+2}{2.5^{\alpha-1}.3^{2\alpha-1}}$
6. $SSD\left(F_n^{(m)}\right)=184$
7. $H\left(F_n^{(m)}\right)=\frac{55}{6}$.
8. $I\left(F_n^{(m)}\right)=\frac{380}{3}$.
9. $A\left(F_n^{(m)}\right)=1530.6$.

Theorem 2.5: The M-polynomial of Kulli path windmill graph is given by

$$M\left[p_{n+1}^m(x,y)\right]=2mx^2y^3+(mn-3m)x^3y^3+2mx^2y^{mn}+(mn-2m)x^3y^{mn}$$

Proof:

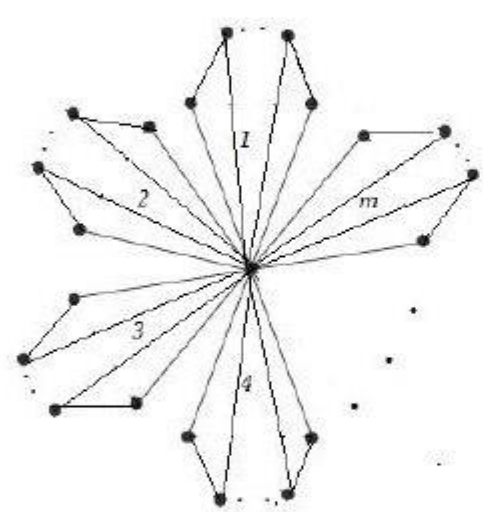

Figure 3: Kulli Path wind mill graph

The Kulli path windmill graph $P_{n+1}^{(m)}$ is the graph obtained by taking $m\geq 2$ copies of the graph $K_1+P_n : n\geq 4$ with a vertex K_1 in common. The Kulli path windmill graph has the vertex set $V(G)$ and edge set $E(G)$ where the cardinality of $V(G)$ is $|V(G)|=mn+1$ and the cardinality of $E(G)$ is $|E(G)|=2mn-m$.From the diagram of Kulli path windmill graph we the three partition of vertex set and four partition of edge set.

The partition of vertex set is as follows.

$V_2=\{v\in V(G):d_G(v)=2\}$ such that $|V_2|=2m$

$V_3=\{v\in V(G:d_G(v)=3)\}$ such that $|V_3|=mn-2m$ and

$V_{mn} = \{v \in V(G) : d_G(v) = mn\}$ such that $|V_{mn}| = 1$

The partition of edge set is as follows.

$E_5 = \{uv \in E(G) : d_G(u) = 2, d_G(v) = 3\}$ such that $|E_5| = 2m$

$E_6 = \{uv \in E(G) : d_G(u) = 3, d_G(v) = 3\}$ such that $|E_6| = mn - 3m$ and

$E_{mn+2} = \{uv \in E(G) : d_G(u) = mn, d_G(v) = 2\}$ such that $|E_{mn+2}| = 2m$

And $E_{mn+3} = \{uv \in E(G) : d_G(u) = mn, d_G(v) = 3\}$ such that $|E_{mn+3}| = mn - 2m$

Now the M-polynomial of Kulli path windmill graph is defined as

$$M\left[P_{n+1}^{(m)}(x,y)\right] = \sum_{\delta \prec i \le j \le \Delta} m_{ij} x^i y^j$$

$$M\left[P_{n+1}^{(m)}(x,y)\right] = \sum_{2\le 3} m_{23} x^2 y^3 + \sum_{3\le 3} m_{3\le 3} x^3 y^3 + \sum_{2\le mn} m_{2.mn} x^2 y^{mn} + \sum_{3\le mn} m_{3.mn} x^3 y^{mn}$$

$$M\left[P_{n+1}^{(m)}(x,y)\right] = |E_5| x^2 y^3 + |E_6| x^3 y^3 + |E_{mn+2}| x^2 y^{mn} + |E_{mn+3}| x^3 y^{mn}$$

$M\left[P_{n+1}^{(m)}(x,y)\right] = 2mx^2 y^3 + (mn - 3m) x^3 y^3 + 2mx^2 y^{mn} + (mn - 2m) x^3 y^{mn}$ is the expression for M-polynomial of Kulli path windmill graph.

Theorem 2.6: Let $P_{n+1}^{(m)}$ be the Kulli path windmill graph, then

1. $M_1\left(P_{n+1}^{(m)}\right) = m^2 n^2 + 9mn - 10m$.
2. $M_2\left(P_{n+1}^{(m)}\right) = 3m^2 n^2 - 2m^2 n + 9mn - 15m$.
3. $R_\alpha\left(P_{n+1}^{(m)}\right) = (3mn)^\alpha (mn - 2m) + 2m(2mn)^\alpha + 9^\alpha (mn - 3m) + 2.6^\alpha m$.
4. ${}^m M_2\left(P_{n+1}^{(m)}\right) = \frac{mn}{9} + \frac{1}{3n} + \frac{1}{3}$.
5. $RR_\alpha\left(P_{n+1}^{(m)}\right) = \frac{2m}{6^\alpha} + \frac{mn - 3m}{9^\alpha} + \frac{2m}{(2mn)^\alpha} + \frac{mn - 2m}{(3mn)^\alpha}$.
6. $SSD\left(P_{n+1}^{(m)}\right) = 3m^2 n^2 - 5m^2 n + 2mn - \frac{5m}{3} + 3 - \frac{2}{n}$.
7. $H\left(P_{n+1}^{(m)}\right) = \frac{4m}{5} + \frac{(mn - 3m)}{3} + \frac{4m}{(mn + 2)} + \frac{2(mn - 2m)}{mn + 3}$.
8. $I\left(P_{n+1}^{(m)}\right) = \frac{4m^2 n}{mn + 2} + \frac{3mn(mn - 2m)}{mn + 3} + \frac{3mn}{2} - \frac{21m}{10}$.
9. $A\left(P_{n+1}^{(m)}\right) = \frac{432m}{243} + \frac{729(mn - 3m)}{1024} + \frac{168m^3 n^4}{(mn)^5} + \frac{27m^3 n^3 (mn - 2m)}{(mn + 1)^5}$.

Proof: From theorem (1) we have M-polynomial of Kulli path Windmill graph is

$M\left[P_{n+1}^{(m)}(x,y)\right] = 2mx^2 y^3 + (mn - 3m) x^3 y^3 + 2mx^2 y^{mn} + (mn - 2m) x^3 y^{mn}$, Then we have

$$D_x\left[M\left(P_{n+1}^{(m)}(x,y)\right)\right]=4mx^2y^3+3(mn-3m)x^3y^3+4mx^2y^{mn}+3(mn-2m)x^3y^{mn}.$$

$$D_y\left[M\left(P_{n+1}^{(m)}(x,y)\right)\right]=6mx^2y^3+3(mn-3m)x^3y^3+2m^2nx^2y^{mn}+mn(mn-2m)x^3y^{mn}.$$

Consider

$$\left[(D_x+D_y)\left(M\left(P_{n+1}^{m}(x,y)\right)\right)\right]=$$
$$10mx^2y^3+3(2mn-6m)x^3y^3+\left(4m+2m^2n\right)x^2y^{mn}+\left[3mn-6m+(mn)^2-2m^2n\right]x^3y^{mn}$$

At $x=1\,\&\,y=1$

The First Zagreb index of Kulli path wind mill graph is $M_1\left[P_{n+1}^{(m)}\right]=m^2n^2+9mn-10m$.

$$D_xD_y\left[M\left(P_{n+1}^{(m)}(x,y)\right)\right]=12mx^2y^3+9(mn-3m)x^3y^3+4m^2nx^2y^{mn}+3mn(mn-2m)x^3y^{mn}.$$

At $x=1\,\&\,y=1$

The Second Zagreb index of Kulli path wind mill graph is $M_2\left(P_{n+1}^{m}\right)=3m^2n^2-2m^2n+9mn-15m$

$$D_x^{\alpha}D_y^{\alpha}\left[M\left(P_{n+1}^{(m)}(x,y)\right)\right]=2.6^{\alpha}x^2y^3+9^{\alpha}(mn-3m)x^3y^3+(2m)^{\alpha+1}n^{\alpha}x^2y^{mn}+(3mn)^{\alpha}(mn-2m)x^3y^{mn}$$

At $x=1\,\&\,y=1$,The Randic index of Kulli path Wind mill graph is

$$R_{\alpha}\left(P_{n+1}^{(m)}\right)=2.6^{\alpha}m+9^{\alpha}(mn-3m)+(2m)^{\alpha+1}n^{\alpha}+(3mn)^{\alpha}(mn-2m).$$

$$S_xS_y\left[M\left(P_{n+1}^{(m)}(x,y)\right)\right]=\frac{mx^2y^3}{3}+\frac{(mn-3m)x^3y^3}{9}+\frac{x^2y^{mn}}{n}+\frac{(mn-2m)x^3y^{mn}}{3mn}.$$

At $x=1\,\&\,y=1$

The second modified Zagreb index of Kulli path windmill graph is ${}^2M_2\left(P_{n+1}^{(m)}\right)=\frac{mn}{9}+\frac{1}{3n}+\frac{1}{3}$.

$$S_x^{\alpha}S_y^{\alpha}\left[M\left(P_{n+1}^{(m)}(x,y)\right)\right]=\frac{2mx^2y^3}{6^{\alpha}}+\frac{(mn-3m)x^3y^3}{9^{\alpha}}+\frac{2mx^2y^{mn}}{(2mn)^{\alpha}}+\frac{(mn-2m)x^3y^{mn}}{(3mn)^{\alpha}}.$$

At $x=1\,\&\,y=1$,Inverse Randic index of Kulli path Windmill graph is

$$RR_{\alpha}\left(P_{n+1}^{(m)}\right)=\frac{2m}{6^{\alpha}}+\frac{(mn-3n)}{9^{\alpha}}+\frac{2m}{(2mn)^{\alpha}}+\frac{mn-2m}{(3mn)^{\alpha}}.$$

$$[S_xD_y+S_yD_x]\left[M\left(P_{n+1}^{(m)}(x,y)\right)\right]=\frac{13mx^2y^3}{3}+2(mn-3m)x^3y^3+\left(\frac{4}{n}+m^2n\right)x^2y^{mn}.$$
$$+\left[\frac{3(mn-2m)}{mn}+3m(mn-2m)\right]x^3y^{mn}$$

At $x=1\,\&\,y=1$, Symmetric division index of Kulli path windmill graph

Is $SSD\left(P_{n+1}^{(m)}\right)=(3mn)^2-5m^2n+2mn-\frac{5m}{3}-\frac{2}{n}+3.$

$$S_xJ\left[M\left(P_{n+1}^{(m)}(x,y)\right]=\frac{2mx^5}{5}+\frac{(mn-3m)x^6}{6}+\frac{2mx^{mn+2}}{mn+2}+(mn-2m)\frac{x^{mn+3}}{mn+3}.$$

At $x=1$

Harmonic index of Kulli path windmill graph is

$$H\left(P_{n+1}^{(m)}\right)=\frac{4m}{5}+\frac{mn-3m}{3}+\frac{4m}{(mn+2)}+\frac{2(mn-2m)}{mn+3}.$$

$$S_x\left\{J\left[D_xD_yM\left(F_n^{(m)}(x,y)\right)\right]\right\}=\frac{12mx^5}{5}+\frac{3(mn-3m)x^6}{2}+\frac{(4m^2n)x^{mn+2}}{mn+2}+\frac{3mn(mn-2m)x^{mn+3}}{mn+3}.$$

At $x=1$

Inverse sum index of Kulli path windmill graph is

$$I\left(P_{n+1}^{(m)}\right)=\frac{12m}{5}+\frac{3(mn-3m)}{2}+\frac{4m^2n}{mn+2}+\frac{3mn(mn-2m)}{mn+3}.$$

$$S_x^3\left\{Q_{-2}J\left[D_x^3D_y^3M\left(D_n^{(m)}(x,y)\right)\right]\right\}=\frac{432x^3}{243}+\frac{729(mn-3m)x^4}{1024}+\frac{168x^{mn}}{m^2n}+\frac{27m^3n^3(mn-2m)x^{mn+1}}{(mn+1)^5}.$$

At $x=1$,

Augmented Zagreb index of Kulli wind mill graph is

$$A\left(P_{n+1}^{(m)}\right)=\frac{432m}{243}+\frac{729(mn-3m)}{1024}+\frac{168m^3n^4}{(mn)^5}+\frac{27m^3n^3(mn-2m)}{(mn+1)^5}.$$

Corollary : For a Kulli path windmill graph having 5copies of path with 3 vertices [First kulli path wind mill graph] we have

1. $M_1\left(F_n^m\right)=310.$
2. $M_2\left(F_n^{(m)}\right)=585.$
3. $R_\alpha\left(F_n^m\right)=10(6)^\alpha+10(30)^\alpha+5(45)^\alpha.$
4. ${}^mM_2\left(F_n^{(m)}\right)=\frac{31}{15}.$
5. $RR_\alpha\left(F_n^{(m)}\right)=\frac{10}{6^\alpha}+\frac{10}{(30)^\alpha}+\frac{5}{(45)^\alpha}.$
6. $SSD\left(F_n^{(m)}\right)=\frac{4864}{15}.$
7. $H\left(F_n^{(m)}\right)=\frac{877}{153}.$
8. $I\left(F_n^{(m)}\right)=42.15.$

9. $A\left(F_n^{(m)}\right)=122.4.$

Theorem 2.8: The M-polynomial of Kulli cycle windmill graph is given

$$M\left[C_{n+1}^{m}(x,y)\right]=mnx^3y^3+mnx^3y^{mn}$$

Proof:

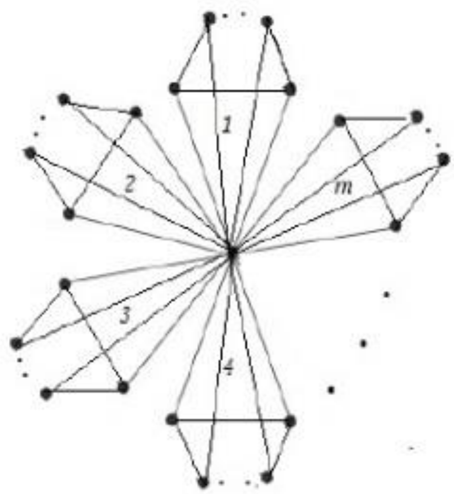

Figure 4: Kulli cycle wind mill graph

The Kulli cycle windmill graph $C_{n+1}^{(m)}$ is the graph obtained by taking m copies of the graph $K_1+C_n : n\geq 3$ with a vertex K_1 in common. The Kulli path windmill graph has the vertex set $V(G)$ and edge set $E(G)$ where the cardinality of $V(G)$ is $|V(G)|=mn+1$ and the cardinality of $E(G)$ is $|E(G)|=2mn$. From the diagram of Kulli cycle windmill graph we the two partition of vertex set and two partition of edge set.

The partition of vertex set is as follows.

$V_3=\{v\in V(G): d_G(v)=3\}$ such that $|V_2|=mn$ and

$V_{mn}=\{v\in V(G): d_G(v)=mn\}$ such that $|V_{mn}|=1$

The partition of edge set is as follows.

$E_6=\{uv\in E(G): d_G(u)=3, d_G(v)=3\}$ such that $|E_6|=mn$ and

$E_{mn+3}=\{uv\in E(G): d_G(u)=mn, d_G(v)=3\}$ such that $|E_{mn+3}=mn|$.

Now the M-polynomial of Kulli Cycle windmill graph is defined as

$$M\left[C_{n+1}^{(m)}(x,y)\right]=\sum_{\delta\prec i\leq j\leq\Delta} m_{ij}x^iy^j$$

$$M\left[C_{n+1}^{(m)}(x,y)\right]=\sum_{3\leq 3} m_{3\leq 3}x^3y^3+\sum_{3\leq mn} m_{3.mn}x^3y^{mn}$$

$M\left[C_{n+1}^{(m)}(x,y)\right]=|E_6|x^3y^3+|E_{mn+3}|x^3y^{mn}$

$M\left[C_{n+1}^{(m)}(x,y)\right]=mnx^3y^3+mnx^3y^{mn}$ is the expression for M-polynomial of Kulli cycle windmill graph.

Theorem 2.9 : Let $C_{n+1}^{(m)}$ be the Kulli cycle windmill graph, then

1. $M_1\left(C_{n+1}^{m}\right)=(mn)^2+9mn$
2. $M_{21}\left(C_{n+1}^{m}\right)=3(mn)^2+9mn$
3. $R_\alpha\left(C_{n+1}^{m}\right)=3^\alpha(mn)^{\alpha+1}+3^{2\alpha}mn$
4. ${}^mM_2\left(C_{n+1}^{m}\right)=\frac{mn}{9}+\frac{1}{3}=\frac{mn+3}{9}$.
5. $RR_\alpha\left(C_{n+1}^{m}\right)=\frac{mn}{3^{2\alpha}}+\frac{mn}{(3mn)^\alpha}$.
6. $SSD\left(C_{n+1}^{m}\right)=2mn+\frac{(mn)^2}{3}+3$
7. $H\left(C_{n+1}^{m}\right)=\frac{(mn)^2+9mn}{3(mn+3)}$.
8. $I\left(C_{n+1}^{m}\right)=\frac{9mn(1+mn)}{2(3+mn)}$.
9. $A\left(C_{n+1}^{m}\right)=\frac{3^6mn}{64}+\frac{3^3(mn)^4}{(mn+1)^3}$.

Proof: From theorem (1) we have M-polynomial of Kulli cycle Windmill graph is

$M\left[C_{n+1}^{(m)}(x,y)\right]=mnx^3y^3+mnx^3y^{mn}$, Then we have

$D_x\left[\left(C_{n+1}^{(m)}(x,y)\right]=3mx^3y^3+3mnx^3y^{mn}\right.$.

$D_y\left[M\left(C_{n+1}^{(m)}(x,y)\right]=3mnx^3y^3+(mn)^2x^3y^{mn}\right.$.

Consider

$\left[(D_x+D_y)\left(M\left(C_{n+1}^{m}(x,y)\right)\right)\right]=6mnx^3y^3+\left[3mn+(mn)^2\right]x^3y^{mn}$

At $x=1\,\&\,y=1$

The First Zagreb index of Kulli cycle windmill graph is $M_1\left[C_{n+1}^{(m)}\right]=m^2n^2+9mn$.

$D_xD_y\left[M\left(C_{n+1}^{(m)}(x,y)\right]=9mnx^3y^3+3(mn)^2x^3y^{mn}\right.$.

At $x=1\,\&\,y=1$

The Second Zagreb index of Kulli cycle win mill graph is $M_2\left(C_{n+1}^{m}\right)=3m^2n^2+9mn$.

$$D_x^{\alpha}D_y^{\alpha}\left[M\left(C_{n+1}^{(m)}(x,y)\right]=3^{2\alpha}mnx^3y^3+3^{\alpha}(mn)^{\alpha+1}x^3y^{mn}$$

At $x=1\,\&\,y=1$

The Randic index of Kulli cycle Windmill graph is $R_{\alpha}\left(C_{n+1}^{(m)}\right)=9^{\alpha}mn+3^{\alpha}(mn)^{\alpha+1}$

$$S_xS_y\left[M\left(C_{n+1}^{(m)}(x,y)\right]=\frac{mnx^3y^3}{9}+\frac{mnx^3y^{mn}}{3mn}.$$

At $x=1\,\&\,y=1$

The second modified Zagreb index of Kulli cycle windmill graph is ${}^2M_2\left(C_{n+1}^{(m)}\right)=\frac{mn}{9}+\frac{1}{3}$.

$$S_x^{\alpha}S_y^{\alpha}\left[M\left(C_{n+1}^{(m)}(x,y)\right)\right]=\frac{mnx^3y^3}{9^{\alpha}}+\frac{mnx^3y^{mn}}{(3mn)^{\alpha}}.$$

At $x=1\,\&\,y=1$

Inverse Randic index of Kulli cycle Windmill graph is $RR_{\alpha}\left(C_{n+1}^{(m)}\right)=\frac{mn}{9^{\alpha}}+\frac{mn}{(3mn)^{\alpha}}$.

$$[S_xD_y+S_yD_x]\left[M\left(C_{n+1}^{(m)}(x,y)\right]=2mnx^3y^3+\left(3+\frac{(mn)^2}{3}\right)x^3y^{mn}.$$

At $x=1\,\&\,y=1$

Symmetric division index of Kulli cycle windmill graph is $SSD\left(C_{n+1}^{(m)}\right)=2mn+\left(3+\frac{(mn)^2}{3}\right)$.

$$S_xJ\left[M\left(P_{n+1}^{(m)}(x,y)\right]=\frac{mnx^6}{6}+\frac{mnx^{mn+3}}{mn+3}.$$

At $x=1$ Harmonic index of Kulli cycle windmill graph is $H\left(C_{n+1}^{(m)}\right)=\frac{9mn+(mn)^2}{3(mn+3)}$.

$$S_x\left\{J\left[D_xD_yM\left(C_n^{(m)}(x,y)\right)\right]\right\}=\frac{3mnx^6}{2}+\frac{3(mn)^2x^{mn+3}}{mn+3}.$$

At $x=1$ Inverse sum index of Kulli cycle windmill graph is $I\left(C_{n+1}^{(m)}\right)=\frac{9mn(mn+1)}{2mn+6}$.

$$S_x^3\left\{Q_{-2}J\left[D_x^3D_y^3M\left(D_n^{(m)}(x,y)\right)\right]\right\}=\frac{729mnx^4}{64}+\frac{27m^4n^4x^{mn+1}}{(mn+1)^3}.$$

At $x=1$, Augmented Zagreb index of Kulli wind mill graph is $A\left(C_{n+1}^{(m)}\right)=\frac{729mn}{64}+\frac{27m^4n^4}{(mn+1)^3}$.

Corollary : For a Kulli cycle windmill graph having 5copies of cycle with 3 vertices we have

1. $M_1\left(C_{n+1}^{m}\right)=360.$
2. $M_2\left(C_{n+1}^{(m)}\right)=810.$
3. $R_\alpha\left(C_{n+1}^{m}\right)=15(3)^\alpha+3^\alpha(15)^{\alpha+1}.$
4. ${}^mM_2\left(C_{n+1}^{(m)}\right)=2.$
5. $RR_\alpha\left(C_{n+1}^{(m)}\right)=\frac{15}{3^{2\alpha}}+\frac{15}{(45)^\alpha}.$
6. $SSD\left(C_{n+1}^{(m)}\right)=108.$
7. $H\left(C_{n+1}^{(m)}\right)=\frac{360}{54}.$
8. $I\left(C_{n+1}^{(m)}\right)=60.$
9. $A\left(C_{n+1}^{(m)}\right)=504.61.$

Conclusion: In this paper we calculate the some topological indices of wind-mill graphs by constructing their M-polynomial, and by using some calculus such as differentiation and integration.

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