IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)
An International Open Access, Peer-reviewed, Refereed Journal

ON REGATIVE SEMI GROUPS

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ABSTRACT

The concept of a semi group being Regative is discussed. The adsorption over a regular semi group has delineated its morphology to be a Regative semi group. Indication of a Regative semi group has been diversified into various branches over the structures of regularity, nilpotency, identity, commutativity, zero-symmetricity. A Regative Semi group in general is examined to possess zero-symmetricity and also the P_4 property. Every ideal of a Regative Semi Group adequates Completely Semi prime property by itself. The property of Regativity over a semi group or in general is found to have an extensive outreach aftermath.

Keywords: Regular semi group, Nilpotency, Identity, Commutativity, Zero-Symmetricity, P_4 property, Completely Semi Prime, Regularity.

I. INTRODUCTION

The concept of semi group is simple and plays an important role in the extensive development of Mathematics. The theory of Semi group is similar to that of Group theory and ring theory. Throughout this paper G stands for a Regative Semi Group with at least two elements and 0 denotes the additive identity and 1 being the multiplicative identity. The ideology and constructive structures of a Semi group is used in formulation of the results.

II. PRELIMINARIES

Definition 2.1

A *semi group* is a non-empty set G with two binary operations "+" and "." such that it is closed and associative with respect to the binary operations.

Definition 2.2

An element a $\in$ G is said to be an *idempotent* element if $a^2 = a$ and the set of all idempotents is denoted by E.

Definition 2.3

An element from the semi near ring is said to be *regular* if for all a $\in$ G there exists x belonging to G such that ax = axa.

Definition 2.4

G is said to be *quasi weak commutative* if efg = feg for all e, f, g $\in$ G.

Definition 2.5

An element a $\in$ G is said to be *nilpotent* if $a^k = 0$, for some least positive integer k.

Definition 2.6

An element e $\in$ G is said to be the *idempotent* element if ae = ea = a.

Definition 2.7

G is said to be *commutative* if ab = ba for all a, b in G.

Definition 2.8

The semi near ring G is said to be *zero symmetric* if g0 = 0 for all g $\in$ G.

Definition 2.9

The function g: G $\rightarrow$ G' is said to be a *homomorphism* if:

- g(a+b) = g(a) + g(b)
- g(ab) = g(a) g(b)

Definition 2.10

A semi group G is said to possess the property P_4 if ab $\in$ G implies, ba $\in$ G.

Definition 2.11

A semi group G is said to be *completely semi prime* if $a^2 \in I$ implies, a $\in$ I.

III. MAIN RESULTS

Theorem 3.1

Any quasi-weak commutative semi group with:

(i) Right identity is weak commutative.

(ii) Left identity is commutative.

Proof:

(i) Let G be the quasi weak commutative semi group.

Let e be the right identity in G.

Since G is quasi weak commutative, we have, abc = bac

abc = (abc)e

= a(bce)

= a(cbe)

= (acb)e

= acb

Thus, G is weak commutative.

(ii) Let G be the quasi weak commutative semi group and e be the left identity in G.

Since G is quasi weak commutative,

we have, abc = bac

Also, ab = (ab)e

= bae

= ba

Thus, G is commutative in general.

Theorem 3.2

In a regular quasi weak commutative semi group, bc = abc.

Proof

Let G be the regular quasi weak commutative semi group.

Then, bac = (ab)c

= (aba)c

= a(bac)

= a(abc)

= a^2bc

Also, bac = a(abc)

= a(abac)

= a^2bac

Thus, bac = a^2bac.

Now, $a^2bc = a^2bac$

Implies, bc = (ba)c

= abc

Hence, bc = abc and this completes the proof.

Theorem 3.3

A regular idempotent quasi weak commutative semi group is commutative in general.

Proof

Let G be the regular idempotent quasi weak commutative semi group.

Then, ab = aba

= baa

= ba^2

Thus, ab = ba and this shows that G is commutative.

IV. REGATIVE SEMI GROUP

On constituting the above proofs, a substructure of the semi group, *The Regative Semi Group* is delineated as follows and thereby G denotes a Regative Semi group hereafter in this paper.

Definition 4.1

A semi group G is said to be *Regative* if aba = ab and ab = ba for all a, b, c in G.

Theorem 4.2

Every Regative Semi group is zero-symmetric.

Proof

Let G be the Regative Semi group.

For every a in G, a.0 = a(00) = (a0)0 = (0a)0 = 0.

Hence, G is zero-symmetric and this completes the proof.

Theorem 4.3

Homomorphism preserves the structure of a Regative Semi group.

Proof

Let G and G' be two Regative Semi groups.

Let g: G → G' be the homomorphism.

For all a, b ∈ G,

f(a) f(b) = f(ab)

= ab = ba

= f(ba)

= f(b) f(a)

Also, f(a) f(b) = f(ab)

= ab = aba

= f(aba)

= f(a) f(b) f(a)

Thus, homomorphic image of a Regative Semi group is also so.

Postulate 4.4

In a quasi weak commutative Regative Semi group, abc = a^nbc

Proof

Let G be the quasi weak commutative Regative Semi group.

Then, aba = ab and ab = ba ⇒ ba = aba

Also, abc = bac

⇒ abc = abac

= aab.c

= a^2bc

Thus, abc = a^2bc

⇒ abc = aabc

= a.bac

= a.aba.c

= a.a^2bc

= a^3bc

Thus, abc = a^3bc

Proceeding this way, we get, abc = a^nbc

This completes the proof.

Theorem 4.5

Every ideal of a Regative Semi group is completely semi prime.

Proof

Let I be an ideal of the Regative Semi group G.

Let $a^2 \in$ I.

G being Regative implies, ab = aba for all b $\in$ G

Then, ab = baa = $ba^2 \in GI \subseteq$ I.

Thus, $a^2 \in I \Rightarrow a \in$ I.

Hence, G is completely semi prime and this completes the proof.

Theorem 4.6

Regative Semi Group G possesses the property P_4.

Proof

Let G be the Regative Semi group.

Consider ab $\in$ I.

$(ba)^2$ = (ba)(ba)

= b(ab)a

$\in GIG \subseteq$ I

$\Rightarrow (ba)^2 \in$ I

Theorem 4.5 gives, ba $\in$ I.

Hence, ab $\in$ I $\Rightarrow$ ba $\in$ I

G possesses the property P_4.

This completes the proof.

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