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PROPERTIES OF Gγ_μ OPEN-SEPARATION AXIOMS IN TOPOLOGY

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Abstract: In this paper our main interest is to introduce a new type of generalized open sets defined in terms of an operation on a generalized topological space. We have studied some properties of these newly defined sets. As an application, we have introduced some weak separation axioms and discussed some of their properties. Finally, we have studied some preservation theorems in terms of some irresolute functions.

Keywords- *μ-open set, γ_μ-open set, γ_μ g-closed set.*

I. INTRODUCTION

In 2018, B. Roy [9] introduced the notation of an operation on a topological space and introduced the concept of $\gamma_\mu$open sets. In 2011 B. Roy [8] defined the concept of γ_μg - closed sets of topological space. In 2002 Császár [1] introduced the concept of generalized open sets. We now recall some notions defined in [1]. Let X be a non-empty set. A sub collection $\mu \subseteq$ P(X) (where P(X) denotes the power set of X) is called a generalized topology [1], (briefly, GT) if $\emptyset \in \mu$ and any union of elements of μ belongs to μ. A set X with a GT μ on the set X is called a generalized topological space (briefly, GTS) and is denoted by (X, μ). If for a GTS (X, μ), X $\in \mu$, then (X, μ) is known as a strong GTS. Throughout the paper, we assume that (X, μ) and (Y, λ) are strong GTS's. The elements of μ are called μ −open sets and μ −closed sets are their complements. The μ −closure of a set A$\subseteq$ X is denoted by $c_\mu(A)$ and defined by the smallest μ −closed set containing A which is equivalent to the intersection of all μ −closed sets containing A. We use the symbol $i_\mu(A)$ to mean the μ −interior of A and it is defined as the union of all μ −open sets contained in A i.e., the largest μ −open set contained in a (see [3, 2, 1]). In this paper, using gγ_μ- open sets, we define and study the notions of gγ_μ −T_0, gγ_μ −T_1, gγ_μ −R_0, gγ_μ −R_1 spaces.

II. PRELIMINARIES

Definition 2.1: [9] Let (X, μ) be a GTS. An operation γμ on a generalized topology μ is a mapping from μ to P(X) with G ⊆ γ_μ(A), for each G ∈ μ. This operation is denoted by $\gamma_\mu : \mu \to P(X)$.

Definition 2.2: [9] Let (X, μ) be a GTS and γ_μ be an operation on μ. A subset G of (X, μ) is called γ_μ -open if for each point x of G, there exists a μ-open set U containing x such that γ_μ (U) ⊆ G.

A subset of a GTS (X, μ) is called γ_μ–closed if its complement is γ_μ–open in (X, μ). We shall use the symbol γ_μ to mean the collection of all γ_μ–open sets of the GTS (X, μ).

Definition 2.3: [9] Let (X, μ) be a GTS and $\gamma_\mu : \mu \rightarrow P(X)$ be an operation. It is easy to see that the family of all γ_μ -open sets forms a GT on X. The γ_μ-closure of a set A of X is denoted by C γ_μ (A) and is defined as C γ_μ (A) = ∩ {F: F is a γ_μ - closed set and A ⊆ F}.

Definition 2.4: [8] Let (X, μ) be a GTS and $\gamma_\mu : \mu \rightarrow P(X)$ be an operation. A subset A of X is said to be γ_μ g -closed if C γ_μ (A) ⊆ U, whenever A ⊆ U and U is a γ_μ -open set in (X, μ).

Example 2.1: let X = {1, 2, 3, 4} and μ –open ={∅, X, {1}, {2}, {3},{1,3},{1,2},{2,3},{1,2,3}}. Then (X, μ) is a GTS. Now $\gamma_\mu : \mu \rightarrow P(X)$ defined by $\gamma_\mu (A) = \begin{Bmatrix} A & if\ 1 \in A \\ \{2\} & Otherwise \end{Bmatrix}$

γ_μ (A) open = { ∅, X, {1}, {2}, {3},{1,3},{1,2},{2,3},{1,2,3}}

γ_μ (A) closed = { ∅, X, {2,3,4}, {1,3,4}, {1,2,4}, {2,4}, {3,4}, {1,4}, {4} }

γ_μ g –closed set ={ ∅, X, {1,2,3}}

(γ_μ, β_λ) - IRRESOLUTE FUNCTION:

Throughout the rest of the paper, (X, μ) and (Y, λ) will denote GTS's and $\gamma_\mu : \mu \rightarrow P(X)$ and $\beta_\lambda : \lambda \rightarrow P(Y)$ will denote operations on μ and λ respectively.

Definition 2.5: A function f: (X, μ) →(Y, λ) is said to be (γ_μ , β_λ)-irresolute if for each x ∈ X and each β_λ -open set V containing f(x), there is a γ_μ -open set U containing x such that f(U) ⊆ V.

Example 2.2: let X = {1, 2, 3} and μ = {∅, X, {1}, {2}, {1, 2}.

γ_μ = {∅, X, {1}, {2}, {1, 2}

Y = {1, 2, 3} and λ = {∅, X, {1}, {2}, {1, 2}.

β_λ = {∅, X, {1}, {2}, {1, 2}

$f(A) = \begin{Bmatrix} A & if\ 1 \in A \\ \{2\} & otherwise \end{Bmatrix}$

f(U) = { ∅, X,{1},{2},{1,2}

f(U) ⊆ V

III. PROPERTIES OF γ_μ G –SEPARATION AXIOMS:

Definition 3.1: A space X is called gγ_μ -T_0 if and only if to each pair of distinct points x, y of X, there exists a gγ_μ-open setcontaining one but not the other.

Example 3.1: Let X={1,2,3} and μ={φ,{1},{1,2},{2,3},X}. Then (X, μ) is a GTS. Now $\gamma_\mu : \mu \rightarrow P(X)$ defined by $\gamma_\mu (A) = \begin{Bmatrix} A, if\ 1 \in A \\ \{2,3\}, \ otherwise \end{Bmatrix}$ Is an operation.

γ_μ -0pen set –{φ,{1},{1,2},{2,3},X}

γ_μ -closed set-{φ,{1},{2,3},{3},X}

γ_μ g-closed set -{φ,{1},{2},{1,2},{1,3},{2,3},X}

γ_μ g-open set-{ φ, {1}, {2}, {3}, {1,3}, {2,3}, X}

1	**3**
{1}	{3}
{1}	{2,3}

2	**3**
{2}	{3}
{2}	{1,3}

1	**2**
{1}	{2}
{1}	{2,3}
{1,3}	{2}

Definition 3.2: A space X is said to be gμ -T_0 space if for each pair of distinct points of X there exists a gμ-openset containing one but not the other.

Clearly, every γ_μ -T_0 is gγ_μ -T_0.

Example 3.2: let X = {1,2,3,4} and μ –open = { ∅, X, {1}, {2}, {3}, {4}, {1,2},{1,3}, {1,4}, {2,3}, {2,4}, {3,4},{1,2,3}, {1,2,4}, {2,3,4}, {1,3,4}}

1	**2**
{1}	{2,3}
{1}	{2,4}
{1}	{2}
{1}	{2,3,4}
{1,3}	{2}
{1,4}	{2}
{1,3,4}	{2}
{1,3}	{2,4}
{1,4}	{2,3}

1	**3**
{1}	{2,3}
{1}	{3,4}
{1}	{3}
{1}	{2,3,4}
{1,2}	{3}
{1,4}	{3}
{1,2,4}	{3}
{1,2}	{3,4}
{1,4}	{2,3}

1	**4**
{1}	{4}
{1}	{2,4}
{1}	{3,4}
{1}	{2,3,4}
{1,2}	{4}
{1,3}	{4}
{1,2,3}	{4}
{1,2}	{3,4}
{1,3}	{2,4}

2	**3**
{2}	{3}
{2}	{1,3}
{2}	{3,4}
{2}	{1,3,4}
{1,2}	{3}
{2,4}	{3}
{1,2,4}	{3}
{1,2}	{3,4}
{2,4}	{1,3}

2	4
{2}	{4}
{2}	{3,4}
{2}	{1,4}
{2}	{1,3,4}
{1,2}	{4}
{2,3}	{4}
{1,2,3}	{4}
{1,2}	{3,4}
{2,3}	{1,4}

3	4
{3}	{4}
{3}	{1,4}
{3}	{2,4}
{3}	{1,2,4}
{1,3}	{4}
{2,3}	{4}
{1,2,3}	{4}
{1,3}	{2,4}
{2,3}	{1,4}

Definition 3.3: A generalized γ_μ-closure of set A is denoted by gγ_μCl(A), is the intersection of all gγ_μ-closed sets that contain A.

We characterize gγ_μ -T_0–spaces in the following

Example 3.3: let X = {1, 2, 3, 4} be any topological space.
μ–open = { ∅,X,{1},{2},{3},{4},{1,2},{1,3},{1,4},{2,3},{2,4},{3,4},{1,2,3},{1,2,4},{2,3,4},{1,3,4}}

γμ open={ ∅,X,{1},{2},{3},{4},{1,2},{1,3}, {1,4}, {2,3},{2,4}, {3,4},{1,2,3}, {1,2,4},{2,3,4},{1,3,4}}

γμ closed= { ∅,X,{1},{2},{3},{4}, {1,2},{1,3},{1,4}, {2,3},{2,4},{3,4},{1,2,3},{1,2,4},{2,3,4},{1,3,4}}

gγμCL(A)={∅,X,{1},{2},{3},{4},{1,2},{1,3},{1,4}, {2,3},{2,4},{3,4},{1,2,3},{1,2,4},{2,3,4},{1,3,4}}

Theorem 3.4: If in any topological space X, g $\gamma_\mu$closures of distinct points are distinct then X is g γ_μ -T_0.
Proof: Let x, y ϵ x, x ≠ y imply g γ_μ cl({x}) ≠ g γ_μ cl({y}). Then there exists a point z ϵ x such that z belongs one of two sets, say, g γ_μ cl({y}) but not to g γ_μ cl({x}). If we suppose that zϵ g γ_μ cl({x}), then z ϵ g γ_μ cl({y}) ⊂ z ϵ g γ_μ cl({x}),which is contradiction. So, y ϵ x- g γ_μ cl({x}),where X- g γ_μ cl({x}) is g γ_μ -open set which does not contain x. Shows that X is g γ_μ -T_0.

Next, We give the following

Example 3.4: let X = {1, 2, 3, 4} be any topological space. Let 1, 2 ∈ X and 1≠2

gγμCL(A)={∅,X,{1},{2},{3},{4},{1,2},{1,3},{1,4}, {2,3}, {2,4},{3,4},{1,2,3},{1,2,4},{2,3,4},{1,3,4}}

gγμCL({1}) = {1}
gγμCL({2}) = {2}
gγμCL({1}) ≠ gγμCL({2})
X is gγ_μ-T_0 Space.

Theorem 3.5: A space X is gγ_μ-T_0 if and only if g γ_μ Cl({x}) ≠ g γ_μ Cl({y}) for every pair of distinct points x, y of X.

Proof follows from th.3.4.

Theorem 3.6: Every sub space of a g γ_μ -T_0 space is g γ_μ -T_0 space.

Proof: Let x be a space and (Y, τ^*) be a subspace of X where τ^* is the relative topology of τ on Y. Let x, y be two distinct points of Y. As Y⊂ X, x and y are distinct points X. Since X is a g γ_μ -T_0 space, There exists a gγ_μ-open set G such that x ∈ G but y ∉ G. Then G∩Y is a g γ_μ -open set in (Y, τ^*) which contains x but does not contain y. Hence (Y, τ^*) is a g γ_μ -T_0 space.

We, give the following

Example 3.5: let X = {1, 2, 3, 4} be any topological space.

$Y \subset X$ and Y = {1, 2, 3}

$\mu = \{\emptyset, X, \{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\}\}$

$\gamma\mu$-open = $\{\emptyset, X, \{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\}\}$

$\gamma\mu$-closed = $\{\emptyset, X, \{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\}\}$

$g\gamma\mu$-closed = $\{\emptyset, X, \{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\}\}$

$g\gamma\mu$-open = $\{\emptyset, X, \{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\}\}$

1	2
{1}	{2}
{1,3}	{2}
{1}	{2,3}

1	3
{1}	{3}
{1,2}	{3}
{1}	{2,3}

2	3
{2}	{3}
{1,2}	{3}
{2}	{1,3}

Definition 3.7: A function f: $X \to Y$ is said to be point g γ_μ closure 1-1 if and only if x, y ϵX such thatg γ_μ Cl({x}) $\neq$ g γ_μ Cl({y}) then f(g γ_μ Cl({x})) $\neq$ f(g γ_μ Cl({y})).

Example 3.6: Let f: R→R is a function. Let X = {1, 2, 3, 4} and Y = {1, 2, 3, 4}. Let 3, 4 ϵ X

$g\gamma_\mu CL(A)=\{\emptyset,X,\{1\},\{2\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,3,4\}\}$

$g\gamma_\mu CL(\{3\}) = \{3\}$

$g\gamma_\mu CL(\{4\}) = \{4\}$

$g\gamma_\mu CL(\{3\}) \neq g\gamma_\mu CL(\{4\})$

f(X) = X

$f(g\gamma_\mu CL(\{3\})) = \{3\}$

$f(g\gamma_\mu CL(\{4\})) = \{4\}$

$f(g\gamma_\mu CL(\{3\})) \neq f(g\gamma_\mu CL(\{4\}))$

A function f: R→R is said to be point g γ_μ closure 1-1.

Theorem 3.8: If function f: X→ Y is point -g γ_μ closure 1-1 and X is g γ_μ -T_0 then f is 1-1

Proof: Let x, y ϵ X with x $\neq$ y. Since X is g γ_μ -T_0, then g γ_μCl({x}) $\neq$ gγ_μCl({y}) by Theorem 3.5. But f is point gγ_μ closure 1-1 implies that f(gγ_μCl({x})) $\neq$f(gγ_μCl({y})). Hence f(x) $\neq$ f(y). Thus, f is 1-1.

Example 3.7: let X = {1, 2, 3, 4} and Y={a, b, c, d} be any topological space. Let f: X→ Y be a function. define, the function f(1) = a, f(2) = b, f(3) = c, f(4) = d and let 1, 4 ϵX

$g\gamma_\mu CL(\{1\}) = \{1\}$

$g\gamma_\mu CL(\{4\}) = \{4\}$

$f(g\gamma_\mu CL(\{1\})) = \{a\}$

. $f(g\gamma_\mu CL(\{4\})) = \{d\}$

$f(g\gamma_\mu CL(\{1\})) \neq f(g\gamma_\mu CL(\{4\}))$

$f(1) \neq f(2)$

f is one-one.

Theorem 3.9: Let f: X→Y is a mapping from $g\gamma_\mu$-T_0 space X into $g\gamma_\mu$-T_0 space Y. Then f is point-$g\gamma_\mu$closure 1-1 if and only if f is 1-1.

Proof follows from th.3.4. Above

Theorem 3.10: Let f: X →Y is an injective $g\gamma_\mu$ -irresolute mapping. If Y is $g\gamma_\mu$-T_0 then X is $g\gamma_\mu$ -T_0.

Proof: Let x, y ϵ X with x ≠ y. Since f is injective and Y is gγ_μ-T_0, there exists a gγ_μ open set V_x inY such that f(x) ϵ V_x and f(y) ∉ V_x or there exists a g$\gamma_\mu$open set V_y in Y such that f(y) ϵ V_y and f(x) ∉ V_y with f(x) ≠ f(y).By gγ_μ irresoluteness of f, $f^{-1}(V_x)$ is g$\gamma_\mu$open set in X such that x ϵ $f^{-1}(V_x)$ and y ∉ $f^{-1}(V_x)$ or $f^{-1}(V_y)$ is g$\gamma_\mu$open set in X such that y ϵ $f^{-1}(V_y)$ and x ∉ $f^{-1}(V_y)$. This shows that X is gγ_μ-T_0.

Definition 3.11: A mapping f: X→Y is said to be always gγ_μ-open, if the image of every gγ_μ-open set of X is gγ_μ-open in Y.

Lemma 3.12: The property of a space being gγ_μ-T_0 is preserved under one-one, onto and always gγ_μ-open mapping.

Proof: Let X be a gγ_μ-T_0 space and Y be any topological space. Let f: X→Y be a one-one, onto always gγ_μ-open mapping from X to Y. Let u, v ∈Y with u ≠ v. Since f is one-one, onto, there exist distinct points x, y∈X. Such that f(x) = u, f(y) = v. Since X is on gγ_μ-T_0 space. There exists gγ_μ-open set G in X such that x ∈ G but y ∉ G. Since f is always gγ_μ-open, f(G) is an gγ_μ-open set containing f(x) = u but not containing f(y) = v. Thus there exists an gγ_μ-open set f(G) in y such that u ∈ f(G) but v ∉ f(G) and hence Y is an gγ_μ-T_0 space.

Next, we give the following

Definition 3.13: A sub set A of a space X is called a gγ_μD-set if there are two gγ_μ-open subsets U and V such that U ≠ X and A = U-X.

Clearly, every gγ_μ–open set in gγ_μD-set.

We, give the following

Definition 3.14: A space X is called a gγ_μ-D_0 if for any disjoint pair of points x and y of X there exists a gγ_μD-set of X containing x but not y or a gγ_μD-set of X containing y but not x.

Clearly, every gγ_μ– T_0 space in g$\gamma_\mu$$D_0$ -space.

We prove the following

Theorem 3.15: If f: X→Y is gγ_μ-irresolute surjective function and A is a gγ_μD-set in Y, then the inverse imageof A is a gγ_μD-set in X.

Proof: Let A be a gγ_μD-set in Y. Then there are gγ_μ-open sets U_1 and U_2 in Y such that A = U_1−U_2 and U_1 ≠ Y. By the gγ_μ-irresoluteness of f, $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are gγ_μ-open set in X. Since U_1 ≠ Y, we have $f^{-1}(U_1)$ ≠X. Hence $f^{-1}(A) = f^{-1}(U_1) - f^{-1}(U_2)$ is a gγ_μD-set.

We define the following

Definition 3.16: A space (X, μ) is gγ_μ-T_1 if and only if for x, y X such that x y, there exists a gγ_μ-open set containing x but not y and there is a gγ_μ-open set containing y but not x.

It is easy to verify the following:

- Every gγ_μ-T_1 space is a gγ_μ-T_0 space.

Theorem 3.17: A space X is a gγ_μ-T_1 space if and only if{x} is gγ_μ-closed in X for every x ∈ X.

Proof: Let x, y be two distinct points X such that {x} and {y} are gγ_μ-closed. Then X$-\{x\}$ and Y$-\{y\}$ are gγ_μ-open in X such that y ∈ X$-\{x\}$ but x ∉ X$-\{x\}$ and x ∈ X$-\{y\}$ but y ∉ X$-\{y\}$. Hence, X is an gγ_μ-T_1 space. Conversely, let X be an gγ_μ-T_1 space and x be any arbitrary point of X. If y ∈ X$-\{x\}$, then y ≠ x. Now the space being gγ_μ-T_1 and y is a point different from x, there exists an gγ_μ-open set G_y such that y ∈G_y but x∉ Gy. Thus for each y ∈ X$-\{x\}$, there exists an gγ_μ-open set G_y such that y ∈ G_y⊂ X$-\{x\}$. Therefore ∪{y| y ≠x} ⊂ ∪{ G_y| y ≠x} ⊂ X$-\{x\}$ which implies that X$-\{x\}$ ⊂∪{ G_y| y ≠x} ⊂ X$-\{x\}$. Therefore, X$-\{x\}$ = ∪{ G_y| y ≠ x}. Since G_y gγ_μ–open in X and the union of gγ_μ-open sets in X is gγ_μ-open in X, X$-\{x\}$ is gγ_μ-open in X and so {x} is gγ_μ-closed.

Recall the following

Definition 3.18: A topological space (X, μ) is γ_μ Symmetric if for x and y in x, x∈ γ_μCl({y}) implies y∈ γ_μCl({x}).

Definition 3.19: A topological space (X, μ) is gγ_μ-symmetric if for any x and y in X, x∈ gγ_μCl({y}) implies y∈ gγ_μCl({x}).

Theorem 3.20: If {x} is gγ_μ-closed for each x in X then a space X is gγ_μ-symmetric.

Proof: Suppose x ∈ g$\gamma_\mu$cl({y}) and y ∉ g$\gamma_\mu$cl({x}). Since {y}⊂ X− g$\gamma_\mu$cl({x}) and {y} is gγ_μ-closed, g$\gamma_\mu$cl({y}) ⊂X −gspcl({x}). Thus x ∈ X−gspcl({y}), a contradiction.

Theorem 3.21: If a space X is extremely disconnected (i.e., closure of every open set is open) and $g\gamma_\mu$-symmetric, then {x} is $g\gamma_\mu$-closed, for each x in X.

Proof: Suppose $\{x\} \subset U$ where U is $g\gamma_\mu$-open and $g\gamma_\mu Cl(\{x\}) \not\subset U$. Then $g\gamma_\mu Cl(\{x\}) \cap (X-U) \neq \emptyset$. Let $y \in g\gamma_\mu Cl(\{x\}) \subset (X-U)$. We have $x \in g\gamma_\mu Cl(\{x\}) \subset (X-U)$ and $x \notin U$, A contradiction. Hence {x} is $g\gamma_\mu$-closed in X.

Corollary 3.22: If X is extremely disconnected, then X is $g\gamma_\mu$-T_1 if and only if X is $g\gamma_\mu$-symmetric.

Proof: Obvious

Next, we have the following invariant properties.

Theorem 3.23: Let f: X→Y be a $g\gamma_\mu$-irresolutes injective map. If Y is $g\gamma_\mu$-T_1, then X is $g\gamma_\mu$-T_1.

Proof: Assume that Y is $g\gamma_\mu$-T_1. Let x, y ∈Y be such that x≠ y. Then there exists a pair of $g\gamma_\mu$-open sets u, v in Y such that f(x) ∈ U, f(y) ∈ V and f(x) ∈ V, f(y) ∈ U. Then $x \in f^{-1}(U)$, $y \in f^{-1}(V)$, $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Since f is $g\gamma_\mu$-irresolute, X is $g\gamma_\mu$-T_1.

Corollary 3.24: A topological space (X, μ) is $g\gamma_\mu$-T_1 if and only if every finite subset of X is $g\gamma_\mu$-closed.

We, define the following

Definition 3.25: A space X is called $g\gamma_\mu$-D_1 if for any distinct pair of points x and y of X there exists a $g\gamma_\mu$-Dset of X containing x but y and a $g\gamma_\mu$-D set of X containing y but not x.

Clearly, every $g\gamma_\mu$–T_1 space is $g\gamma_\mu$–D_1 space.

Theorem 3.26: If Y is a $g\gamma_\mu$-D_1 and f: X→Y is $g\gamma_\mu$-irresolute and bijective, then X is $g\gamma_\mu$-D_1.

Proof: Suppose that Y is a $g\gamma_\mu$-D_1 space. Let x and y be any pair of distinct points in X. Since f is injective and Y is $g\gamma_\mu$-D_1, there exist $g\gamma_\mu$-D sets G_x and G_y of Y containing f(x) and f(y) respectively, such that $f(y) \notin G_x$ and $f(x) \notin G_y$. By Theorem 3.15, $f^{-1}(G_x)$ and $f^{-1}(G_y)$ are $g\gamma_\mu$-D sets in X containing x and y respectively. This impliesthat X is a $g\gamma_\mu$-D_1 space.

We, define and study the concept of $g\gamma_\mu$-R_0 spaces in the following:

Definition 3.27: Let X be a topological space and A ⊂ X.Then the generalized μ -kernel of A denoted by $g\gamma_\mu$- ker(A), is defined to be the set $g\gamma_\mu\text{-ker}(A) = \cap\{G \in g\gamma_\mu O(X) | A \subset G\}$.

Lemma 3.28: Let X be a topological space and x ∈ X. Then $y \in g\gamma_\mu\text{-ker}(\{x\})$ if and only if $x \in g\gamma_\mu Cl(\{y\})$

Proof: Suppose that $y \in g\gamma_\mu\text{-ker}(\{x\})$. Then there exists a $g\gamma_\mu$-open set V containing x such that y ∈ V. Therefore, we have $x \in g\gamma_\mu Cl(\{y\})$. Conversely, Suppose that $x \in g\gamma_\mu\text{-ker}(\{y\})$. Then there exists a $g\gamma_\mu$-open set V containing y such that x ∈V.Therefore, we have $y \in g\gamma_\mu Cl(\{x\})$.

Lemma 3.29: Let X be a topological space and A be a subset of X. Then $g\gamma_\mu\text{-ker}(A) = \{x \in X | g\gamma_\mu Cl(\{x\}) \cap A \neq \emptyset\}$.

Proof: Let $x \in g\gamma_\mu\text{-ker}(A)$ and suppose $g\gamma_\mu Cl(\{x\}) = \emptyset$. Hence $x \in X | g\gamma_\mu Cl(\{x\})$ which is a $g\gamma_\mu$-open set containing A. This is absurd. Since $x \in g\gamma_\mu\text{-ker}(A)$. Consequently, $g\gamma_\mu Cl(\{x\}) \cap A \neq \emptyset$. Next, let $g\gamma_\mu Cl(\{x\}) \cap A \neq \emptyset$ and suppose that x $g\gamma_\mu$-ker(A). Then there exists $g\gamma_\mu$-open set U containing A and $x \in U$. Let $y \notin g\gamma_\mu Cl(\{x\}) \subset A$. hence, U is a $g\gamma_\mu$-neighbourhood of y where $x \notin U$. But this is a contradiction, Therefore $x \in g\gamma_\mu\text{-ker}(A)$ and the claim.

Now, we define the following

Definition 3.30: A space X is said to be $g\gamma_\mu$-R_0 space if every $g\gamma_\mu$-open set contains the $g\gamma_\mu$-closure of each of its singletons.

Clearly, every $g\gamma_\mu$–R_0 space id $g\gamma_\mu$–T_1 space.

We recall the following:

Definition 3.31: A topological space (X, μ) is said to be $g\mu$ -R_0 space if every $g\mu$ -open set contains the $g\mu$ -closure of each of its singletons.

Theorem 3.32: For any topological space X the following properties are equivalent:

(i) X is $g\gamma_\mu$-R_0 space;

(ii) For any $F \in g\gamma_\mu C(X, \mu)$ $x \notin F \Rightarrow F \subset U$ and $x \notin U$ for some $U \in g\gamma_\mu C(X, \mu)$;

(iii) For any $F \in g\gamma_\mu C(X, \mu)$ $x \notin F \Rightarrow F \cap g\gamma_\mu Cl(\{x\}) = \phi$;

(iv) For any distinct points x and y either $g\gamma_\mu Cl(\{x\}) = g\gamma_\mu Cl(\{y\})$ or $g\gamma_\mu Cl(\{x\})$

$g\gamma_\mu Cl(\{y\}) = \phi$.

Proof: (i) ⇒ (ii): Suppose $F \in g\gamma_\mu C(X, \mu)$ and $x \notin F$. Then by (i) $g\gamma_\mu Cl(\{x\}) \subset X|F$. Set $U = X\ g\gamma_\mu(\{x\})$ then $U \cup \in g\gamma_\mu O(X, \mu)$ $F \subset U$ and $x \notin U$

(ii) ⇒ (iii): Let $F \in g\gamma_\mu C(X, \mu)$, $x \notin F$. Therefore, there exists $U \in g\gamma_\mu O(X, \mu)$ Such that $F \subset U$ and $x \notin U$. Since $U \in g\gamma_\mu O(X, \mu)$ $U \cap g\gamma_\mu Cl(\{x\}) = \phi$. and $F \cap g\gamma_\mu Cl(\{x\}) = \phi$.

(iii) ⇒ (iv): Suppose that $g\gamma_\mu Cl(\{x\}) \neq g\gamma_\mu Cl(\{y\})$ for distinct points $x, y \in X$. There exist $z \in g\gamma_\mu Cl(\{x\})$ such that $z \notin g\gamma_\mu Cl(\{y\})$. One can also assume that $z \in g\gamma_\mu Cl(\{y\})$ such that $z \notin g\gamma_\mu Cl(\{x\})$. There exists $V \in g\gamma_\mu O(X, \mu)$ such that $y \notin V$ and $z \in V$. Hence $x \in V$. Therefore we obtain $x \notin g\gamma_\mu Cl(\{y\})$. By (iii) we obtain $g\gamma_\mu Cl(\{x\}) \cap g\gamma_\mu Cl(\{y\}) = \phi$. The proof of otherwise is similar.

(iv)⇒(i): Let $V \in g\gamma_\mu O(X, \mu)$ and $x \in V$. For each $y \notin V$, $x \neq y$ and $x \notin g\gamma_\mu Cl(\{y\})$. This show that $g\gamma_\mu Cl(\{x\}) \neq g\gamma_\mu Cl(\{y\})$. By (iv) $g\gamma_\mu Cl(\{x\}) \cap g\gamma_\mu Cl(\{y\}) = \phi$ for each $y \in X|V$. Hence $g\gamma_\mu Cl(p\{x\}) \cap (U\{ g\gamma_\mu Cl(\{y\})|\ y\ X|V\} = \phi$. On the other hand, since $V \in g\gamma_\mu O(X, \mu)$ and $y \notin X|V$. We have $g\gamma_\mu Cl(\{y\}) \subset X|V$. Therefore $X|V = U\{ g\gamma_\mu Cl(\{y\})|\ y\ X|V\}$. Therefore we obtain $(X|V) \cap g\gamma_\mu Cl(\{x\}) = \phi$ and $g\gamma_\mu Cl(\{x\}) \subseteq V$. Hence (X, μ) is $g\gamma_\mu$-R_0 space.

Finally, we define and study the following.

Definition 3.33: A space X is said to be a $g\gamma_\mu$-R_1 if for x, y in X with $g\gamma_\mu Cl(\{x\}) \neq g\gamma_\mu Cl(\{y\})$, there exists disjoint $g\gamma_\mu$-open sets U and V such that $g\gamma_\mu Cl(\{x\}) \subset U$ and $g\gamma_\mu Cl(\{y\}) \subset V$.

Definition 3.34: A topological space X is said to be $g\mu$-R_1 space if for x, y in X with $g\mu Cl(\{x\})$ $g\mu Cl(\{y\})$, there exist disjoint $g\mu$-open sets U and V such that $g\mu Cl(\{x\})$ is a subset of U and $g\mu Cl(\{y\})$ is asubset of V.

Theorem 3.35: If X is gsp-R_1, then X is gsp-R_0-space.

Proof: Let U be a $g\gamma_\mu$-open and $x \in U$. If $y \notin U$ then since $x \subset g\gamma_\mu Cl(\{y\})$, $g\gamma_\mu Cl(\{x\}) \subset g\gamma_\mu Cl(\{y\})$. Hence there exists a $g\gamma_\mu$-open V such that $g\gamma_\mu Cl(\{x\}) \subset V$ and $x \notin V$, which implies $y \subset g\gamma_\mu Cl(\{x\})$. Thus $g\gamma_\mu Cl(\{y\}) \subset U$. Therefore (X, μ) is $g\gamma_\mu$-R_0 space.

Conclusion: If we replace $\boldsymbol{\mu}$ by different GT's or $\boldsymbol{\gamma_\mu}$ by different operators, we can obtain various forms of generalized closed sets and related continuous functions.

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