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Interval-Valued Fermatean Neutrosophic Sets Of Second Type And Its Properties

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Abstract: Impreciseness lies in almost every field we deal with. In almost all of the real-world problems that we consider, there is always an inconsistency in the values that are predicted. Many techniques have been proposed in order to handle with the inconsistent information. Many sets have been introduced in the neutrosophic field to overcome the unpredictability. In this paper, we have introduced a new set called the Interval-valued fermatean Neutrosophic Set of Second . Also, certain basic properties of interval-valued fermatean neutrosophic sets of second type are discussed.

***Index Terms* - Fermatean, neutrosophic, interval-values, fermatean fuzzy set, interval-valued fermatean neutrosophic set of second type.**

I. INTRODUCTION

Crisp values do not give an accuracy in real world problems. Hence the theory of fuzzy sets came into light with the mastermind of Zadeh [10]. This theory considered only the positive values of the problem and the negative part of it was neglected. Hence Atanassov [1] emerged with the theory of intuitionistic sets wherein the positive and negative memberships of the values were considered. Intuitionistic fuzzy sets dealt with some more accuracy in results. Researchers were always ever active in analyzing the real world problems with all its pros and cons. The step-by-step improvement in achieving accurate results made Smarandache [5] to introduce Neutrosophic sets which took into account, the neutrality in the problems. Yager [9] developed ideas of Pythagorean membership and their properties. Stephy et al [6,7] introduced the interval-valued neutrosophic Pythagorean sets and defined some of its properties and aggregation operators and applied the same in decision making problems. Peng and Yang [2] applied the Pythagorean fuzzy operators in MCDM methods to assess stock investment. Senapati and Yager [4] came forward with the notion of fermatean fuzzy sets and found their applications in real world situations using TOPSIS method and also defined distance measures. Sweety et al [8] developed Fermatean neutrosophic sets and defined some basic operations. Broumi et al [2] introduced the concept of interval-valued fermatean neutrosophic sets and interval-valued fermatean neutrosophic graphs.

Here, in this paper, we have defined Interval-valued fermatean neutrosophic sets and we have defined its properties like union, intersection, direct sum, direct product, complement with their respective proofs.

II. PRELIMINARIES

DEFINITION 2.1 [9]: Pythagorean Fuzzy set

Let U be a universe. We define a Pythagorean fuzzy set as an object of the form $R = \{(x, (\mu(x), \nu(x))/x \in U\}$satisfying the condition $0 \leq (\mu(x))^2 + (\nu(x))^2 \leq 1$. Here the functions $\mu(x): 0 \to [0,1]$ and $\nu(x): 0 \to [0,1]$ and they represent the membership degree and the non-membership degree respectively.

DEFINITION 2.2 [4]: Fermatean Fuzzy set

Let U be a universe. We define a Fermatean fuzzy set as an object of the form $R = \{(x, (\mu(x), \nu(x))/x \in U\}$satisfying the condition $0 \leq (\mu(x))^3 + (\nu(x))^3 \leq 1$. Here the functions $\mu(x): 0 \to [0,1]$ and $\nu(x): 0 \to [0,1]$ and they represent the membership degree and the non-membership degree respectively.

DEFINITION 2.3[8]: Fermatean Neutrosophic set

Let U be a universe. We define a Fermatean neutrosophic set as an object of the form $R = \{(x, (\mu(x), \nu(x))/x \in U\}$satisfying the condition $0 \leq (\mu(x))^3 + (\nu(x))^3 \leq 1 \text{ and } (\lambda(x))^3 \leq 1$. Here the functions $\mu(x): 0 \to [0,1]$ and $\nu(x): 0 \to [0,1]$ and $\lambda(x): 0 \to [0,1]$, they represent the membership degree, the non-membership degree and the degree of indeterminacy respectively.

Also $(\mu(x))^3 + (\nu(x))^3 + (\lambda(x))^3 \leq 2$

DEFINITION 2.4[2]: Interval-valued Fermatean Neutrosophic set

Let U be an universal set. An interval-valued fermatean neutrosophic set (IVNFS) is an object that takes the form $R = \{(x, [\mu_R^-, \mu_R^+], [\nu_R^-, \nu_R^+], [\lambda_R^-, \lambda_R^+])/x \in U\}$ satisfying the condition $\left(\left[\frac{\mu_R^- + \mu_R^+}{2}\right]^3 + \left[\frac{\nu_R^- + \nu_R^+}{2}\right]^3 + \left[\frac{\lambda_R^- + \lambda_R^+}{2}\right]^3\right) \leq 2$.

Here $[\mu_R^-, \mu_R^+], [\nu_R^-, \nu_R^+], [\lambda_R^-, \lambda_R^+]$ denotes the interval values of the membership values of truth, indeterminant and false degrees.

Also $[\mu_R^-, \mu_R^+]$ *and* , $[\lambda_R^-, \lambda_R^+]$ are dependent components and $[\nu_R^-, \nu_R^+]$ is independent component.

III. INTERVAL-VALUED FERMATEAN NEUTROSOPHIC SET OF SECOND TYPE

DEFINITION 3.1:

Let U be an universal set. An interval-valued fermatean neutrosophic set of second type (IVNFSST) is an object that takes the form $R = \{(x, [\mu_R^-, \mu_R^+], [\nu_R^-, \nu_R^+], [\lambda_R^-, \lambda_R^+])/x \in U\}$ satisfying the condition $\left(\left[\frac{\mu_R^- + \mu_R^+}{2}\right]^3 + \left[\frac{\nu_R^- + \nu_R^+}{2}\right]^3 + \left[\frac{\lambda_R^- + \lambda_R^+}{2}\right]^3\right)^2 \leq 2$.

Here $[\mu_R^-, \mu_R^+], [\nu_R^-, \nu_R^+], [\lambda_R^-, \lambda_R^+]$ denotes the interval values of the membership values of truth, indeterminant and false degrees.

Also $[\mu_R^-, \mu_R^+]$ *and* , $[\lambda_R^-, \lambda_R^+]$ are dependent components and $[\nu_R^-, \nu_R^+]$ is independent component.

DEFINITION 3.2:

Let $R = \{(x, [\mu_R^-, \mu_R^+], [\nu_R^-, \nu_R^+], [\lambda_R^-, \lambda_R^+])/x \in U\}$ be an interval-valued fermatean neutrosophic set of second type, then its complement is defined as $R^c = \{(x, [\lambda_R^-, \lambda_R^+], [1 - \nu_R^-, 1 - \nu_R^+], [\mu_R^-, \mu_R^+])/x \in U\}$

DEFINITION 3.3:

Let us consider two interval-valued fermatean neutrosophic sets of second type R and S defined as $R = \{(x, [\mu_R^-, \mu_R^+], [\nu_R^-, \nu_R^+], [\lambda_R^-, \lambda_R^+])/x \in U\}$ and $S = \{(y, [\mu_S^-, \mu_S^+], [\nu_S^-, \nu_S^+], [\lambda_S^-, \lambda_S^+])/y \in U\}$, then the distance between two IVNFS is defined as

$$D(R,S) = \frac{1}{2}\left\{\begin{matrix} [|(\mu_R^-)^3 - (\mu_S^-)^3|, |(\mu_R^+)^3 - (\mu_S^+)^3|], \\ [|(\nu_R^-)^3 - (\nu_S^-)^3|, |(\nu_R^+)^3 - (\nu_S^+)^3|], \\ [|(\lambda_R^-)^3 - (\lambda_S^-)^3|, |(\lambda_R^+)^3 - (\lambda_S^+)^3|] \end{matrix}\right\}$$

DEFINITION 3.4:

Let $R = \{(x, [\mu_R^-, \mu_R^+], [\nu_R^-, \nu_R^+], [\lambda_R^-, \lambda_R^+])/x \in U\}$ and $S = \{(y, [\mu_S^-, \mu_S^+], [\nu_S^-, \nu_S^+], [\lambda_S^-, \lambda_S^+])/y \in U\}$ be two interval-valued fermatean neutrosophic sets of second type, then

$$R + S = \left\{\begin{matrix} x + y, & [\mu_R^- + \mu_S^- - \mu_R^- \mu_S^-, \mu_R^+ + \mu_S^+ - \mu_R^+ \mu_S^+], \\ & [\nu_R^- \nu_S^-, \nu_R^+ \nu_S^+], \\ & [\lambda_R^- \lambda_S^-, \lambda_R^+ \lambda_S^+] \end{matrix}\right\}$$

$$R.S = \left\{\begin{matrix} xy, \quad [\mu_R^- \mu_S^-, \mu_R^+ \mu_S^+], \\ [\nu_R^- + \nu_S^- - \nu_R^- \nu_S^-, \nu_R^+ + \nu_S^+ - \nu_R^+ \nu_S^+], \\ [\lambda_R^- + \lambda_S^- - \lambda_R^- \lambda_S^-, \lambda_R^+ + \lambda_S^+ - \lambda_R^+ \lambda_S^+] \end{matrix}\right\}$$

DEFINITION 3.5:

For an interval-valued fermatean neutrosophic set of second type $R = ([\mu_R^-, \mu_R^+], [\nu_R^-, \nu_R^+], [\lambda_R^-, \lambda_R^+])$, the score function of R be defined as

$$Score(R) = \frac{1}{2}[(\mu_R^-)^3 + (1 - \nu_R^-)^3 + (1 - \lambda_R^-)^3 - (\mu_R^+)^3 - (1 - \nu_R^+)^3 - (1 - \lambda_R^+)]$$

Where Score (R) $\in [-1,1]$.

Consider any two IVFNNSTs R and S,

- If Score(R) < Score (S), then R<S.
- If Score(R) > Score(S), then R>S.
- If Score(R) = Score(S), then R=S.

IV. BASIC OPERATIONS OF INTERVAL-VALUED FERMATEAN NEUTROSOPHIC SET OF SECOND TYPE

DEFINITION 4.1:

Consider $R = ([\mu_R^-, \mu_R^+], [\nu_R^-, \nu_R^+], [\lambda_R^-, \lambda_R^+])$, $S = ([\mu_S^-, \mu_S^+], [\nu_S^-, \nu_S^+], [\lambda_S^-, \lambda_S^+])$, $T = ([\mu_T^-, \mu_T^+], [\nu_T^-, \nu_T^+], [\lambda_T^-, \lambda_T^+])$ be three interval-valued fermatean neutrosophic numbers of second type and $\rho > 0$, then the following equations are defined:

1. $$S \cup T = \left\{ \begin{matrix} [\max(\mu_S^-, \mu_T^-), \max(\mu_S^+, \mu_T^+)], \\ [\min(\nu_S^-, \nu_T^-), \min(\nu_S^+, \nu_T^+)], \\ [\min(\lambda_S^-, \lambda_T^-), \min(\lambda_S^+, \lambda_T^+)] \end{matrix} \right\}$$

2. $$S \cap T = \left\{ \begin{matrix} [\min(\mu_S^-, \mu_T^-), \min(\mu_S^+, \mu_T^+)], \\ [\max(\nu_S^-, \nu_T^-), \max(\nu_S^+, \nu_T^+)], \\ [\max(\lambda_S^-, \lambda_T^-), \max(\lambda_S^+, \lambda_T^+)] \end{matrix} \right\}$$

3. $$S \oplus T = \left\{ \begin{matrix} \left[\sqrt[3]{(\mu_S^-)^3 + (\mu_T^-)^3 - (\mu_S^-)^3(\mu_T^-)^3}, \sqrt[3]{(\mu_S^+)^3 + (\mu_T^+)^3 - (\mu_S^+)^3(\mu_T^+)^3}\right], \\ [\nu_S^- \nu_T^-, \nu_S^+ \nu_T^+], \\ [\lambda_S^- \lambda_T^-, \lambda_S^+ \lambda_T^+] \end{matrix} \right\}$$

4. $$S \otimes T = \left\{ \begin{matrix} [\mu_S^- \mu_T^-, \mu_S^+ \mu_T^+], \\ \left[\sqrt[3]{(\nu_S^-)^3 + (\nu_T^-)^3 - (\nu_S^-)^3(\nu_T^-)^3}, \sqrt[3]{(\nu_S^+)^3 + (\nu_T^+)^3 - (\nu_S^+)^3(\nu_T^+)^3}\right], \\ \left[\sqrt[3]{(\lambda_S^-)^3 + (\lambda_T^-)^3 - (\lambda_S^-)^3(\lambda_T^-)^3}, \sqrt[3]{(\lambda_S^+)^3 + (\lambda_T^+)^3 - (\lambda_S^+)^3(\lambda_T^+)^3}\right] \end{matrix} \right\}$$

5. $$\rho R = \left\{ \begin{matrix} \left[\sqrt[3]{1 - (1 - (\mu_R^-)^3)^\rho}, \sqrt[3]{1 - (1 - (\mu_R^+)^3)^\rho}\right], \\ [(\nu_R^-)^\rho, (\nu_R^+)^\rho], \\ [(\lambda_R^-)^\rho, (\lambda_R^+)^\rho] \end{matrix} \right\}$$

6. $$R^\rho = \left\{ \begin{matrix} [(\mu_R^-)^\rho, (\mu_R^+)^\rho], \\ \left[\sqrt[3]{1 - (1 - (\nu_R^-)^3)^\rho}, \sqrt[3]{1 - (1 - (\nu_R^+)^3)^\rho}\right], \\ \left[\sqrt[3]{1 - (1 - (\lambda_R^-)^3)^\rho}, \sqrt[3]{1 - (1 - (\lambda_R^+)^3)^\rho}\right] \end{matrix} \right\}$$

THEOREM 4.1:

Let $R = ([\mu_R^-, \mu_R^+], [\nu_R^-, \nu_R^+], [\lambda_R^-, \lambda_R^+])$, $S = ([\mu_S^-, \mu_S^+], [\nu_S^-, \nu_S^+], [\lambda_S^-, \lambda_S^+])$, $T = ([\mu_T^-, \mu_T^+], [\nu_T^-, \nu_T^+], [\lambda_T^-, \lambda_T^+])$ be three interval-valued fermatean neutrosophic numbers of second type and let $\rho > 0$,

1. $(R^c)^\rho = (\rho R)^c$
2. $\rho(R^c) = (R^\rho)^c$
3. $S \cup T = T \cup S$
4. $S \cap T = T \cap S$
5. $\rho(S \cup T) = \rho S \cup \rho T$
6. $(S \cup T)^\rho = S^\rho \cup T^\rho$
7. $R \cup (S \cup T) = (R \cup S) \cup T$
8. $R \cap (S \cap T) = (R \cap S) \cap T$

Proof: We shall prove for 1, 3 and 5 and the remaining shall follow.

1. $$(R^c)^\rho = \left\{ \begin{matrix} [\lambda_R^-, \lambda_R^+], \\ [1 - \nu_R^-, 1 - \nu_R^+], \\ [\mu_R^-, \mu_R^+] \end{matrix} \right\}^\rho$$

$$= \left\{ \begin{matrix} [(\lambda_R^-)^\rho, (\lambda_R^+)^\rho], \\ \left[\sqrt[3]{1 - (1 - (\nu_R^-)^3)^\rho}, \sqrt[3]{1 - (1 - (\nu_R^+)^3)^\rho}\right], \\ \left[\sqrt[3]{1 - (1 - (\mu_R^-)^3)^\rho}, \sqrt[3]{1 - (1 - (\mu_R^+)^3)^\rho}\right] \end{matrix} \right\}$$

$$(\rho R)^c = \begin{Bmatrix} [(\mu_R^-)^\rho, (\mu_R^+)^\rho], \\ \left[\sqrt[3]{1-(1-(\nu_R^-)^3)^\rho}, \sqrt[3]{1-(1-(\nu_R^+)^3)^\rho}\right], \\ \left[\sqrt[3]{1-(1-(\lambda_R^-)^3)^\rho}, \sqrt[3]{1-(1-(\lambda_R^+)^3)^\rho}\right] \end{Bmatrix}^c$$

$$= \begin{Bmatrix} [(\lambda_R^-)^\rho, (\lambda_R^+)^\rho], \\ \left[\sqrt[3]{1-(1-(\nu_R^-)^3)^\rho}, \sqrt[3]{1-(1-(\nu_R^+)^3)^\rho}\right], \\ \left[\sqrt[3]{1-(1-(\mu_R^-)^3)^\rho}, \sqrt[3]{1-(1-(\mu_R^+)^3)^\rho}\right] \end{Bmatrix}$$

$$3. S \cup T = \begin{Bmatrix} [\max(\mu_S^-, \mu_T^-), \max(\mu_S^+, \mu_T^+)], \\ [\min(\nu_S^-, \nu_T^-), \min(\nu_S^+, \nu_T^+)], \\ [\min(\lambda_S^-, \lambda_T^-), \min(\lambda_S^+, \lambda_T^+)] \end{Bmatrix}$$

$$= \begin{Bmatrix} [\max(\mu_T^-, \mu_S^-), \max(\mu_T^+, \mu_S^+)], \\ [\min(\nu_T^-, \nu_S^-), \min(\nu_T^+, \nu_S^+)], \\ [\min(\lambda_T^-, \lambda_S^-), \min(\lambda_T^+, \lambda_S^+)] \end{Bmatrix}$$

$$= T \cup S$$

$$5. \rho(S \cup T) = \rho \begin{Bmatrix} [\max(\mu_S^-, \mu_T^-), \max(\mu_S^+, \mu_T^+)], \\ [\min(\nu_S^-, \nu_T^-), \min(\nu_S^+, \nu_T^+)], \\ [\min(\lambda_S^-, \lambda_T^-), \min(\lambda_S^+, \lambda_T^+)] \end{Bmatrix}$$

$$= \begin{Bmatrix} \left[\sqrt[3]{1-(1-max(\mu_S^-, \mu_T^-)^3)^\rho}, \sqrt[3]{1-(1-max(\mu_S^+, \mu_T^+)^3)^\rho}\right], \\ [\min(\nu_S^-, \nu_T^-)^\rho, \min(\nu_S^+, \nu_R^+)^\rho], \\ [\min(\lambda_S^-, \lambda_T^-)^\rho, \min(\lambda_S^+, \lambda_T^+)^\rho] \end{Bmatrix}$$

$$\rho S \cup \rho T = \begin{Bmatrix} \left[\sqrt[3]{1-(1-(\mu_S^-)^3)^\rho}, \sqrt[3]{1-(1-(\mu_S^+)^3)^\rho}\right], \\ [(\nu_S^-)^\rho, (\nu_S^+)^\rho], \\ [(\lambda_S^-)^\rho, (\lambda_S^+)^\rho] \end{Bmatrix} \cup \begin{Bmatrix} \left[\sqrt[3]{1-(1-(\mu_T^-)^3)^\rho}, \sqrt[3]{1-(1-(\mu_T^+)^3)^\rho}\right], \\ [(\nu_T^-)^\rho, (\nu_T^+)^\rho], \\ [(\lambda_T^-)^\rho, (\lambda_T^+)^\rho] \end{Bmatrix}$$

$$= \begin{Bmatrix} \left[\max\left(\sqrt[3]{1-(1-(\mu_S^-)^3)^\rho}, \sqrt[3]{1-(1-(\mu_T^-)^3)^\rho}\right), \max\left(\sqrt[3]{1-(1-(\mu_S^+)^3)^\rho}, \sqrt[3]{1-(1-(\mu_T^+)^3)^\rho}\right)\right], \\ [\min((\nu_S^-)^\rho, (\nu_T^-)^\rho), \min((\nu_S^+)^\rho, (\nu_T^+)^\rho)], \\ [\min((\lambda_S^-)^\rho, (\lambda_T^-)^\rho), \min((\lambda_S^+)^\rho, (\lambda_T^+)^\rho)] \end{Bmatrix}$$

$$= \begin{Bmatrix} \left[\sqrt[3]{1-(1-\max((\mu_S^-)^3, (\mu_T^-)^3)^\rho}, \sqrt[3]{1-(1-\max((\mu_S^+)^3, (\mu_T^+)^3)^\rho}\right], \\ [\min(\nu_S^-, \nu_T^-)^\rho, \min(\nu_S^+, \nu_T^+)^\rho], \\ [\min(\lambda_S^-, \lambda_T^-)^\rho, \min(\lambda_S^+, \lambda_T^+)^\rho] \end{Bmatrix}$$

THEOREM 4.2:

Let us consider $S = ([\mu_S^-, \mu_S^+], [\nu_S^-, \nu_S^+], [\lambda_S^-, \lambda_S^+])$, $T = ([\mu_T^-, \mu_T^+], [\nu_T^-, \nu_T^+], [\lambda_T^-, \lambda_T^+])$ be three interval-valued fermatean neutrosophic numbers of second type, then we define and prove the following,

1. $S^c \cup T^c = (S \cap T)^c$
2. $S^c \cap T^c = (S \cup T)^c$
3. $S^c \oplus T^c = (S \otimes T)^c$
4. $S^c \otimes T^c = (S \oplus T)^c$

Proof: We shall prove for 1 and 3 and the proof is similar for 2 and 4.

$$1. \quad S^c \cup T^c = \begin{Bmatrix} [\lambda_S^-, \lambda_S^+], \\ [1-\nu_S^-, 1-\nu_S^+], \\ [\mu_S^-, \mu_S^+] \end{Bmatrix} \cup \begin{Bmatrix} [\lambda_T^-, \lambda_T^+], \\ [1-\nu_T^-, 1-\nu_T^+], \\ [\mu_T^-, \mu_T^+] \end{Bmatrix}$$

$$= \begin{Bmatrix} [\max(\lambda_S^-, \lambda_T^-), \max(\lambda_S^+, \lambda_T^+)], \\ [\min(1-\nu_S^-, 1-\nu_T^-), \min(1-\nu_S^+, 1-\nu_T^+)], \\ [\min(\mu_S^-, \mu_T^-), \min(\mu_S^+, \mu_T^+)] \end{Bmatrix}$$

$$(S\cap T)=\begin{Bmatrix}[\min(\mu_S^-,\mu_T^-),\min(\mu_S^+,\mu_T^+)],\\ [\max(\nu_S^-,\nu_T^-),\max(\nu_S^+,\nu_T^+)],\\ [\max(\lambda_S^-,\lambda_T^-),\max(\lambda_S^+,\lambda_T^+)]\end{Bmatrix}$$

$$(S\cap T)^c=\begin{Bmatrix}[\max(\lambda_S^-,\lambda_T^-),\max(\lambda_S^+,\lambda_T^+)],\\ [\min(1-\nu_S^-,1-\nu_T^-),\min(1-\nu_S^+,1-\nu_T^+)],\\ [\min(\mu_S^-,\mu_T^-),\min(\mu_S^+,\mu_T^+)]\end{Bmatrix}$$

$$3.\ S^c\oplus T^c=\begin{Bmatrix}[\lambda_S^-,\lambda_S^+],\\ [1-\nu_S^-,1-\nu_S^+],\\ [\mu_S^-,\mu_S^+]\end{Bmatrix}\oplus\begin{Bmatrix}[\lambda_T^-,\lambda_T^+],\\ [1-\nu_T^-,1-\nu_T^+],\\ [\mu_T^-,\mu_T^+]\end{Bmatrix}$$

$$=\begin{Bmatrix}\left[\sqrt[3]{(\lambda_S^-)^3+(\lambda_T^-)^3-(\lambda_S^-)^3(\lambda_T^-)^3},\sqrt[3]{(\lambda_S^+)^3+(\lambda_T^+)^3-(\lambda_S^+)^3(\lambda_T^+)^3}\right],\\ [(1-\nu_S^-)(1-\nu_T^-),(1-\nu_S^+)(1-\nu_T^+)],\\ [\mu_S^-\mu_T^-,\mu_S^+\mu_T^+]\end{Bmatrix}$$

$$(S\otimes T)^c=\begin{Bmatrix}[\mu_S^-\mu_T^-,\mu_S^+\mu_T^+],\\ \left[\sqrt[3]{(\nu_S^-)^3+(\nu_T^-)^3-(\nu_S^-)^3(\nu_T^-)^3},\sqrt[3]{(\nu_S^+)^3+(\nu_T^+)^3-(\nu_S^+)^3(\nu_T^+)^3}\right],\\ \left[\sqrt[3]{(\lambda_S^-)^3+(\lambda_T^-)^3-(\lambda_S^-)^3(\lambda_T^-)^3},\sqrt[3]{(\lambda_S^+)^3+(\lambda_T^+)^3-(\lambda_S^+)^3(\lambda_T^+)^3}\right]\end{Bmatrix}^c$$

$$=\begin{Bmatrix}\left[\sqrt[3]{(\lambda_S^-)^3+(\lambda_T^-)^3-(\lambda_S^-)^3(\lambda_T^-)^3},\sqrt[3]{(\lambda_S^+)^3+(\lambda_T^+)^3-(\lambda_S^+)^3(\lambda_T^+)^3}\right],\\ [(1-\nu_S^-)(1-\nu_T^-),(1-\nu_S^+)(1-\nu_T^+)],\\ [\mu_S^-\mu_T^-,\mu_S^+\mu_T^+]\end{Bmatrix}$$

THEOREM 4.3:

Let $R=([\mu_R^-,\mu_R^+],[\nu_R^-,\nu_R^+],[\lambda_R^-,\lambda_R^+])$, $S=([\mu_S^-,\mu_S^+],[\nu_S^-,\nu_S^+],[\lambda_S^-,\lambda_S^+])$, $T=([\mu_T^-,\mu_T^+],[\nu_T^-,\nu_T^+],[\lambda_T^-,\lambda_T^+])$ be three interval-valued fermatean neutrosophic numbers of second type, then

1. $(R\cap S)\oplus T=(R\oplus T)\cap(S\oplus T)$
2. $(R\cup S)\oplus T=(R\oplus T)\cup(S\oplus T)$
3. $(R\cap S)\otimes T=(R\otimes T)\cap(S\otimes T)$
4. $(R\cup S)\otimes T=(R\otimes T)\cup(S\otimes T)$
5. $(R\cup S)\oplus(R\cap S)=(R\oplus S)$
6. $(R\cup S)\otimes(R\cap S)=R\otimes S$

Proof: We shall prove for 1, 3, 5 and 6.

1. $$(R\cap S)\oplus T=\begin{Bmatrix}[\min(\mu_R^-,\mu_S^-),\min(\mu_R^+,\mu_S^+)],\\ [\max(\nu_R^-,\nu_S^-),\max(\nu_R^+,\nu_S^+)],\\ [\max(\lambda_R^-,\lambda_S^-),\max(\lambda_R^+,\lambda_S^+)]\end{Bmatrix}\oplus\begin{Bmatrix}[\mu_T^-,\mu_T^+],\\ [\nu_T^-,\nu_T^+],\\ [\lambda_T^-,\lambda_T^+]\end{Bmatrix}$$

$$=\begin{Bmatrix}\left[\begin{matrix}\sqrt[3]{\min((\mu_R^-)^3,(\mu_S^-)^3)+(\mu_T^-)^3-(\mu_T^-)^3\min((\mu_R^-)^3,(\mu_S^-)^3)},\\ \sqrt[3]{\min((\mu_R^+)^3,(\mu_S^+)^3)+(\mu_T^+)^3-(\mu_T^+)^3\min((\mu_R^+)^3,(\mu_S^+)^3)}\end{matrix}\right],\\ [\nu_T^-\max(\nu_R^-,\nu_S^-),\nu_T^+\max(\nu_R^+,\nu_S^+)],\\ [\lambda_T^-\max(\lambda_R^-,\lambda_S^-),\lambda_T^+\max(\lambda_R^+,\lambda_S^+)]\end{Bmatrix}$$

$$R\oplus T=\begin{Bmatrix}\left[\sqrt[3]{(\mu_R^-)^3+(\mu_T^-)^3-(\mu_R^-)^3(\mu_T^-)^3},\sqrt[3]{(\mu_R^+)^3+(\mu_T^+)^3-(\mu_R^+)^3(\mu_T^+)^3}\right],\\ [\nu_R^-\nu_T^-,\nu_R^+\nu_T^+],\\ [\lambda_R^-\lambda_T^-,\lambda_R^+\lambda_T^+]\end{Bmatrix}$$

$$S\oplus T=\begin{Bmatrix}\left[\sqrt[3]{(\mu_S^-)^3+(\mu_T^-)^3-(\mu_S^-)^3(\mu_T^-)^3},\sqrt[3]{(\mu_S^+)^3+(\mu_T^+)^3-(\mu_S^+)^3(\mu_T^+)^3}\right],\\ [\nu_S^-\nu_T^-,\nu_S^+\nu_T^+],\\ [\lambda_S^-\lambda_T^-,\lambda_S^+\lambda_T^+]\end{Bmatrix}$$

$$(R \oplus T) \cap (S \oplus T) = \left\{ \begin{array}{c} \left[\begin{array}{c} \sqrt[3]{\min((\mu_R^-)^3, (\mu_S^-)^3) + (\mu_T^-)^3 - (\mu_T^-)^3 \min((\mu_R^-)^3, (\mu_S^-)^3)}, \\ \sqrt[3]{\min((\mu_R^+)^3, (\mu_S^+)^3) + (\mu_T^+)^3 - (\mu_T^+)^3 \min((\mu_R^+)^3, (\mu_S^+)^3)} \end{array} \right], \\ [\nu_T^- \max(\nu_R^-, \nu_S^-), \nu_T^+ \max(\nu_R^+, \nu_S^+)], \\ [\lambda_T^- \max(\lambda_R^-, \lambda_S^-), \lambda_T^+ \max(\lambda_R^+, \lambda_S^+)] \end{array} \right\}$$

3. $$(R \cap S) \otimes T = \left\{ \begin{array}{c} [\min(\mu_R^-, \mu_S^-), \min(\mu_R^+, \mu_S^+)], \\ [\max(\nu_R^-, \nu_S^-), \max(\nu_R^+, \nu_S^+)], \\ [\max(\lambda_R^-, \lambda_S^-), \max(\lambda_R^+, \lambda_S^+)] \end{array} \right\} \otimes \left\{ \begin{array}{c} [\mu_T^-, \mu_T^+], \\ [\nu_T^-, \nu_T^+], \\ [\lambda_T^-, \lambda_T^+] \end{array} \right\}$$

$$= \left\{ \begin{array}{c} [\mu_T^- \min(\mu_R^-, \mu_S^-), \mu_T^+ \min(\mu_R^+, \mu_S^+)], \\ \left[\begin{array}{c} \sqrt[3]{\max((\nu_R^-)^3, (\nu_S^-)^3) + (\nu_T^-)^3 - (\nu_T^-)^3 \max((\nu_R^-)^3, (\nu_S^-)^3)}, \\ \sqrt[3]{\max((\nu_R^+)^3, (\nu_S^+)^3) + (\nu_T^+)^3 - (\nu_T^+)^3 \max((\nu_R^+)^3, (\nu_S^+)^3)} \end{array} \right], \\ \left[\begin{array}{c} \sqrt[3]{\max((\lambda_R^-)^3, (\lambda_S^-)^3) + (\lambda_T^-)^3 - (\lambda_T^-)^3 \max((\lambda_R^-)^3, (\lambda_S^-)^3)}, \\ \sqrt[3]{\max((\lambda_R^+)^3, (\lambda_S^+)^3) + (\lambda_T^+)^3 - (\lambda_T^+)^3 \max((\lambda_R^+)^3, (\lambda_S^+)^3)} \end{array} \right] \end{array} \right\}$$

$$R \otimes T = \left\{ \begin{array}{c} [\mu_R^- \mu_T^-, \mu_R^+ \mu_T^+], \\ \left[\sqrt[3]{(\nu_R^-)^3 + (\nu_T^-)^3 - (\nu_R^-)^3 (\nu_T^-)^3}, \sqrt[3]{(\nu_R^+)^3 + (\nu_T^+)^3 - (\nu_R^+)^3 (\nu_T^+)^3} \right], \\ \left[\sqrt[3]{(\lambda_R^-)^3 + (\lambda_T^-)^3 - (\lambda_R^-)^3 (\lambda_T^-)^3}, \sqrt[3]{(\lambda_R^+)^3 + (\lambda_T^+)^3 - (\lambda_R^+)^3 (\lambda_T^+)^3} \right] \end{array} \right\}$$

$$S \otimes T = \left\{ \begin{array}{c} [\mu_S^- \mu_T^-, \mu_S^+ \mu_T^+], \\ \left[\sqrt[3]{(\nu_S^-)^3 + (\nu_T^-)^3 - (\nu_S^-)^3 (\nu_T^-)^3}, \sqrt[3]{(\nu_S^+)^3 + (\nu_T^+)^3 - (\nu_S^+)^3 (\nu_T^+)^3} \right], \\ \left[\sqrt[3]{(\lambda_S^-)^3 + (\lambda_T^-)^3 - (\lambda_S^-)^3 (\lambda_T^-)^3}, \sqrt[3]{(\lambda_S^+)^3 + (\lambda_T^+)^3 - (\lambda_S^+)^3 (\lambda_T^+)^3} \right] \end{array} \right\}$$

$$(R \otimes T) \cap (S \otimes T) = \left\{ \begin{array}{c} [\mu_T^- \min(\mu_R^-, \mu_S^-), \mu_T^+ \min(\mu_R^+, \mu_S^+)], \\ \left[\begin{array}{c} \sqrt[3]{\max((\nu_R^-)^3, (\nu_S^-)^3) + (\nu_T^-)^3 - (\nu_T^-)^3 \max((\nu_R^-)^3, (\nu_S^-)^3)}, \\ \sqrt[3]{\max((\nu_R^+)^3, (\nu_S^+)^3) + (\nu_T^+)^3 - (\nu_T^+)^3 \max((\nu_R^+)^3, (\nu_S^+)^3)} \end{array} \right], \\ \left[\begin{array}{c} \sqrt[3]{\max((\lambda_R^-)^3, (\lambda_S^-)^3) + (\lambda_T^-)^3 - (\lambda_T^-)^3 \max((\lambda_R^-)^3, (\lambda_S^-)^3)}, \\ \sqrt[3]{\max((\lambda_R^+)^3, (\lambda_S^+)^3) + (\lambda_T^+)^3 - (\lambda_T^+)^3 \max((\lambda_R^+)^3, (\lambda_S^+)^3)} \end{array} \right] \end{array} \right\}$$

5. $$(R \cup S) \oplus (R \cap S) = \left\{ \begin{array}{c} [\max(\mu_R^-, \mu_S^-), \max(\mu_R^+, \mu_S^+)], \\ [\min(\nu_R^-, \nu_S^-), \min(\nu_R^+, \nu_S^+)], \\ [\min(\lambda_R^-, \lambda_S^-), \min(\lambda_R^+, \lambda_S^+)] \end{array} \right\} \oplus \left\{ \begin{array}{c} [\min(\mu_R^-, \mu_S^-), \min(\mu_R^+, \mu_S^+)], \\ [\max(\nu_R^-, \nu_S^-), \max(\nu_R^+, \nu_S^+)], \\ [\max(\lambda_R^-, \lambda_S^-), \max(\lambda_R^+, \lambda_S^+)] \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left[\sqrt[3]{(\mu_R^-)^3 + (\mu_S^-)^3 - (\mu_R^-)^3 (\mu_S^-)^3}, \sqrt[3]{(\mu_R^+)^3 + (\mu_S^+)^3 - (\mu_R^+)^3 (\mu_S^+)^3} \right], \\ [\nu_R^- \nu_S^-, \nu_R^+ \nu_S^+], \\ [\lambda_R^- \lambda_S^-, \lambda_R^+ \lambda_S^+] \end{array} \right\}$$

$$= R \oplus S$$

6. $$(R \cup S) \otimes (R \cap S) = \left\{ \begin{array}{c} [\max(\mu_R^-, \mu_S^-), \max(\mu_R^+, \mu_S^+)], \\ [\min(\nu_R^-, \nu_S^-), \min(\nu_R^+, \nu_S^+)], \\ [\min(\lambda_R^-, \lambda_S^-), \min(\lambda_R^+, \lambda_S^+)] \end{array} \right\} \otimes \left\{ \begin{array}{c} [\min(\mu_R^-, \mu_S^-), \min(\mu_R^+, \mu_S^+)], \\ [\max(\nu_R^-, \nu_S^-), \max(\nu_R^+, \nu_S^+)], \\ [\max(\lambda_R^-, \lambda_S^-), \max(\lambda_R^+, \lambda_S^+)] \end{array} \right\}$$

$$= \left\{ \begin{array}{c} [\mu_R^- \mu_S^-, \mu_R^+ \mu_S^+], \\ \left[\sqrt[3]{(\nu_R^-)^3 + (\nu_S^-)^3 - (\nu_R^-)^3 (\nu_S^-)^3}, \sqrt[3]{(\nu_R^+)^3 + (\nu_S^+)^3 - (\nu_R^+)^3 (\nu_S^+)^3} \right], \\ \left[\sqrt[3]{(\lambda_R^-)^3 + (\lambda_S^-)^3 - (\lambda_R^-)^3 (\lambda_S^-)^3}, \sqrt[3]{(\lambda_R^+)^3 + (\lambda_S^+)^3 - (\lambda_R^+)^3 (\lambda_S^+)^3} \right] \end{array} \right\}$$

$$= R \otimes S$$

THEOREM 4.4:

Consider $R = ([\mu_R^-, \mu_R^+], [\nu_R^-, \nu_R^+], [\lambda_R^-, \lambda_R^+])$, $S = ([\mu_S^-, \mu_S^+], [\nu_S^-, \nu_S^+], [\lambda_S^-, \lambda_S^+])$, $T = ([\mu_T^-, \mu_T^+], [\nu_T^-, \nu_T^+], [\lambda_T^-, \lambda_T^+])$ to be three interval-valued fermatean neutrosophic sets of second type, then

1. $R \cup S \cup T = R \cup T \cup S$
2. $R \cap S \cap T = R \cap T \cap S$
3. $R \oplus S \oplus T = R \oplus T \oplus S$
4. $R \otimes S \otimes T = R \otimes T \otimes S$

Proof: The proof for 1 and 3 will be proved where as 2 and 4 follows analogously.

1. $$R \cup S \cup T = \left\{ \begin{array}{c} [\max(\mu_R^-, \mu_S^-), \max(\mu_R^+, \mu_S^+)], \\ [\min(\nu_R^-, \nu_S^-), \min(\nu_R^+, \nu_S^+)], \\ [\min(\lambda_R^-, \lambda_S^-), \min(\lambda_R^+, \lambda_S^+)] \end{array} \right\} \cup \left\{ \begin{array}{c} ([\mu_T^-, \mu_T^+], \\ [\nu_T^-, \nu_T^+], \\ [\lambda_T^-, \lambda_T^+]) \end{array} \right\}$$

$$= \left\{ \begin{array}{c} [\max(\mu_R^-, \mu_S^-, \mu_T^-), \max(\mu_R^+, \mu_S^+, \mu_T^+)], \\ [\min(\nu_R^-, \nu_S^-, \nu_T^-), \min(\nu_R^+, \nu_S^+, \nu_T^+)], \\ [\min(\lambda_R^-, \lambda_S^-, \lambda_T^-), \min(\lambda_R^+, \lambda_S^+, \lambda_T^+)] \end{array} \right\}$$

$$= \left\{ \begin{array}{c} [\max(\mu_R^-, \mu_T^-, \mu_S^-), \max(\mu_R^+, \mu_T^+, \mu_S^+)], \\ [\min(\nu_R^-, \nu_T^-, \nu_S^-), \min(\nu_R^+, \nu_T^+, \nu_S^+)], \\ [\min(\lambda_R^-, \lambda_T^-, \lambda_S^-), \min(\lambda_R^+, \lambda_T^+, \lambda_S^+)] \end{array} \right\}$$

$$= R \cup T \cup S$$

2. $$R \oplus S \oplus T = \left\{ \begin{array}{c} \left[\sqrt[3]{(\mu_R^-)^3 + (\mu_S^-)^3 - (\mu_R^-)^3 (\mu_S^-)^3}, \sqrt[3]{(\mu_R^+)^3 + (\mu_S^+)^3 - (\mu_R^+)^3 (\mu_S^+)^3} \right], \\ [\nu_R^- \nu_S^-, \nu_R^+ \nu_S^+], \\ [\lambda_R^- \lambda_S^-, \lambda_R^+ \lambda_S^+] \end{array} \right\} \oplus \left\{ \begin{array}{c} ([\mu_T^-, \mu_T^+], \\ [\nu_T^-, \nu_T^+], \\ [\lambda_T^-, \lambda_T^+]) \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left[\sqrt[3]{(\mu_R^-)^3 + (\mu_S^-)^3 + (\mu_T^-)^3 - (\mu_R^-)^3 (\mu_S^-)^3 (\mu_T^-)^3}, \sqrt[3]{(\mu_R^+)^3 + (\mu_S^+)^3 + (\mu_T^+)^3 - (\mu_R^+)^3 (\mu_S^+)^3 (\mu_T^+)^3} \right], \\ [\nu_R^- \nu_S^- \nu_T^-, \nu_R^+ \nu_S^+ \nu_T^+], \\ [\lambda_R^- \lambda_S^- \lambda_T^-, \lambda_R^+ \lambda_S^+ \lambda_T^+] \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left[\sqrt[3]{(\mu_R^-)^3 + (\mu_T^-)^3 + (\mu_S^-)^3 - (\mu_R^-)^3 (\mu_T^-)^3 (\mu_S^-)^3}, \sqrt[3]{(\mu_R^+)^3 + (\mu_T^+)^3 + (\mu_S^+)^3 - (\mu_R^+)^3 (\mu_T^+)^3 (\mu_S^+)^3} \right], \\ [\nu_R^- \nu_T^- \nu_S^-, \nu_R^+ \nu_T^+ \nu_S^+], \\ [\lambda_R^- \lambda_T^- \lambda_S^-, \lambda_R^+ \lambda_T^+ \lambda_S^+] \end{array} \right\}$$

$$= R \oplus T \oplus S$$

V. CONCLUSION

In this paper, we have introduced the notion of Interval-valued fermatean neutrosophic sets of second type. This interval valued concept of fermatean sets of second type gives some more preciseness and it resolves the inconsistency that existed earlier. Also, we have developed the properties of union, intersection, direct sum, direct product, complement and scalar multiplication and power. Having defined these concepts, some theorems have been proved to justify the same.

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