A STUDY ON Comparision of NS- Method with MODI and VAM Method

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ABSTRACT:

In this paper, an advanced method named NS- method is proposed for finding an optimal solution for a wide range of transportation problems directly.

Key words:

Operations research , Linear Programming, Transportation problem, NS method

OPERATIONS RESEARCH

The activity "Operations Research" (abbreviated as O.R.) has become increasingly important in the face of fast moving technology and increasing complexities in business and industry. This chapter starts with the development of Operations Research, properties of Linear Programming Problems (LPP) and the special class of linear programming problems – Transportation Problem.

Operations research is a scientific approach to problem solving for executive decision making which requires the formulation of mathematical, economic and statistical models for decision and control problems to deal with situations arising out of risk and uncertainty. According to Operations Research Society of America (ORSA), it is a tool which is concerned with the design and operation of the man-machine system scientifically, usually under conditions requiring the optimum allocation of limited sources. As per the Operations Research Society of Great Britain, operations research is the application of the scientific methods to complex problems arising in the direction and management of large systems of men, machines, materialsand money in industry, business and government.

The term Operational Research, was first coined in 1940 by McClosky and Trefthen in a small town, Bowdsey, of the United Kingdom. This new science came into existence as a result of research on military operations during World War II. During the war there were strategic and tactical problems which were generally complicated, to expect adequate solutions from individuals or specialists was unrealistic. Therefore, the military management called on scientists from various disciplines and organized them into teams to assist in solving strategic and tactical problems, i.e., to discuss, evolve and suggest ways and means to improve the suggested certain approaches that showed remarkable results. This new approach to systematic and scientific study of the operations of the system was called the Operations Research or Operational Research (abbreviated as O.R.).

LINEAR PROGRAMMING

Linear programming is a mathematical programming technique to optimize performance (e.g. profit or cost) under a set of resource constraints (e.g. machine- hours, man-hours, money, materials, etc.) as specified by an organization. This involves the allocation of limited resources in an optimal manner, on the basis of a given criterion of optimality. Programming is just another word for 'planning' and refers to the process of determining a particular plan of action from amongst several alternatives.

Linear Programming Problem (LPP) consist of three components, namely

- The decision variables (activities)
- The objective (goal)
- The constraints (restrictions).

The four basic assumptions necessary for all linear programming problems are:

a) Certainty – means that all the coefficients in the objective function as well as in the constraints are completely known with certainty and do not change during the period of study.

b) Divisibility (or continuity) – implies that solution values of the decision variables and resources can take on any negative-values, including fractional values of the decision variables.

c) Proportionality – This requires the contribution of each decision variable in both the objective function and the constraints to be directly proportional to the value of the variable.

d) Additivity – means that the total output for a given combination of activity levels is the algebraic sum of the output of each individual process.

The properties of linear programming solutions are:

- Feasible solution
- Optimal solution
- Alternate optimum solution
- Unbounded solution
- Infeasible solution
- Degenerate solution

TRANSPORTATION PROBLEM

The Transportation Problem is one of the subclasses of Linear Programming Problem. It is one of the most important applications of linear programming problem which deals with the distribution of single commodity from various sources of supply to various destination of demand in such a manner that the total transportation cost is minimized. To achieve this objective, we must know the amount and location of available supplies and the quantities demanded. In addition, we must know the costs that result from transporting one unit of commodity from various origins to various destinations. Each source has a limited supply (i.e.maximum number of products that can be sent from it) while each destination has a demand to be satisfied (i.e. minimum number of products that need to be shipped to it). Transportation problems whose main objective is to minimize the cost of shipping goods are called minimizing. An alternative objective is to maximize the profit of shipping goods, in which case the problems are called maximizing.

Table 1 Examples of Transportation Problem

Source	Destination	Commodity	Objective
Plants	Markets	Finished goods	Minimizing total cost of shipping
Plants	Finished goods warehouses	Finished goods	Minimizing total cost of shipping
Finishes goods warehouses	Markets	Finished goods	Minimizing total cost of shipping
Suppliers	Plants	Raw materials	Minimizing total cost of shipping
Suppliers	Raw material warehouses	Raw materials	Minimizing total cost of shipping
Raw material warehouses	Plants	Raw materials	Minimizing total cost of shipping

Basic notations:

Let

m be the number of sources

n be the number of destinations

a_i be the supply at the source i

b_j be the demand at the destination j

c_{ij} be the cost of transportation per unit from source I to the destination j

X_{ij} be the number of units to be transported from the source I to the destination j.

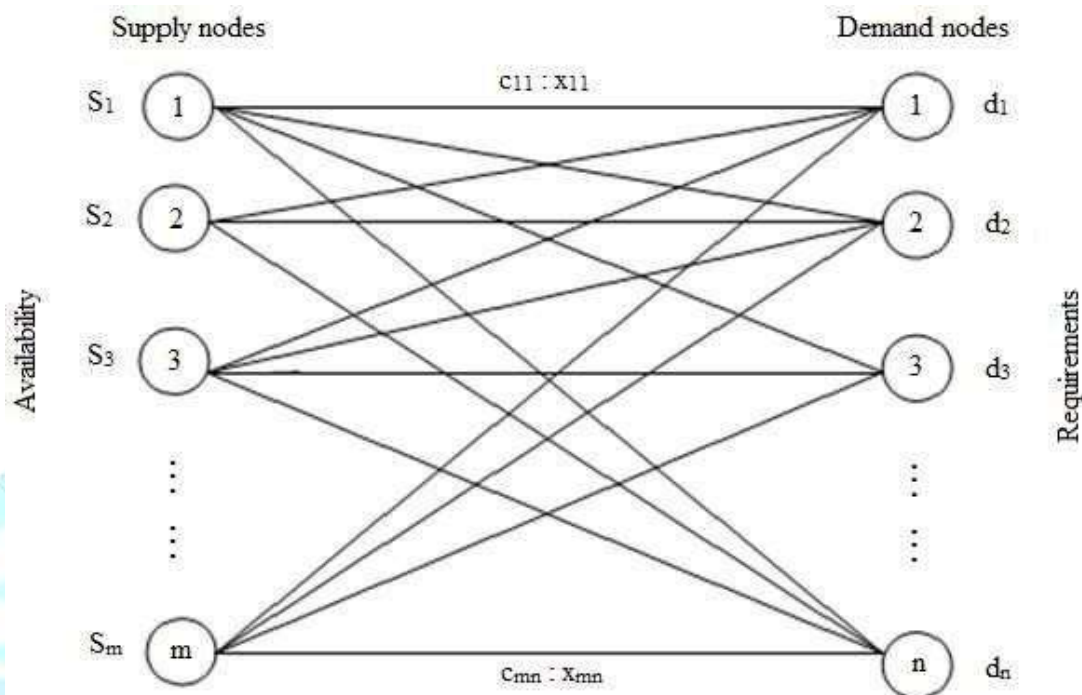A schematic representation of transportation problem is shown in Figure 1:

Figure 1 Schematic diagram of simple transportation problem.

In 1941, Hitchcook developed the basic transportation problem and later in 1949 Koopmans discussed the problem in detail. Again in 1951, the transportation problem has been formulated as linear programming problem by Dantzig. Later in 1954, Charnes and Cooper developed Stepping stone method for solving a large-scale transportation problem.

It is required to solve a transportation problem in two stages so as to obtain optimal solution. In the first stage, we obtain initial basic feasible solutions by methods like 'North West Corner", "Matrix Minima", "Column Minima" and "Vogel's Approximation Method" etc. in the next stage, an optimal solution to the transportation problem is obtained using ModiMethod.

This paper presents a new simple approach to solve the transportation problem. The proposed method helps to directly get the optimal solution with lesser number of iterations and very easy computations.

EXISTENCE OF SOLUTION IN TRANSPORTATION PROBLEM

EXISTENCE OF FEASIBLE SOLUTION:

Theorem

A necessary and sufficient condition for the existence of a feasible solution to the general transportation problem is that

Total Supply = Total Demand

i.e.,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \lambda(say)$$

Proof

The condition is necessary. Let there exist a feasible solution to the transportation problem. Then, we have

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} a_i$$

and

$$\sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} b_j$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \lambda(say)$$

Sufficiency:

We assert that there exists a feasible solution given by $x_{ij} = a_i b_j / \lambda$ for all i and j.

Clearly, $x_{ij} \geq 0$ since $a_i > 0, b_j > 0$ for all i and j.

Also,

$$\sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n} (a_i \, b_j \, / \lambda) = \frac{a_i}{\lambda} \sum_{j=1}^{n} b_j = a_i \, , \qquad for \; i = 1,2,\dots,m$$

and

$$\sum_{i=1}^{m} x_{ij} = \sum_{i=1}^{m} (a_i \, b_j \, / \lambda) = \frac{b_j}{\lambda} \sum_{i=1}^{m} a_i = b_j \, , \qquad for \; j = 1,2,\dots,n$$

Thus, x_{ij} satisfies all the constraints of the transportation problem and hence is a feasible solution.

EXISTENCE OF AN OPTIMUM SOLUTION:

Corollary

There always exists an optimum solution to a transportation problem.

Proof

Let

$$\sum_{i} a_i = \sum_{j} b_j,$$

so that a feasible solution x_{ij} exists. It follows from the constraints of the problem that each x_{ij} is bounded, viz.,

$$0 \leq x_{ij} \leq \text{min. } (a_i, b_j)$$

Thus, the feasible region of the problem is closed, bounded and non- empty and hence there exists an optimum solution.

BASIC FEASIBLE SOLUTION

Theorem

The number of basic (decision) variables of the general transportation problem at any stage of feasible solution must be m+n-1.

Proof

Consider the m+n constraints of the transportation problem:

$$\sum_{i=1}^{m} x_{ij} = b_j \quad and \quad \sum_{j=1}^{n} x_{ij} = a_i, \qquad (j = 1,2\ldots,n; i = 1,2,\ldots,m)$$

Taking summation on both sides, these yield

$$\sum_{j=1}^{n-1}\sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n-1} b_j \quad and \quad \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} a_i$$

Subtracting the two, we have

$$\sum_{j=1}^{n-1}\sum_{i=1}^{m} x_{ij} - \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n-1} b_j - \sum_{i=1}^{m} a_i$$

$$\sum_{i=1}^{m}\left[\sum_{j=1}^{n-1} x_{ij} - \sum_{j=1}^{n} x_{ij}\right] = \sum_{j=1}^{n-1} b_j - \sum_{j=1}^{n} b_j \qquad \left(\because \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j\right)$$

$$\sum_{i=1}^{m} x_{in} = b_n$$

which happens to be the last constraint of the problem.

This indicates that if the first m+n-1 constraints are satisfied, then $\sum_i a = \sum_j b$, ensures that the (m + n)th constraint will be automatically satisfied.

Thus, out of m+n equations, we have only (m+n-1) linearly independent equations. Therefore, a basic feasible solution will consist of at most (m+n-1) positive variable, the rest being zero.

Further, a feasible solution involving exactly (m+n-1) positive variables is known as non-degenerate basic feasible solution, otherwise it is said to be degenerate basic feasible.

FINDING THE INITIAL BASIC FEASIBLE SOLUTION

The solution of transportation problem can be obtained in two stages, namely initial solution and optimum solution.

Initial solution can be obtained by any one of the three methods:

1. North- West Corner Method (NWCM)
2. Least Cost Method (LCM) or Matrix Minima Method
3. Vogel's Approximation Method (VAM) or Penalty Method

VAM is preferred over the other two methods, since the initial basic feasible solution by this method is either optimal or very close to the optimal solution.

The three techniques mentioned above are in the increasing order of their solution accuracy. The cost of the initial basic feasible solution through VAM will be least among all the three techniques.

The improved solution of the initial basic feasible solution is called optimal solution which is the second stage of solution, that can be obtained by MODI Method (Modified Distribution Method).

Algorithm for each of the three techniques for finding the initial basic feasible solution is given below.

Algorithm for North- West Corner Method

Step 1: Find the minimum of the supply and demand values with respect to the current north-west corner cell of the matrix.

Step 2: Allocate this minimum value to the current northwest corner cell and subtract this minimum from the supply and demand values with respect to the current northwest corner cell.

Step 3: Check whether exactly one of the row/ column corresponding to the northwest corner cell has zero supply/demand, respectively. If so, go to step 4 otherwise go to step 5.

Step 4: Delete that row/ column with respect to the current northwest corner cell which has the zero supply/ demand and go to step 6.

Step 5: Delete both the row and the column with respect to the current northwest corner cell.

Step 6: Check whether exactly one row or column is left out. If yes, go to step 7 otherwise go to step 1.

Step 7: Match the supply/ demand of that row/ column with the remaining demands/ supplies of the undeleted columns/ rows.

Step 8: Go to phase 2.

Algorithm for Least Cost Method

Step 1: Find the minimum of the (undeleted) values in the cost matrix (i.e., find the matrix minimum).

Step 2: Find the minimum of the supply and demand values (X) with respect to the cell corresponding to the matrix minimum.

Step 3: Allocate X units to the cell with the matrix minimum. Also, subtract X units from the supply and the demand values corresponding to the cell with the matrix minimum.

Step 4: Check whether exactly one of the row/ column corresponding to the cell with the matrix minimum has zero supply/ zero demand, respectively. If yes, go to step 5 otherwise, go to step 6.

Step 5: Delete that row/ column with respect to the cell with the matrix minimum which has the zero supply/ zero demand and go to step 7.

Step 6: Delete both the row and the column with respect to the cell with the matrix minimum.

Step 7: Check whether exactly one row or column is left out. If yes, go to step 8 otherwise, go to step 1.

Step 8: Match the supply/demand of that row/column with the remaining demands/supplies of the undeleted columns/rows.

Step 9: Go to phase 2.

Algorithm for Vogel's Approximation Method

Step 1: Find row penalties, i.e., the difference between the first minimum and the second minimum in each row. If the two minimum values are equal, then the row penalty is zero.

Step 2: Find column penalties, i.e., the difference between the first minimum and the second minimum in each column. If the two minimum values are equal, then the column penalty is zero.

Step 3: Find the maximum amongst the row penalties and column penalties and identify whether it occurs in a row or in a column (break tie randomly). If the maximum penalty is in a row, go to step 4 otherwise, go to step 7.

Step 4: Identify the cell for allocation which has the least cost in that row.

Step 5: Find the minimum of the supply and demand values with respect to the selected cell.

Step 6: Allocate this minimum value to that cell and subtract this minimum from the supply and demand values with respect to the selected cell and go to step 10.

Step 7: Identify the cell for allocation which has the least cost in that column.

Step 8: Find the minimum of the supply and demand values with respect to the selected cell.

Step 9: Allocate this minimum value to the selected cell and subtract this minimum from the supply and demand values with respect to the selected cell.

Step 10: Check whether exactly one of the row/ column corresponding to the selected cell has zero supply/demand, respectively. If yes, go to step 11; otherwise go to step 12.

Step 11: Delete the row/column which has the zero supply/zero demand and revise the corresponding row/column penalties. Then, go to step 13.

Step 12: Delete both the row and the column with respect to the selected cell. Then, revise the row and the column penalties.

Step 13: Check whether exactly one row/column is left out. If yes, go to step 14, otherwise, go to step 3.

Step 14: Match the supply/demand of the left-out row/column with the remaining demands/supplies of the undeleted rows/columns.

Step 15: Go to phase 2.

OPTIMALITY TEST

Once the initial basic feasible solution has been computed, the next step in the problem is to determine whether the solution is optimum or not.

Optimality test can be conducted to any initial basic feasible solution of a transportation problem provided such allocations has exactly m+n-1 non- negative allocations, where m is the number of origins and n is the number of destinations. Also, these allocations must be in independent positions.

To perform this optimality test, we use the Modified Distribution Method (MODI).Steps

involved in the MODI method for performing optimality is given below:

Step 1: For an initial basic feasible solution with m + n -1 occupied cells, calculate u_i and v_j for rows and columns. The initial solution can be obtained by any of the three methods mentioned above.

To start with, any one of u_i 's or v_j 's is assigned the value zero. It is better to assign zero for a particular u_i or v_j where there are maximum number of allocation in a row or column respectively, as it will reduce arithmetic work considerably.

Then complete the calculation of u_i 's and v_j 's for other rows and columns by using the relation $c_{ij} = u_i + v_j$ for all occupied cells (i, j).

Step 2: For unoccupied cells, calculate opportunity cost (the difference that indicates the per unit cost reduction that can be achieved by an allocation in the unoccupied cell) by using the relationship $d_{ij} = c_{ij} - (u_i + v_j)$ for all i and j.

Step 3: Examine sign of each d_{ij}

i) If $d_{ij} > 0$, then current basic feasible solution is optimal.

ii) If $d_{ij} = 0$, then current basic feasible solution will remain unaffected but an alternative solution exists.

iii) If one or more $d_{ij} < 0$, then an improved solution can be obtained by entering unoccupiedcell (i, j) in the basis.

An occupied cell having the largest negative value of d_{ij} is chosen for entering into the solution mix (new transportation schedule).

Step 4: Construct a closed-path (or loop) for the unoccupied cell with largest negative opportunity cost. Start the closed path with the selected unoccupied cell and mark a plus sign (+) in this cell, trace a path along the rows (or column) to an occupied cell, mark the corner with minus sign (-) and continue down the column (or row) to an occupied cell andmark the corner

with plus (+) sign and minus sign (-) alternatively. Close the path back to the selected unoccupied cell.

Step 5: Select the smallest quantity amongst the cell marked with minus sign on the corners of closed loop. Allocate this value to the selected unoccupied cell and add it to other occupied cells marked with plus sign and subtract it from the occupied cells marked with minus sign.

Step 6: Obtain a new improved solution by allocating units to the unoccupied cell according to Step 5 and calculate the new total transportation cost.

Step 7: Test the revised solution further for optimality. The procedure terminates when all $d_{ij} \geq 0$, for unoccupied cells.

NS- METHOD

NS- method is proposed for finding an optimal solution for a wide range of transportation problems directly. However, this method gives either optimal solution or near to optimal solution.

PROCEDURE OF NS- METHOD

Step 1: Construct the transportation matrix of order mxn for given problem.

Step 2: Select minimum cost from all costs in the matrix. If maximum cost is 0 or 1 then allocate minimum of supply/demand to the cell with cost 0 and then 1.

Step 3: Select minimum even cost from all costs in the matrix.

Step 4: Subtract selected even cost only from even costs in the matrix. Now there will be at least one zero and remaining all costs become odd.

Step 5: Allocate the minimum of supply/demand to the cell with 0 cost in the matrix. If there is more than one cell with 0 cost, then select the cell having maximum sum of costs in corresponding row or column and allocate the minimum of supply/demand to that cell.

Step 6: Repeat steps 3,4 and 5 for remaining supplies and demands till m+n-1 cells are allocated.

Step 7: Finally, the total minimum cost is calculated as

$$\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \, x_{ij}$$

NUMERICAL EXAMPLES

The proposed method is illustrated by the following transportation problems.

Example 1:

Consider the following transportation problem having three origins and five destinations.

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	4	1	2	4	4	60
O_2	2	3	2	2	3	35
O_3	3	5	2	4	4	40
Demand	22	45	20	18	30	135

Solution:

Since,

$$\sum_i a_i = \sum_j b_j = 135$$

there exists a feasible solution to the transportation problem.

North-West Corner Rule

Step 1: Create a transportation table for the given transportation problem.

	D_1	D_2	D_3	D_4	D_5	
O_1	4	1	2	4	4	60
O_2	2	3	2	2	3	35
O_3	3	5	2	4	4	40
	22	45	20	18	30	135

Step 2: Allocate the minimum supply value or demand value to the left most corner cell (1,1).Here,

minimum value is the demand value= 22.

4	1	2	4	4	
22					60 38
2	3	2	2	3	
---					35
3	5	2	4	4	
---					40
22 45	20	18	30		

Here the left most corner cell is allocated with minimum demand value 22 and thus the remaining

cells in the column gets unallocated. The corresponding supply value becomes 38 (i.e., 60-22= 38).

Step 3: Now allocate the next left most corner cell (1,2) with corresponding minimum supply value

or demand value.

Here the minimum value is the supply value= 38.

4	1	2	4	4	
22	38	---	---	---	38
	3	2	2	3	
					35
	5	2	4	4	
					40
--	45 7	20	18	30	

Here the 2nd cell from 1st row is allocated. Since the value is totally allocated, the remainingcells

in the row gets unallocated. The demand value then becomes 7.

Step 4: Next to consider the left most corner cell in 2nd row and 2nd column (i.e., 2,2). Then,

allocate the cell with the corresponding minimum demand value or supply value.

The minimum value is the demand value= 7.

4 22	1 38			--
	3 7	2	2	3
	5 ---	2	4	4

35 28

40

-- 7 20 18 30

The 2nd cell in second column is allocated and the remaining cell in the column gets unallocated. The corresponding supply value becomes 28.

Step 5: Consider the next left most corner cell (2,3) and then allocate the correspondingminimum supply value or demand value to that cell.

Here the minimum value is the demand value= 20.

4 22	1 38			--
	3 7	2 20	2	3
		2 ---	4	4

28 8

40

-- -- 20 18 30

The remaining cell in the 3rd column gets unallocated and the supply value becomes 8.

Step 6: Allocate the minimum supply value or demand value to the next left most corner cell(2,4).

The minimum value here is the supply value= 8

4 22	1 38				--
	3 7	2 20	2 8	3 ---	8
			4	4	40
--	--	--	18	30	

10

The remaining cell in the 2ⁿᵈ row gets unallocated. Now, only 2 cells are remaining from the 3ʳᵈ row.

Step 7: Again, choose the leftmost corner cell(3,4) and then allocate the corresponding minimum supply value or demand value.

Here the minimum value is the demand value= 10.

4 22	1 38				--
	3 7	2 20	2 8		--
			4 10	4	40
--	--	--	10	30	

Thus, we have only one unallocated cell.

Step 8: Now allocate the value (30) to the final cell (3,5).

4 22	1 38				--
	3 7	2 20	2 8		--
			4 10	4 30	30
--	--	--	--	30	

The final transportation table is:

4 22	1 38	⧅	⧅	⧅
⧅	3 7	2 20	2 8	⧅
⧅	⧅	⧅	4 10	4 30

The transportation cost is: (4x22) + (1x38) + (3x7) + (2x20) + (2x8) + (4x10) + (4x30)

$$= 88+38+21+40+16+40+120$$

$$= 363$$

Least Cost Method

Step 1: Determine the minimum cost from the transportation table and then allocate the corresponding supply value or demand value.

The minimum cost is 1 and the minimum value is the demand value= 45

4	1 45	2	4	4	
2	3 ---	2	2	3	60
3	5 ---	2	4	4	35
					40
22	45	20	18	30	

The cell (1,2) is allocated and the remaining cell in the column is unallocated. The corresponding supply value becomes 15.

Step 2: Now choose the next minimum cost =2 and then allocate the corresponding minimum supply value or demand value. If there exists more than one, then choose either the row or column.

The minimum value is supply value= 15

4 ---	1 45	2 15	4 ---	4 ---	15
2		2	2	3	35
3		2	4	4	40
22	--	20	18	30	

The cell (1,3) is now allocated leaving the other cells in the row unallocated. The corresponding demand value becomes 5

Step 3: Determine the next minimum cost. Here the cost= 2 in cell (2,1). Then, allocate the corresponding minimum supply value or demand value.

The supply value is minimum= 22.

1 45	2 15			--
2 22	2	2	3	35 13
3 ---	2	4	4	40
22	--	5	18	30

The cell (2,1) is now allocated with 22 and the remaining cell in the 1st column is unallocated.

The supply value changes to 13.

Step 4: Consider the next minimum cost, again 2, and then allocate the corresponding minimum supply value or demand value.

Let the cell we consider be (2,3) and the minimum value is demand value= 5.

	1	2			--
	45	15			
2		2	2	3	13̶ 8
22		5			
		2	4	4	40

--	--	5̶	18	30	

5 is allocated to the cell (2,3) and the remaining in the particular column is unallocated.

The supply value then becomes 8.

Step 5: Next choose the minimum cost=2 from the table and allocate the corresponding minimum supply value= 8.

	1	2			--
	45	15			
2		2	2	3	8̶
22		5	8	---	
			4	4	40
--	--	--	18̶	30	
			10		

The cell (2,4) is allocated with the minimum value 8 leaving the remaining cell unallocated. The corresponding demand value changes to 10.

Step 6: The only cost left is 4. Choose the cell (3,4) and allocate the corresponding minimum demand value= 10.

	1 45	2 15			--
2 22		2 5	2 8		--
			4 10	4	40 30
--	--	--	1	30	

The cell (3,4) gets allocated with value 10.

Step 7: The last cell is to be allocated with the value 30.

	1 45	2 15			--
2 22		2 5	2 8		--
			4 10	4 30	30
--	--	--	--	30	

Thus, the final transportation table is:

	1	2		
	45	15		
2		2	2	
22		5	8	
			4	4
			10	30

The transportation cost= (1x45) +(2x15) +(2x22) +(2x5) +(2x8) +(4x10) +(4x30)

$$= 45+ 30+ 44+ 10+ 16+ 40+ 120$$

$$= 305$$

Vogel's Approximation Method

Iteration 1: Find the penalties for all the rows and columns. Then choose the maximum penalty value and then determine the minimum cost in that particular column or row. Then, allocate the corresponding minimum supply or demand value to that chosen cell.

Here the maximum penalty value we get is 2 and the corresponding minimum cost=1 with demand value= 45.

4	1	2	4	4	60̶ 15 1
	45				
2	3	2	2	3	35 0

3	5	2	4	4	40 1

1	2	0	2	1	

| 22 | 4̶5̶ | 20 | 18 | 30 |

The cell (1,2) is allocated with the corresponding demand value= 45 and the remaining cells in that particular row becomes unallocated. The supply value changes to 15.

Iteration 2: Repeat step 1.

					new
4	1	2	4	4	15 1 2
	45		---		
2	╲	2	2	3	3̶5̶ 17 0 0
			18		
3	╲	2	4	4	40 1 1

1	2	0	2	1	

new 1 | -- | 0 | 2 | 1

| 22 | - | 20 | 1̶8̶ | 30 |

Here the maximum penalty is again 2 and the corresponding cell (2,4) is allocated leaving the other cells in the column unallocated.

Iteration 3: Repeat step1.

The maximum penalty becomes 2 (column). The cell (1,3) gets allocated with the minimum value 15.

new

4	1	2		4			
---	45	15		---	15	2	2
2		2	2	3			
			18		17	0	0
3		2		4			
					40	1	1
1	--	0	2	1			

new 1   --   0   1

22   --   20 5   30

The cell (1,3) is allocated with value 15 and the remaining cells in the row are unallocated.

Iteration 4: Repeat step 1.

The maximum penalty becomes 1(row). So, the cell (2,1) gets allocated.

new

	1	2					
	45	15		--	--	--	
2		2	2	3			
17		---	18	---	17	0	0
3		2		4			
					40	1	1
1	--	0	--	1			

new 1   --   0   --   1

22 5   --   5   --   30

Now, the cell (2,1) is allocated the remaining cells are left unallocated.

Iteration 5: Repeat step 1.

Here, the cell (3,1) gets allocated with supply value 5, cell(3,3) with supply value 5 and cell (3,5) with 30.

	1 / 45	2 / 15			--
2 / 17			2 / 18		--
3 / 5		2 / 5		4 / 30	4
5	--	5	--	30	

Thus, the final transportation table is:

	1 / 45	2 / 15		
2			2 / 18	
	17			
3 / 5		2 / 5		4 / 30

The transportation cost= (1x45) + (2x15) + (2x17) + (2x18) + (3x5) + (2x5) + (4x30)

$$= 45 + 30 + 34 + 36 + 15 + 10 + 120$$

$$= 290$$

MODI Method

Step 1: Find a feasible solution to the transportation problem using any of the methods given above.

Here, choosing the solution by Vogel's Approximation Method.

So, the final transportation table for the VAM is used here.

	1	2		
	45	15		
2			2	
17			18	
3		2		4
5		5		30

Step 2: Check whether there is m+n-1 allocations in the final feasible solution of VAM.

m+n-1= 3+5-1=7, where m is the number of rows and n is the number of columns.

The condition is satisfied.

Step 3: Now, introducing variables u and v such that

$$c_{ij}= u_{ij}+v_{ij}. \text{---------------------------- (A)}$$

Here, u_1, u_2, u_3 are introduced row wise and v_1, v_2, v_3, v_4, v_5 are introduced column wise.

By default, $u_1= 0$.

	v_1	v_2	v_3	v_4	v_5
$u_1= 0$	4	1 / 45	2 / 15	4	4
u_2	2 / 17	3	2	2 / 18	3
u_3	3 / 5	5	2 / 5	4	4 / 30

Step 4: Next, to find the values for u_2, u_3, v_1, v_2, v_3, v_4 and v_5.

We have from (A), $v_{ij}= c_{ij} - u_{ij}$ and $u_{ij}= c_{ij}- v_{ij}$.

Since, u_1 and c_{13} are known, we have $v_3 = c_{13} - u_1 = 2 - 0 = 2$.

Following in the same manner with known values, we find the unknown values.

$u_3 = c_{33} - v_3 = 2 - 2 = 0,$ $u_3 = 0$

$v_2 = c_{12} - u_1 = 1 - 0 = 1,$ $v_2 = 1$

$v_5 = c_{35} - u_3 = 4 - 0 = 4,$ $v_5 = 4$

$v_1 = c_{31} - u_3 = 3 - 0 = 3,$ $v_1 = 3$

$u_2 = c_{21} - v_1 = 2 - 3 = -1,$ $u_2 = -1$

$v_4 = c_{24} - u_2 = 2 - (-1) = 2 + 1 = 3,$ $v_4 = 3$

Step 5: Now we find some values for the unallocated cells using $c_{ij} - (u_{ij} + v_{ij})$.

	$v_1 = 3$	$v_2 = 1$	$v_3 = 2$	$v_4 = 3$	$v_5 = 4$
$u_1 = 0$	4 (1)	1 45	2 15	4 (1)	4 (0)
$u_2 = -1$	2 17	3 (3)	2 (1)	2 18	3 (0)
$u_3 = 0$	3 5	5 (4)	2 5	4 (1)	4 30

The values of all unallocated cells are positive and thus the iteration ends here. So,

the optimal solution is same as that of Vogel's Approximation Method= 290.NS-

Method

Step 1: Construct a transportation table for the given transportation problem. Then selectminimum cost from all costs and allocate the minimum supply or demand to the cell.

Here, minimum cost is 1 and the corresponding minimum value is the demand value= 45.

4	1	2	4	4	60̶ 15
	45				
2	3	2	2	3	35

3	5	2	4	4	40

22	4̶5̶	20	18	30	

Step 2: Choose the minimum even cost and then subtract it from all the even costs.

Here, the minimum even cost= 2.

2	1	0	2	2	15
	45				
0		0	0	3	35
3		0	2	2	40
22	--	20	18	30	

Step 3: Allocate the minimum of supply or demand to the cell with 0 cost.

Here, we have more than one cell with 0 cost, so we find the maximum sum of costs in corresponding row or column and then allocate the minimum supply or demand.

The first row has only one 0 and thus minimum supply is allocated to cell (1,3).

2	1	0	2	2	
---	45	15	---	---	15
0		0	0	3	
					35
3		0	2	2	
					40
22	--	20 5	18	30	

Step 4: Repeat step 2.

Consider the 5th column and allocate the minimum demand value to cell (3,5)

Consider the 4th column and allocate the minimum demand value to cell (2,4)

	1	0			
	45	15			--
0		0	0	3	
			18	---	35 17
3		0	0	0	
			---	30	40 10
22	--	5	18	30	

Step 5: Next, choose the cell (2,1) and allocate minimum value= 17.

Allocate minimum value= 5 in cell (3,3)

	1	0			--
		45	15		
0		0	0		1̶7̶
17		---	18		
3		0		0	1̶0̶ 5
			5		30
2̶2̶ 5	--	5̶	--	--	

Step 6: Finally, allocate the value 5 to cell (3,1).

	1	0			--
		45	15		
0			0		--
17			18		
3		0		0	5̶
	5		5		30
5̶	--	--	--	--	

The final transportation table is:

	1	0		
		45	15	
0			0	
17			18	
3		0		0
	5		5		30

Thus, the transportation cost = (1x45) + (2x15) + (2x17) + (2x18) + (3x5) + (2x5) + (4x30)

$$= 45 + 30 + 34 + 36 + 15 + 10 + 120$$

$$= 290$$

Example 2

Consider the following transportation problem having three origins and four destinations.

	D_1	D_2	D_3	D_4	Supply
O_1	5	2	4	3	22
O_2	4	8	1	6	15
O_3	4	6	7	5	8
Demand	7	12	17	9	45

Solution:

Since,

$$\sum_i a_i = \sum_j b_j = 135$$

there exists a feasible solution to the transportation problem.

Northwest corner rule

Follow the steps as in Example 1.

We have the final transportation table as:

5	2	4		
7	12	3		22 5 3
		1	6	
		14	1	15
			5	
			8	8
7	12	17 14	9 8	

The transportation cost = 131

<u>Least Cost Method</u>

Follow as in Example 1.

The final transportation table is:

	2	4	3	
	12	1	9	22 10 1
		1		
		15		15
4		7		
7		1		8 1
7	12	17 21	9	

The transportation cost= 105

Vogel's Approximation Method

Steps to be followed as in Example 3.1.

The final transportation table is:

	2	4	3	22 ~~10~~ 8	1	1	1	2
	12	2	8					
		1		~~15~~	3	3	-	-
		15						
4			5	~~8~~ 1	1	1	1	1
7			1					

~~7~~   ~~12~~   ~~17~~ ~~23~~   ~~9~~ 1

0	4	3	2
0	-	3	2
1	-	3	2
1	-	-	2

Transportation cost= 104

MODI Method

Steps to be followed as in Example 3.1.

The final transportation table is:

	$v_1= 2$	$v_2= 2$	$v_3= 4$	$v_4= 3$
$u_1= 0$	5 (3)	2 — 12	4 — 2	3 — 8
$u_2= -3$	4 (5)	8 (9)	1 — 15	6 (6)
$u_3= 2$	4 — 7	6 (2)	7 — 1	5 — 1

The transportation cost= 104

NS- Method

Follow the steps as in Example 3.1.

The final transportation table is:

	0	0	3	
	12	2	8	22 10 1
		1		
		15		15
2				8 1
	7	1		
	7	12	17 21	9

The transportation cost= 104

COMPARISON OF VARIOUS METHODS OF SOLVING TRANSPORTATION PROBLEM

After solving the two examples, here we compare all the methods.

Example	NWCM	LCM	VAM	MODI	NS
Example 1	363	305	290	290	290
Example 2	131	105	104	104	104

CONCLUSION

In this paper, an introduction to Operations Research, Linear Programming and Transportation Problem is specified and the basic definitions are stated along with the theorems and remarks. Also, the predefined methods to find an initial basic feasible solution is explained using algorithms.

Finally, an advanced method for solving transportation problem has been developed. This method is easy and simple to understand. The optimal solution obtained by Vogel's Approximation and Modi method can be obtained easily by the method presented.

A comparison between all the methods are listed in a tabular form from which it is clear that the solution obtained by NS- Method is similar to that of Vogel's Approximation Method and MODI Method.

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