



Bianchi Type -V Cosmological Model in a $f(R, T)$ Theory of Gravitation

A. P. Wasnik¹, S. A. Sadar²

¹Assistant Professor

¹Department of mathematics, Bharatiya Mahavidyalaya Amravati (M.S.), India

²Research scholars at S.G.B. Amravati University Amravati (M.S.), India

Abstract: A cosmological model is a mathematical description of the universe, which tries to explain the reason of current aspect, and to describe its evolution during time. The universe is homogeneous and isotropic. In this paper we have studied Bianchi Type- V cosmological model in presence of perfect fluid within the frame work of $f(R, T)$ gravity, where R is Ricci scalar and T is the trace of the source matter. Here we consider first case of the $f(R, T)$ model i.e. $f(R, T) = R + 2f(T)$, it is also possible to make progress towards solving the cosmological constant problem.

Keywords: Bianchi type-V universe, $f(R, T)$ theory, deceleration parameter

Introduction: The new evidence has confirmed that the expansion of the universe is accelerating under the influence of a gravitationally repulsive form of energy that makes up two-thirds of the cosmos [1-5]. It is assumed that this cosmic acceleration is due to some kind of negative- pressure form of matter known as dark energy (DE) and positive density [6,-8]. The nature of dark energy and cosmological origin remain unknown. The main first component of the acceleration universe is dark energy. The universe contains 73 percent of dark energy. The second component is dark matter, the universe contain 23 percent of dark matter. The dark matter is responsible to explain the structure about the formation of universe. The universe contains 4 percent of usual baryonic matter. The cosmological constant Λ , introduced by Albert Einstein the vacuum energy is the most simple and natural candidate for DE, which appears to fit the observational data [9].

To explain the mechanism of the late-time acceleration, dark matter, dark energy and many modified theories of gravity have been studied. The modification in Einstein-Hilbert action on large cosmological scales may be a correct explanation of a late time cosmic acceleration of the expanding universe [10, 11].

$f(R, T)$ Modified theory of gravity proposed by Harko et al [12], where the gravitational Lagrangian is given by an arbitrary function of the Ricci Scalar R and of the trace of the stress-energy tensor T. Many authors have investigated various aspects of the Bianchi type model in $f(R, T)$ gravity will now be mentioned.

Anisotropic Bianchi type-VI_h model investigated by Rao et al. [13] Bianchi type -V cosmology with $\Lambda(T)$ investigated by Ahmed and Pradhan [14]. A spatially homogeneous and anisotropic Bianchi type-V cosmological model in a scalar-tensor theory of gravitation when the source for energy momentum tensor is a bulk viscous fluid containing one dimensional cosmic strings investigates by Naidu, Reddy, Ramprasad and Ramana [15]. Variability of the Bianchi type-V universe in $f(R, T)$ theory of gravitation investigates by Sharma, Singh, Yadav [16]. Shamir [17] investigate explore the exact solutions of Bianchi type- V space time in $f(R, T)$ theory of gravity, two exact solutions are investigated using assumptions of the variation

law of Hubble parameter and constant deceleration parameter. A self-consistent system of gravitational field with a Binary mixture of perfect fluid and dark energy given by cosmological constant has been considered in Bianchi type- V universe investigated by Singh and Chaubey [18]. LRS Bianchi Type- V cosmological model filled with perfect fluid in the $f(R, T)$ gravity investigated by Nath and Sahu [19]. The spatially homogeneous and anisotropic Bianchi type- V cosmological solution of massive string have been investigated in the presence of a magnetic field within the framework of $f(R, T)$ gravity theory found by Ram and Chandel [20]. Bhardwaj and Yadav [21] studied the transition and physical behavior of Bianchi type-V cosmological models within the formalism of $f(R, T)$ gravity. Zubair et al. [22] investigate the Bianchi type I and V space-time in $f(R, T)$ gravity with the help of special law of deceleration parameter in connection to $f(R, T)$ gravity. Non-singular Bianchi type I and V cosmological models in the presence of bulk viscous fluid within the framework of $f(R, T)$ gravity theory investigated by Shriram and Kumari [23]. Bianchi type-V cosmological models with quark matter distribution and domain walls with observational constraints in $f(R, T)$ theory of gravity investigated by Maurya [24]. $f(R, T)$ Gravity for Bianchi type-V metric in Lyra geometry investigated by Brahma and Dewri [25]. Agrawal and Pawar [26] investigate Bianchi type-V space-time using magnetic domain wall in $f(R, T)$ theory of gravity and deciphered the exact solution of the corresponding field equation. An anisotropic Bianchi Type-V cosmological space-time is discussed within the framework of Lyra's manifold investigated by Ram et al. [27]. The transition of the universe from the early decelerating phase to the current accelerating phase with viscous fluid and time dependent cosmological constant Λ as source of matter in Bianchi type-V space time investigated by Yadav [28].

Inspired by above discussion, in the present paper, Bianchi Type-V cosmological model in the presence of perfect fluid within the framework of $f(R, T)$ gravity has been studied.

2. Modified $f(R, T)$ Gravity:

The action of $f(R, T)$ gravity is given by

$$S = \int \left[\sqrt{-g} \left(\frac{-1}{16\pi G} f(R, T) + L_m \right) \right] d^4x \quad (1)$$

$$S = \frac{1}{16\pi} \int f(R, T) \sqrt{-g} d^4x + \int L_m \sqrt{-g} d^4x$$

where $f(R, T)$ an arbitrary function of Ricci scalar R is and trace of stress-energy tensor T . g is the determinant of the metric tensor g_{ij} , and L_m is the matter Lagrangian density. The stress energy tensor T_{ij} of matter source is given by

$$T_{ij} = \frac{-2}{\sqrt{-g}} \frac{\delta \sqrt{-g} L_m}{\delta g^{ij}} \quad (2)$$

Here we consider that the dependence of matter Lagrangian merely on the metric tensor g_{ij} rather than its derivatives, we get

$$T_{ij} = g_{ij} L_m - \frac{\partial L_m}{\partial g^{ij}} \quad (3)$$

For a perfect fluid distribution, the energy-momentum tensor of the matter has the form

$$T_{ij} = (p + \rho) u_i u_j + p g_{ij} \quad (4)$$

where p is the pressure and ρ is the energy density of the fluid. Here u^i is the four-velocity vector satisfying $u^i u_i = -1$ and $u^i \nabla_j u_i = 0$.

The $f(R, T)$ gravity field equation are obtained by varying the action S in (2) with respect to g_{ij}

$$f_R(R, T) R_{ij} - \frac{1}{2} f(R, T) g_{ij} + (g_{ij} \square - \nabla_i \nabla_j) f_R(R, T) = 8\pi T_{ij} - f_T(R, T) \theta_{ij} \quad (5)$$

where

$$\theta_{ij} = -2T_{ij} + g_{ij} L_m - 2g^{\alpha\beta} \frac{\partial^2 L_m}{\partial g^{ij} \partial g^{\alpha\beta}} \quad (6)$$

Here

$$f_R(R, T) = \frac{\partial f(R, T)}{\partial R}, f_T(R, T) = \frac{\partial f(R, T)}{\partial T} \text{ and } \square = \nabla^u \nabla_u, \text{ where } \nabla_u \text{ denotes the covariant derivative.}$$

From the equation (4), we get

$$f_R(R, T)R + 3 \square f_R(R, T) - 2f(R, T) = 8\pi T - (T + \theta)f_T(R, T) \quad (7)$$

where $\theta = \theta_j^i$. Equation (7) gives a relation between Ricci Scalar R and the trace of energy momentum tensor T .

Now,

using the fact that $L_m = -p$ since, there is no unique definition of the matter Lagrangian.

Equation (6) can be rewriting as

$$\theta_{ij} = -2T_{ij} - pg_{ij} \quad (8)$$

Harko et al [12] have considered three possible forms of the function $f(R, T)$:

$$f(R, T) = \begin{cases} R + 2f_1(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases} \quad (9)$$

In this paper, we shall consider the first form of $f(R, T)$ i.e. $f(R, T) = R + 2f_1(T)$ and we choose

$$f_1(T) = -\lambda T,$$

where λ is an arbitrary constant. For this consideration and energy-momentum tensor (4) and (6) is reduced to the form:

$$R_{ij} - \frac{1}{2}Rg_{ij} = -(1 + 2\lambda)T_{ij} - \lambda(T + 2p)g_{ij} \quad (10)$$

Now, Einstein field equations with the cosmological term can be written as

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} + \Lambda g_{ij} \quad (11)$$

By comparing equation (10) and (11) and by taking the parameter Λ to be small, we can make the identification

$$\Lambda = \Lambda(T) = \lambda(T + 2p)$$

Therefore, in the $f(R, T)$ theory of gravity, the field equation with a variable cosmological parameter $\Lambda(T)$ can be expressed as,

$$R_{ij} - \frac{1}{2}Rg_{ij} = -(1 + \lambda)T_{ij} + \Lambda g_{ij} \quad (12)$$

In the case of perfect fluid, the trace T of the energy- momentum tensor can be written as:

$$T = \rho - 3p \quad (13)$$

The cosmological parameter can be written as:

$$\Lambda = \lambda(\rho - p) \quad (14)$$

The usual energy conservation law does not hold in the $f(R, T)$ theory. That the non-conservation of energy from the thermodynamic point of view implies an irreversible matter creation process investigate by Shabani et al [29]. Again Shabani et al [30] investigate in another paper consequence of energy conservation violation they investigate if there is energy conservation in $f(R, T)$ gravity then late-time stable accelerating solution are not a general feature. However, with energy non-conservation, it is possible to find a large class of solution with dynamic $\Lambda(T)$ which have late-time acceleration and are stable. Tiwari et al [31] investigated the anisotropic Bianchi type-I cosmological model by assuming a particular form for the deceleration parameter as a function of Hubble parameter in $f(R, T)$ theory of gravity.

In this paper we will investigate a Bianchi type-V cosmological model with perfect fluid, as matter content in $f(R, T)$ gravity proposed by Harko et al.

3. Metric and Field Equations:

We consider the spatially homogeneous Bianchi type-V space-time described by the line element

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x}(dy^2 + dz^2) \quad (15)$$

where A and B are the function of cosmic time t only for Bianchi type-V space-time (14) the field equation (13) in $f(R, T)$ gravity, can be explicitly written as

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B} - \frac{1}{A^2} = \Lambda - (1 + 2\lambda)p \quad (16)$$

$$\frac{\ddot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \Lambda - (1 + 2\lambda)p \quad (17)$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B} - \frac{3}{A^2} = \Lambda + (1 + 2\lambda)\rho \quad (18)$$

$$\frac{\dot{B}}{B} - \frac{\dot{A}}{A} = 0 \quad (19)$$

Here an overhead dot indicates differentiation with respect to cosmic time t. We assume that the matter content obey the usual equation of state

$$p = w\rho, \quad -1 \leq w \leq 1 \quad (20)$$

We shall now define the dynamical parameters which will be useful in solving the field equation and in the physical discussion of the solution.

The average scale factor $a(t)$ of the Bianchi type-V space-time is defined as

$$a(t) = (AB^2)^{\frac{1}{3}} \quad (21)$$

The spatial volume of the metric is

$$v = a^3(t) = AB^2 \quad (22)$$

The directional Hubble Parameters are

$$H_x = \frac{\dot{A}}{A}, \quad H_y = H_z = \frac{\dot{B}}{B} \quad (23)$$

The average Hubble parameter is

$$H = \frac{1}{3}(H_x + H_y + H_z) \quad (24)$$

$$H = \frac{\dot{a}}{a} = \frac{1}{3} \frac{\dot{v}}{v} = \frac{1}{3} \left(\frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right)$$

The expansion scalar (θ), shear scalar (σ) are defined as

$$\theta = 3H = \left(\frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) \quad (25)$$

$$\sigma^2 = \frac{1}{2} \sigma^{ij} \sigma_{ij} = \frac{1}{2} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right)^2 \quad (26)$$

The average anisotropic parameter A_m is define as

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2 \quad (27)$$

The deceleration parameter is

$$q = -1 + \frac{d}{dt} \left(\frac{1}{H} \right)$$

4. A Solution of field equation:

Now the field equation (15)-(19) reduces to a system of four independent equations in five unknowns $A, B, \rho_\Lambda, p_\Lambda, \rho_m$. Hence to find the determinate solution of the system we require one condition and this condition we get from equation (19)

$$A = kB$$

Where k is integrating constant and simplicity we choose $k = 1$

$$\therefore A = B \quad (28)$$

From equation (15)-(16) and (28) we can easily obtain the metric potentials

$$A = B = c_1 + c_2 e^t \quad (29)$$

where c_1 and c_2 are integrating constant. By equation (29), we can write the metric in the form

$$ds^2 = -dt^2 + (c_1 + c_2 e^t)^2 dx^2 + (c_1 + c_2 e^t)^2 e^{2x} (dy^2 + dz^2) \quad (30)$$

The deceleration parameter (q) is defined as,

$$q = \frac{-a\ddot{a}}{\dot{a}^2} \quad (31)$$

$$q = -\frac{c_1 + c_2 e^t}{c_2 e^t} \quad (32)$$

Then, equation (13)-(16) can be expressed in terms of H , q and σ as

$$3H^2 - \sigma^2 = \Lambda + (1 + 2\lambda) \quad (34)$$

$$H^2(2q - 1) - \sigma^2 = (1 + 2\lambda)p - \Lambda$$

4.1 Dynamical parameters and their physical discussion

We discuss the physical properties of the cosmological model and develop a cosmological theory in $f(R, T)$ of gravity.

The average scalar factor $a(t)$ found to be

$$a(t) = c_1 + c_2 e^t \quad (35)$$

where c_1 and c_2 are constant.

The directional Hubble parameters H_x, H_y and H_z are given by,

$$H_x = H_y = H_z = \frac{c_2 e^t}{c_1 + c_2 e^t} \quad (36)$$

The average Hubble parameter is

$$H = \frac{c_2 e^t}{c_1 + c_2 e^t} \quad (37)$$

The spatial volume of the metric is

$$v = a^3(t) = (c_1 + c_2 e^t)^3 \quad (38)$$

The dynamical scalar expansion θ and shear scalar σ are

$$\theta = \frac{3c_2 e^t}{c_1 + c_2 e^t} \quad (39)$$

$$\sigma^2 = 0 \quad (40)$$

The average anisotropic parameter A_m is

$$A_m = 0 \quad (41)$$

The deceleration parameter is

$$q = -\frac{c_1 + c_2 e^t}{c_2 e^t} \quad (42)$$

The field equation (12) is basically used to determine the dynamical quantities, viz. the energy density ρ , pressure p and cosmological parameter Λ .

From equation (15) we determine pressure p

$$p = \frac{-1}{1+2\lambda} \left[\frac{2c_1 c_2 e^t + 3c_2^2 e^{2t} - 1}{(c_1 + c_2 e^t)^2} \right] \quad (43)$$

From equation (18) we determine energy density ρ

$$\rho = \frac{1}{1+2\lambda} \left[\frac{2(c_2 e^t)^2 + (c_2 e^t)^3 + c_1 (c_2 e^t)^2 - 3}{(c_1 + c_2 e^t)^2} \right] \quad (44)$$

The cosmological parameter $\Lambda = \lambda(\rho - p)$ is given by

$$\Lambda = \frac{1}{(1+2\lambda)} \left[\frac{-4 + (5+c_1)(c_2 e^t)^2 + (c_2 e^t)^3 + 2c_1 c_2 e^t}{(c_1 + c_2 e^t)^2} \right] \quad (45)$$

We observed that the model (30) represent an expanding, non-shearing, non-rotating and accelerates universe. This model has no initial singularity. At $t = 0$ the metric potential of the universe $A(t)$ and $B(t)$ are constant as $t \rightarrow \infty, q \rightarrow -1$ where q is deceleration parameter. The evaluation shoes that the universe is expanding and accelerating exponentially.

4.2 Some Cosmological Distance Parameters and Cosmological Red-shift:

Hubble parameter defined the age and size of the universe. From the equation (37) the Hubble parameter is

$$H = \frac{c_2 e^t}{c_1 + c_2 e^t}$$

From this equation, we obtain

$$\frac{H}{H_0} = \frac{e^t(c_1 + c_2 e^{t_0})}{e^{t_0}(c_1 + c_2 e^t)} \quad (46)$$

where H_0 is the present value of the Hubble parameter and t_0 is the present time

$$a = \frac{a_0}{1+z} \quad (47)$$

This equation represents the relation between the scalar factor a and red-shift z , where a_0 is the present value of the scalar factor. Here, we assume $a_0 = 1$ from equation (34)

$$a = \frac{1}{1+z} = c_1 + c_2 e^t \quad (48)$$

$$H = \frac{H_0}{c_2} [1 - c_1(1+z)][c_1 e^{-t_0} + c_2] \quad (49)$$

This equation (49) represents the value of the Hubble parameter in terms of the red-shift parameter.

The distance modulus μ is given by

$$\mu(z) = 5 \log d_L + 25 \quad (50)$$

where d_L denoted for the luminosity distance, which is defined by

$$d_L = r_1(1+z)a_0 \quad (51)$$

A source emits a photon at $r = r_1$ in time $t = t_0$ and an observer receives it at time t , located at $r = 0$, then we can calculate, from the following equation

$$r_1 = \int_t^{t_0} \frac{dt}{a} = \int_t^{t_0} \frac{dt}{c_1 + c_2 e^t} \quad (52)$$

By solving this integral, we obtain the value of r_1 as

$$r_1 = \frac{1}{c_1} \left[(t_0 - t) - \log \left(\frac{c_1 + c_2 e^{t_0}}{c_1 + c_2 e^t} \right) \right] + c_3 \quad (53)$$

We get the expression for the luminosity distance as

$$d_L = \left(\frac{1}{c_1} \left[(t_0 - t) - \log \left(\frac{c_1 + c_2 e^{t_0}}{c_1 + c_2 e^t} \right) \right] + c_3 \right) (1+z) \quad (54)$$

5. Conclusion:

We have investigated Bianchi type-V cosmological model in $f(R, T)$ gravity. We consider the first case of the $f(R, T)$ model i.e. $f(R, T) = R + 2f_1(t)$ where $f_1(t) = -\lambda T$. In this model volume $\neq 0$ and density $\neq \infty$, it means that Bianchi type-V cosmological model in $f(R, T)$ gravity has no physical singularity and universe starts its expansion with finite volume. The solar system tests do not place any restriction on the value of λ .

Hubble parameter H and the expansion scalar all these parameter are function of cosmic time t . These parameter are finite at the initial stage ($t=0$) increase with increase in value of time t , for $t \rightarrow \infty$ it becomes constant. Hence the universe expands with the influence of dark energy. Hubble parameter is positive and increase as time increase. But Samants et.al [32] investigates Hubble parameter is always positive and decies as time increase. If the value of $\ddot{a} > 0$ where a is the scale factor of the universe then the deceleration parameter q is negative. The evolution indicates that the universe is expanding and accelerating.

From the equation (40) and (41) the scalar sigma $\sigma=0$ and anisotropic parameter $A_m = 0$ respectively, which indicates that Bianchi type-V model can approach isotropy in the presence of an anisotropic dark energy. Bianchi type-V model is isotropic throughout the evolution of the universe.

References:

1. Riess, A. G., Kirshner, R. P., Schmidt, B. P., Jha, S., Challis, P., Garnavich, P. M., Esin, A. A., Carpenter, C., Grashius, R., Schild, R. E. et al. [1998], BVRI light curves for 22 type in supernovae. *Astron. J.* 116, 1009.
2. Perlmutter, S., Aldering, G., Goldhaber, G., Knop, R. A., Nugent, P., Castro, P.G., Deustua, S., Fabbro, S., Goobar, A., Groom, D. E. et al. [1999], Measurements of Ω and Λ from 42 High-Redshift supernovae, *Astrophys. J.* 517, 565.
3. Tegmark, M., Strauss, M. A., Blanton, M. R., Abazajian, K., Dodelson, S., Sandvik, H., Wang, X., Weinberg, D. H., Zehavi, I., Bahcall, N. A. et al. [2004], Cosmological parameters from SDSS and WMAP, *Phys. Rev. D* 69, 103501.
4. Allen, S. W., Schmidt, R. W., Ebeling, H., Fabian, A.C., Van Speybroeck, L. [2004], Constraints on dark energy from Chandra observations of the largest relaxed clusters, *Mon. Not. R. Astron. Soc.* 353, 457.
5. Bennett, C., Halpern, M., Hinshaw, G., Jarosik, N., Kogut, A., Limon, M., et al. [2003] First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Preliminary Maps and basic Results, *Astrophys. J. suppl. Ser.* 148, 1-27.
6. Nojiri, S., Odintsov, S. D. [2007], Introduction to modified gravity and gravitational alternative for dark energy, *Int. J. Geom. Methods Mod. Phys.* 4,115-145.
7. Copeland, E. J., Sami, M., Tsujikawa, S. [2006], Dynamics of dark energy, *Int. J. Mod. Phys. D*15,1753.
8. Ade, P. A., Aghanim, N., Armitage-Caplan, C., Arnaud, M., Ashdown, M., Atrio-Barandela, F., Aumont, J. et al. [2014], Planck 2013 result. XVI. Cosmological parameters, *Astron. Astron. Astrophys.* 571, A71.
9. Peebles, P. J. E., Ratra, B. [2003], the cosmological constant and dark energy, *Rev. Mod. Phys.* 75, 559-606.
10. Capozziello, S., Cardone, V. F., Troisi, A. [2005], Reconciling dark energy models with $f(R)$ theories, *Phys. Rev. D*71, 43503.
11. Nojiri, S., Odintsov, S. D., Sami, M. [2006], Dark energy cosmology from higher-order, string-inspired gravity, and its reconstruction, *Phys. Rev.*74, 046004.
12. Harko, T., Lobo, F. S. N., Nojiri, S., Odintsov, S. D. [2011], $f(R, T)$ theory of gravity, *Phys. Rev. D* 84, 24020.
13. Rao, V. U. M., Santh, M. V., Aditya, Y. [2015], Bianchi Type-VI_h perfect fluid cosmological model in a Modified theory of Gravity, *Pre space time J.* 6(10), 947-960.
14. Ahmed, N., Pradhan, A. [2014], Bianchi Type-V cosmological model in gravity, *inter. J. of theoretical phys.* 53(1), 289-306.
15. Naidu, R. L., Reddy, D. R. K., Ramprasad, T., Ramana, K. V. [2013], Bianchi Type-V bulk viscous string cosmological model in $f(R, T)$ gravity, *Astro and space sci.*, 348(1), 247-257.
16. Sharma, L. K., Singh, B. K., Yadav, A. K. [2020], Viability of Bianchi Type-V universe in $f(R, T) = f_1(R) + f_2(R)f_3(T)$ gravity, *Int. J. Geo. Methods in modern Phys.*, 17(07), 2050111.
17. Shamir, M. F. [2015], Exact Solution of Bianchi Type-V spacetime in $f(R, T)$ gravity, *Int. J. of theoretical phys.*, 54, 1304-1315.
18. Singh, T., Chaubey, R. [2006], Bianchi Type-V model with a perfect fluid and A-term, *Pramana*, 67, 415-428.
19. Nath, A., Sahu, S. K. [2019], LRS Bianchi Type-V perfect fluid cosmological model in $f(R, T)$ theory, *Candian J. phys.*, 4, 443-449.
20. Ram, S., Chandel, S. [2015], Dynamics of magnetized string cosmological model in $f(R, T)$ gravity theory, *Ast. Space sci.*, 355(1), 195-202.

21. Bhardwaj, V. K., Yadav, A. K. [2020], Some Bianchi type-V accelerating cosmological modes in $f(R, T) = f_1(R) + f_2(T)$ formalism, Int. J. of Geometric methods in modern Phys., 17(10), 2050159.
22. Zubair, M., Hussan, S. M. A., Abbas, G. [2016], Bianchi type-I and V solution in $f(R, T)$ gravity with time dependent deceleration parameter, Canadian J. of Phys., 94(12), 1289-1296.
23. Shriram, Kumari, P. [2014], Bianchi type-I and V bulk viscous fluid cosmological modes in $f(R, T)$ gravity theory, Center Euro. J. of Phys., 12, 744-754.
24. Maurya, D. C., Pradhan, A., Dixit, A. [2020], Domain walls and quark matter in Bianchi type-V universe eighth observational constraints in $f(R, T)$ gravity, Int. J. of geometric methods in modern Phys., 17(10), 2050014.
25. Brahma, B. P., Dewri, M. [2021], Bianchi type-V modified $f(R, T)$ gravity model in Lyra geometry with varying deceleration parameter, J. Math Comput. Sci., 11(1), 1018-1028.
26. Agrawal, P. K., Pawar, D. D.[2017], Magnetic domain wall in $f(R, T)$ theory of gravity, new Astronomy, 54, 56-60.
27. Ram. S., Zeyauddin, M., Singh, C. P. [2010], Anisotropic Bianchi type-V perfect fluid cosmological models in Lyra's geometry, J. of Geometry and phys. 60(11), 1671-1680.
28. Yadav, A. K. [2012], Cosmological constant Dominated Transit universe from the early deceleration phase to the current acceleration phase in Bianchi type-V spacetime, chine. Phys. letter, 29, 9801.
29. Shabani, H., Ziaie, A. H. [2018], interpretation of $f(R, T)$ gravity in terms of conserved effective fluid, int. J. Mod. Phys., A, 33, 1850050.
30. Shabani, H., Ziaie, A. H. [2017], Consequences of energy conservation violation: Late time solution of $\Lambda(T)$ CDM subclass of $f(R, T)$ gravity using dynamical system approach, Eur. Phys. J., C77, 282.
31. Tiwari, R. K., Beesham, A., Mishra, S., Dubey, V.[2021], Anisotropic cosmological model in a modified theory of gravitation, Universe, 7(7), 226.
32. Samanta, G. C., Myrzakulov, R. [2017], Imperfect fluid cosmological model in modified gravity, Chinese J. of Phys., 55(3), 1044-1054.