



Truth Table- Smarandachley product cordial labeling of Rooted Product Graph

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Abstract- In this paper the researcher study and work on the labeling of Corona Product Graph by admitting the certain condition of Smarandachely product cordial labeling.

Smarandachely product cordial labeling on G is such a labeling $f : V(G) \rightarrow \{0, 1\}$ with induced labeling $f(u)f(v)$ on edge $uv \in E(G)$ that $|v_f(0) - v_f(1)| \geq 2$ and $|e_f(0) - e_f(1)| \geq 2$.

Key words- Cordial labeling, Smarandachely product cordial labeling, and Corona product graph.

Introduction- A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. Labeling helps to distinguish between any two adjacent vertices or edges. Graph labelling was first introduced in the year 1967 by Rosa [1]. Rosa defined a function as $f : V(G) \rightarrow \{0, 1, 2, 3, \dots, q\}$, f is an injection such that, when each edge xy is assigned the label $|f(x) - f(y)|$, the resulting edge labels are distinct.

Elvi Khairunnisa and Kiki Ariyanti Sugeng [2] worked on the graceful labelling of Corona Product of Aster flower Graph. Chiranlal [3] introduced the Proper d-Lucky Labeling of Corona Products of Certain Graph. And He [4] also proved and worked on Proper d-Lucky Labeling of Rooted Products and Corona Products of Certain Graphs.

Now the researcher continues his work as labeling on the graph which is known as Corona Product graph. Graphs can be used to model interconnection networks in which vertices correspond to processors of the network and the edges correspond to communication links. A new interconnection network topology which is called the bloom graph has been introduced by truth table, which satisfies the condition of Smarandachley Product Cordial labelling.

Def. Corona Product graph- The corona product of two graphs G and H is defined as the graph obtained by taking one copy of G and $|V(G)|$ copies of H and joining the i -th vertex of G to every vertex in the i -th copy of H .

Theorem 1 – The $m \times n$ dimensional Corona product graph $C_m \circ C_n$ admits Smarandachely Product Cordial labeling $3 \leq m, n \geq 10$.

Proof- It admits the condition of Smarandachely Product Cordial Labeling when

Case1. When m and n both are even-

Truth Table-

$C_m \circ C_n$	V(0)	V(1)	$ \sum v(0) - \sum v(1) \geq 2$	e(0)	e(1)	$ \sum e(0) - \sum e(1) \geq 2$
$C_4 \circ C_4$	4	12	8	8	12	4
$C_4 \circ C_6$	8	16	8	16	12	4
$C_4 \circ C_8$	12	20	8	24	12	12
$C_4 \circ C_{10}$	16	24	8	32	12	20

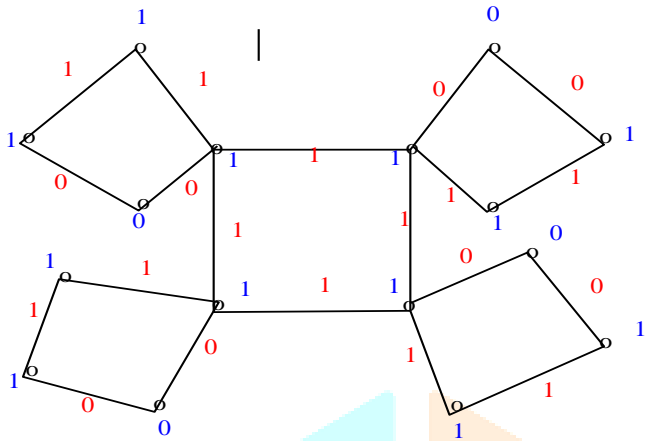
$C_m \circ C_n$	V(0)	V(1)	$ \sum v(0) - \sum v(1) \geq 2$	e(0)	e(1)	$ \sum e(0) - \sum e(1) \geq 2$
$C_6 \circ C_4$	6	18	12	12	18	6
$C_6 \circ C_6$	12	24	12	24	18	6
$C_6 \circ C_8$	18	30	12	36	18	18
$C_6 \circ C_{10}$	24	36	12	48	18	30

$C_m \circ C_n$	V(0)	V(1)	$ \sum v(0) - \sum v(1) \geq 2$	e(0)	e(1)	$ \sum e(0) - \sum e(1) \geq 2$
$C_8 \circ C_4$	8	24	16	16	24	8
$C_8 \circ C_6$	16	32	16	32	24	8
$C_8 \circ C_8$	24	40	16	48	24	24
$C_8 \circ C_{10}$	32	48	16	64	24	40

$C_m \circ C_n$	V(0)	V(1)	$ \sum v(0) - \sum v(1) \geq 2$	e(0)	e(1)	$ \sum e(0) - \sum e(1) \geq 2$
$C_{10} \circ C_4$	10	30	20	20	30	10
$C_{10} \circ C_6$	20	40	20	40	30	10
$C_{10} \circ C_8$	30	50	20	60	30	30
$C_{10} \circ C_{10}$	40	60	20	80	30	50

All the vertices of m are labelled with $(1,1,1, \dots)$ and 3 vertices of n are labelled with $(0,1,1)$ respectively and rest will be labelled with $(0,1,0,1, \dots)$ alternatively.

Figure 1.1

 $C_4^0 C_4$ 

$$v(0) = 4, v(1) = 12$$

$$\left| \sum v(0) - \sum v(1) \right| \geq 2$$

$$|4 - 12| \geq 2$$

$$8 \geq 2$$

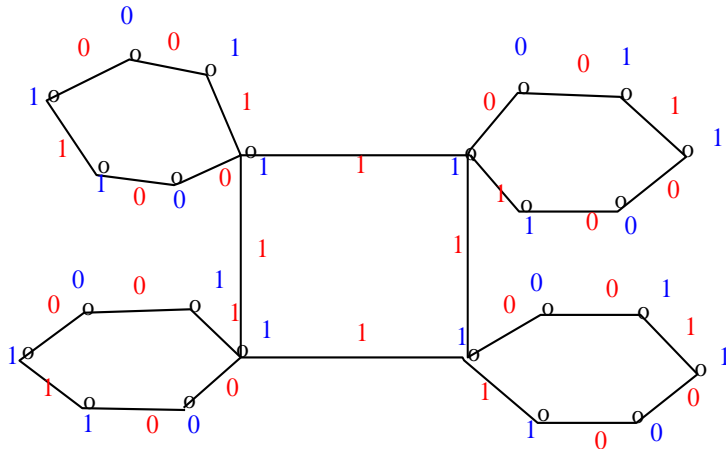
$$e(0) = 8, e(1) = 12$$

$$\left| \sum e(0) - \sum e(1) \right| \geq 2$$

$$|8 - 12| \geq 2$$

$$4 \geq 2$$

Figure 1.2

 $C_4^0C_6$ 

$$v(0) = 8, v(1) = 16$$

$$|\sum v(0) - \sum v(1)| \geq 2$$

$$|8 - 16| \geq 2$$

$$8 \geq 2$$

$$e(0) = 16, e(1) = 12$$

$$|\sum e(0) - \sum e(1)| \geq 2$$

$$|16 - 12| \geq 2$$

$$4 \geq 2$$

Case 2- when m is odd, n is even-

Truth Table-

$C_m^0C_n$	V(0)	V(1)	$ \sum v(0) - \sum v(1) \geq 2$	e(0)	e(1)	$ \sum e(0) - \sum e(1) \geq 2$
$C_3^0C_4$	3	9	6	6	9	3
$C_3^0C_6$	6	12	6	12	9	3
$C_3^0C_8$	9	15	6	18	9	9
$C_3^0C_{10}$	12	18	6	24	9	15

$C_m o C_n$	V(0)	V(1)	$ \sum v(0) - \sum v(1) \geq 2$	e(0)	e(1)	$ \sum e(0) - \sum e(1) \geq 2$
$C_5 o C_4$	5	15	10	10	15	5
$C_5 o C_6$	10	20	10	20	15	5
$C_5 o C_8$	15	25	10	30	15	15
$C_5 o C_{10}$	20	30	10	40	15	25

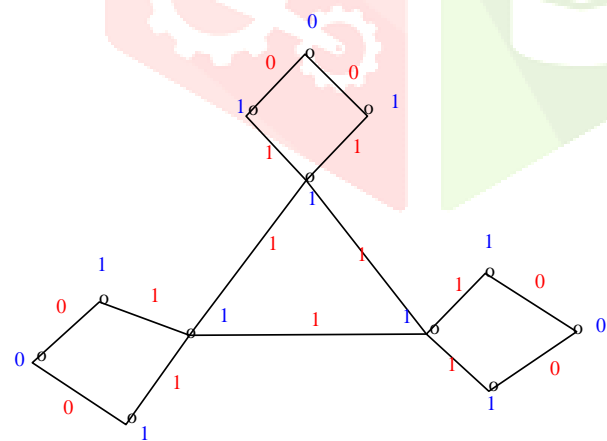
$C_m o C_n$	V(0)	V(1)	$ \sum v(0) - \sum v(1) \geq 2$	e(0)	e(1)	$ \sum e(0) - \sum e(1) \geq 2$
$C_7 o C_4$	7	21	14	14	21	7
$C_7 o C_6$	14	28	14	28	21	7
$C_7 o C_8$	21	35	14	42	21	21
$C_7 o C_{10}$	28	42	14	56	21	35

$C_m o C_n$	V(0)	V(1)	$ \sum v(0) - \sum v(1) \geq 2$	e(0)	e(1)	$ \sum e(0) - \sum e(1) \geq 2$
$C_9 o C_4$	9	27	18	18	27	9
$C_9 o C_6$	18	36	18	36	27	9
$C_9 o C_8$	27	45	18	54	27	27
$C_9 o C_{10}$	36	54	18	72	27	45

All the vertices of m are labelled with (1, 1, 1.....) and all vertices of n are labelled with (1,0,1,0,1.....) alternatively.

Figure 2.1

$C_3 o C_4$



$$v(0) = 3, v(1) = 9$$

$$|\sum v(0) - \sum v(1)| \geq 2$$

$$|3 - 9| \geq 2$$

$$6 \geq 2$$

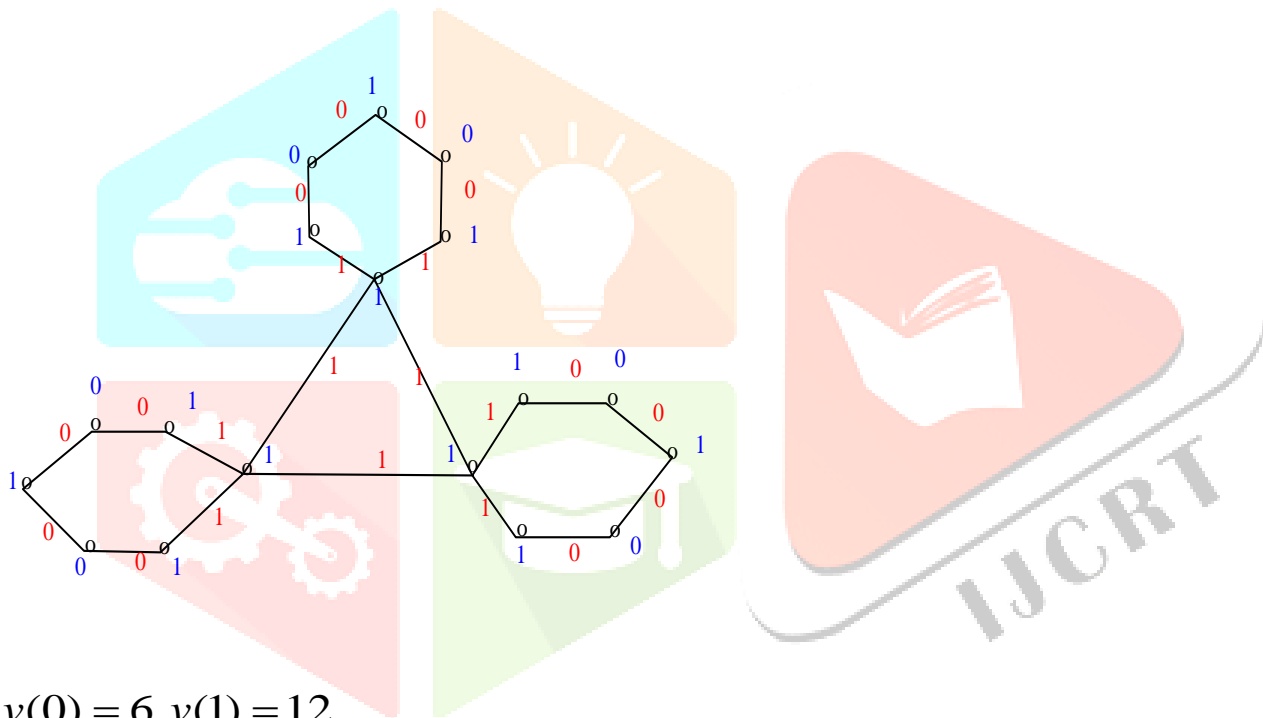
$$e(0) = 6, e(1) = 9$$

$$\left| \sum e(0) - \sum e(1) \right| \geq 2$$

$$|6 - 9| \geq 2$$

$$3 \geq 2$$

Figure 2.1

$$C_3 \circ C_6$$


$$v(0) = 6, v(1) = 12$$

$$\left| \sum v(0) - \sum v(1) \right| \geq 2$$

$$|6 - 12| \geq 2$$

$$6 \geq 2$$

$$e(0) = 12, e(1) = 9$$

$$\left| \sum e(0) - \sum e(1) \right| \geq 2$$

$$|12 - 9| \geq 2$$

$$3 \geq 2$$

Case -3- When m is even, n in odd-

Truth Table-

$C_m O C_n$	V(0)	V(1)	$ \sum v(0) - \sum v(1) \geq 2$	e(0)	e(1)	$ \sum e(0) - \sum e(1) \geq 2$
$C_4 O C_3$	4	8	4	12	4	8
$C_4 O C_5$	8	12	4	20	4	16
$C_4 O C_7$	12	16	4	28	4	24
$C_4 O C_9$	16	20	4	36	4	32

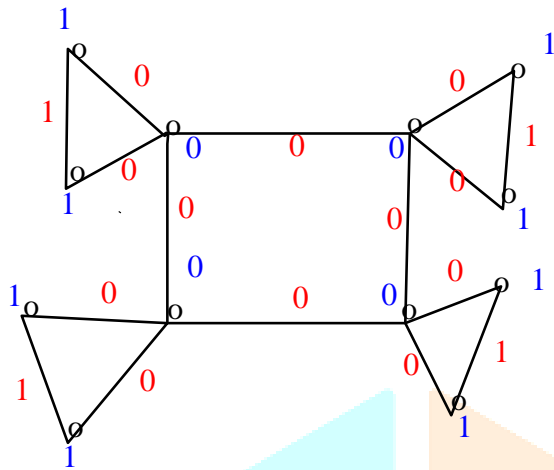
$C_m O C_n$	V(0)	V(1)	$ \sum v(0) - \sum v(1) \geq 2$	e(0)	e(1)	$ \sum e(0) - \sum e(1) \geq 2$
$C_6 O C_3$	6	12	6	18	6	12
$C_6 O C_5$	12	18	6	30	6	24
$C_6 O C_7$	18	24	6	42	6	36
$C_6 O C_9$	24	30	6	54	6	48

$C_m O C_n$	V(0)	V(1)	$ \sum v(0) - \sum v(1) \geq 2$	e(0)	e(1)	$ \sum e(0) - \sum e(1) \geq 2$
$C_8 O C_3$	8	16	8	24	8	16
$C_8 O C_5$	16	24	8	40	8	32
$C_8 O C_7$	24	32	8	56	8	48
$C_8 O C_9$	32	40	8	72	8	64

$C_m O C_n$	V(0)	V(1)	$ \sum v(0) - \sum v(1) \geq 2$	e(0)	e(1)	$ \sum e(0) - \sum e(1) \geq 2$
$C_{10} O C_3$	10	20	10	30	10	20
$C_{10} O C_5$	20	30	10	50	10	40
$C_{10} O C_7$	30	40	10	70	10	60
$C_{10} O C_9$	40	50	10	90	10	80

All the vertices of m are labelled with (0,0,0.....) and 2 vertices of n are labelled with (1,1) and rest will be labelled with (0,1,0,1.....) alternatively.

Figure 3.1

$$C_4^0 C_3$$


$$v(0) = 4, v(1) = 8$$

$$\left| \sum v(0) - \sum v(1) \right| \geq 2$$

$$|4 - 8| \geq 2$$

$$4 \geq 2$$

$$e(0) = 12, e(1) = 4$$

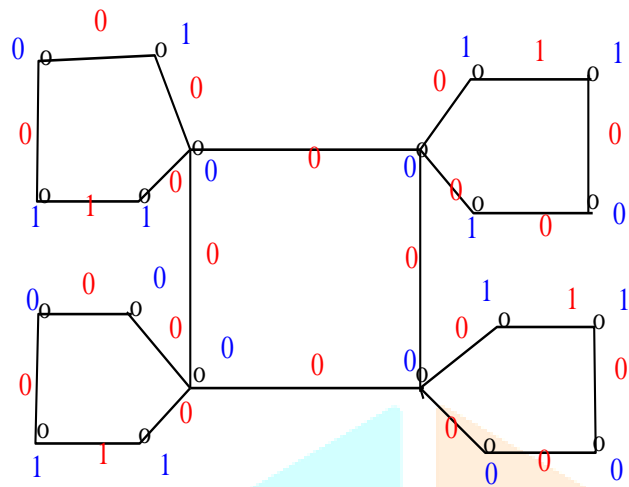
$$\left| \sum e(0) - \sum e(1) \right| \geq 2$$

$$|12 - 4| \geq 2$$

$$8 \geq 2$$

Figure 3.2

$C_4 \circ C_5$



$v(0) = 8, v(1) = 12$

$|\sum v(0) - \sum v(1)| \geq 2$

$|8 - 12| \geq 2$

$4 \geq 2$

$e(0) = 20, e(1) = 4$

$|\sum e(0) - \sum e(1)| \geq 2$

$|20 - 4| \geq 2$

$16 \geq 2$

Case 4 – When m, n both are odd –

Truth Table-

$C_m \circ C_n$	V(0)	V(1)	$ \sum v(0) - \sum v(1) \geq 2$	e(0)	e(1)	$ \sum e(0) - \sum e(1) \geq 2$
$C_3 \circ C_3$	3	6	3	9	3	6
$C_3 \circ C_5$	6	9	3	15	3	12
$C_3 \circ C_7$	9	11	3	21	3	18
$C_3 \circ C_9$	12	15	3	27	3	24

$C_m^o C_n$	V(0)	V(1)	$ \sum v(0) - \sum v(1) \geq 2$	e(0)	e(1)	$ \sum e(0) - \sum e(1) \geq 2$
$C_5^o C_3$	5	10	5	14	5	9
$C_5^o C_5$	10	15	5	24	5	19
$C_5^o C_7$	15	20	5	34	5	29
$C_5^o C_9$	20	25	5	44	5	39

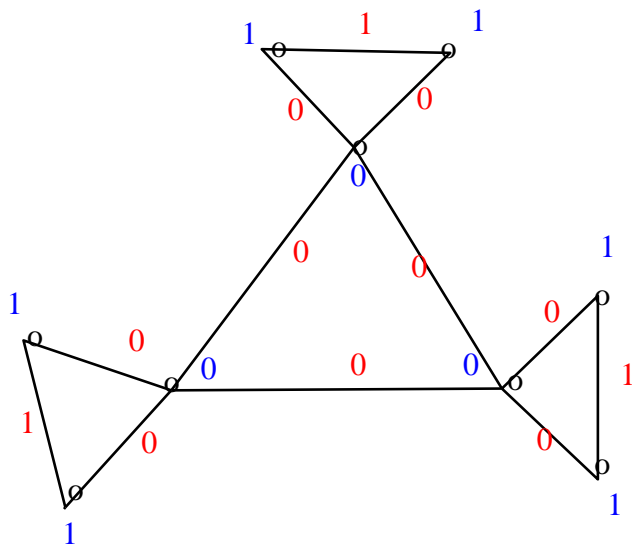
$C_m^o C_n$	V(0)	V(1)	$ \sum v(0) - \sum v(1) \geq 2$	e(0)	e(1)	$ \sum e(0) - \sum e(1) \geq 2$
$C_7^o C_3$	7	14	7	21	7	14
$C_7^o C_5$	14	21	7	35	7	28
$C_7^o C_7$	21	28	7	49	7	42
$C_7^o C_9$	28	35	7	63	7	56

$C_m^o C_n$	V(0)	V(1)	$ \sum v(0) - \sum v(1) \geq 2$	e(0)	e(1)	$ \sum e(0) - \sum e(1) \geq 2$
$C_9^o C_3$	9	18	9	27	9	18
$C_9^o C_5$	18	27	9	45	9	36
$C_9^o C_7$	27	36	9	63	9	54
$C_9^o C_9$	36	45	9	81	9	72

All the vertices of m are labelled with (0,0,0.....) and 2 vertices of n are labelled with (1,1) and rest will be labelled with (0,1,0,1.....) alternatively

Figure 4.1

$C_3^o C_3$



$$v(0) = 3, v(1) = 6$$

$$\left| \sum v(0) - \sum v(1) \right| \geq 2$$

$$|3 - 6| \geq 2$$

$$3 \geq 2$$

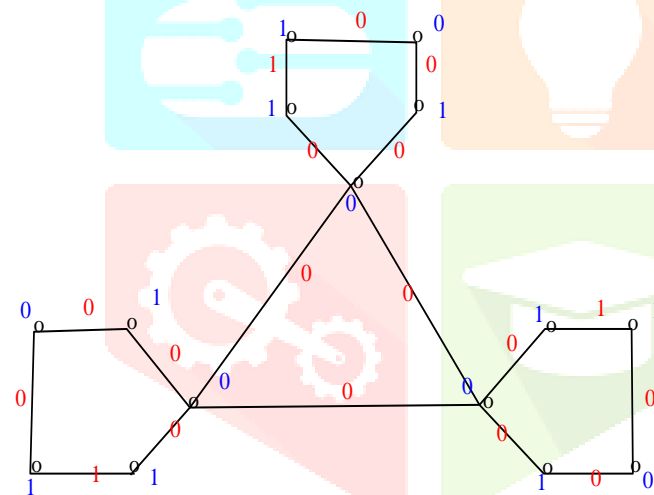
$$e(0) = 9, e(1) = 3$$

$$\left| \sum e(0) - \sum e(1) \right| \geq 2$$

$$|9 - 3| \geq 2$$

$$6 \geq 2$$

Figure 4.2

$$C_3 \circ C_5$$


$$v(0) = 6, v(1) = 9$$

$$\left| \sum v(0) - \sum v(1) \right| \geq 2$$

$$|6 - 9| \geq 2$$

$$3 \geq 2$$

$$e(0) = 3, e(1) = 15$$

$$\left| \sum e(0) - \sum e(1) \right| \geq 2$$

$$|3 - 15| \geq 2$$

$$12 \geq 2$$

5. Result Table-

Case -	$ \sum v(0) - \sum v(1) \geq 2$	$ \sum e(0) - \sum e(1) \geq 2$
1. When m, n both are even.	<i>even number</i> ≥ 2	<i>even number</i> ≥ 2
2. When m is odd, n is even.	<i>even number</i> ≥ 2	<i>odd number</i> ≥ 2
3. When m is even, n is odd.	<i>even number</i> ≥ 2	<i>even number</i> ≥ 2
4. When m, n both are odd.	<i>odd number</i> ≥ 2	<i>even number</i> ≥ 2

6. Conclusion – Here we computed the truth table and labelled the Corona product Graph, which satisfies the SmaranDachley Product Cordial Labeling that is a labeling $f : V(G) \rightarrow \{0, 1\}$ with induced labeling $f(u)f(v)$ on edge $uv \in E(G)$ that $|\sum v(0) - \sum v(1)| \geq 2$ and $|\sum e(0) - \sum e(1)| \geq 2$

A lot of work has been accomplished in this area by many researcher and still work is being carried out for this Graph.

7. References-

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