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AVAILABILITY FORECAST FOR A REDUNDANT COMPLEX SYSTEM WITH WAITING

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ABSTRACT: In this modern society, systems are complex enough and are designed to be operative for a specified period. This specified period is called the mission time. Our aim is to get improvement in this mission time. Availability analysis of each unit of equipment under given operating conditions is helpful to design the units for a minimum failure and to develop a plan in advance for scheduled maintenance or preventive maintenance. the authors have considered in this paper, a multi - component redundant complex system for evaluation of various reliability parameters. Supplementary variable technique and Laplace transform have used to formulate and solve the mathematical model. Laplace transform of various state probabilities, reliability, availability and M.T.T.F. of the system have obtained. Some particular cases and steady-state behaviour of the system are also given at the end. A numerical example with graphical illustration has also been mentioned in last to highlight the important results. We may utilize this general approach to the similar systems used in any industry or elsewhere.

Key Words: Non-Markovian system, supplementary variables technique, Laplace transform, asymptotic behaviour etc.

1. INTRODUCTION

There should be no failure in any unit or part of unit of considered equipment, under specified operating conditions during the whole period. This whole period consists of operating period, administrative period and repair period. We may also increase the availability of any equipment by introducing redundancy at design stage. This redundancy is generally of two type as standby and parallel redundancy.

Keeping all the above facts in view, the authors have considered in this paper, a multi - component redundant complex system for evaluation of various reliability parameters. The whole system comprises of two subsystems namely A and B, connected in series. The subsystem A consists of n-identical units in series and a similar set of nidentical units in standby redundancy. On failure of 1 A-set of units we may change over the 2A-set of units on line with the help of an imperfect switching device. This subsystem A is of 1-out-of-n: F nature.

The subsystem B has two identical units in parallel redundancy and on failure of any one unit, the system works in reduced efficiency state. The whole system can fail due to failure of either subsystem, due to critical human-error and due to environmental reasons. If we are repairing both the units of subsystem B, the system has to wait for repair. On the other hand, in repair of any single unit (from A or B) the repair facilities are always available. Head-of-line policy has been used for repair purpose. This policy is nothing but the "first come first served" policy. All the failures, waiting and switching follow exponential time distribution whereas all the repairs follow general time distribution.

2. ASSUMPTIONS

The following assumptions have been associated with this chapter:

1. Initially, all the units are good.
2. In one step only one change can take place.
3. Failures are statistically independent.
4. All repairs follow general time distribution whereas all failures and waiting follow exponential time distribution.
5. The switching device used, is imperfect.
6. The whole system can fail due to environmental failure and critical human - error.
7. The head-of-line policy has been used to repair purpose.
8. The system has to wait for repair, in case, both the units of subsystem B are failed.
9. Repair has given only if the system is either in degraded state or in failed state.
10. After repair, system works like a new and never damages anything.

3. NOTATIONS

The following notations have been used throughout this chapter:

α / β	: Failure rate of a unit of subsystem A / B.
γ_{e_1}, γ_{e_2}	: Failure rates due to environmental reasons.
γ_{h_1}, γ_{h_2}	: Failure rates due to critical human-error.
$(1-\lambda)$	: Failure rate of switching device.
w	: Waiting rate
$\mu_1(x)\Delta$ / $\mu_2(x)\Delta$	: The first order probability that the subsystem 1A /2A will be repaired in the time interval $(x, x+\Delta)$ conditioned that it was not repaired up to the time x .
$\xi(y)\Delta$ / $\xi_1(z)\Delta$	: The first order probability that the single unit / both units of subsystem B will be repaired in the time interval $(y, y+\Delta)$ / $(z, z+\Delta)$, conditioned that it was not repaired up to the time y / z.
$\eta(r)\Delta$ / $\mu_e(u)\Delta$ / $\mu_h(v)\Delta$	: The first order probability that the switching device /environmental failure / human-error will be repaired in the time interval $(r, r+\Delta)$ / $(u, u+\Delta)$ / $(v, v+\Delta)$ conditioned that it was not repaired up to the time r / u / v.
$P_0(t)$/ $P_1(t)$	: The probability that at time 't', the system is in operable state due to working of 1A/2A set of units.
$P_3(r,t)\Delta$, $P_8(r,t)\Delta$	: The probability that at time 't', the system is in failed state due to failure of switching device. The elapsed repair time lies in the interval $(r, r+\Delta)$.
$P_e(u,t)\Delta$ / $P_h(v,t)\Delta$	: The probability that at time 't', the system is in failed state

due to environmental reasons / human-error and elapsed repair time lies in the interval (u, u + Δ) / (v, v + Δ).

$P_2(x, t)\,\Delta$ / $P_9(y, t)\,\Delta$: The probability that at time 't', the system is in failed state due to failure of subsystem A / subsystem A and 1B unit. The elapsed repair time lies in the interval (x, x + Δ) / (y, y + Δ).

$P_4(y, t)\,\Delta$ / $P_7(t)$: The probability that at time 't', the system is in degraded state due to failure of 1B unit / 1B and 1A-set of unit and elapsed repair time lies in the interval (y, y + Δ).

$P_5(t)$ / $P_6(z, t)\,\Delta$: The probability that at time 't', the system is in failed state due to failure of subsystem B and is waiting / ready for repair. The elapsed repair time lies in the interval(z, z + Δ).

$P_{10}(x, t)\,\Delta$: The probability that at time 't', the system is in failed state due to failure of subsystem B and 1A- set of units. The elapsed repair time lies in the interval (x, x + Δ).

s : Laplace transform variable.

$\bar{A}(s)$: Laplace transform of function A (t).

6.4 FORMULATION OF MATHEMATICAL MODEL:

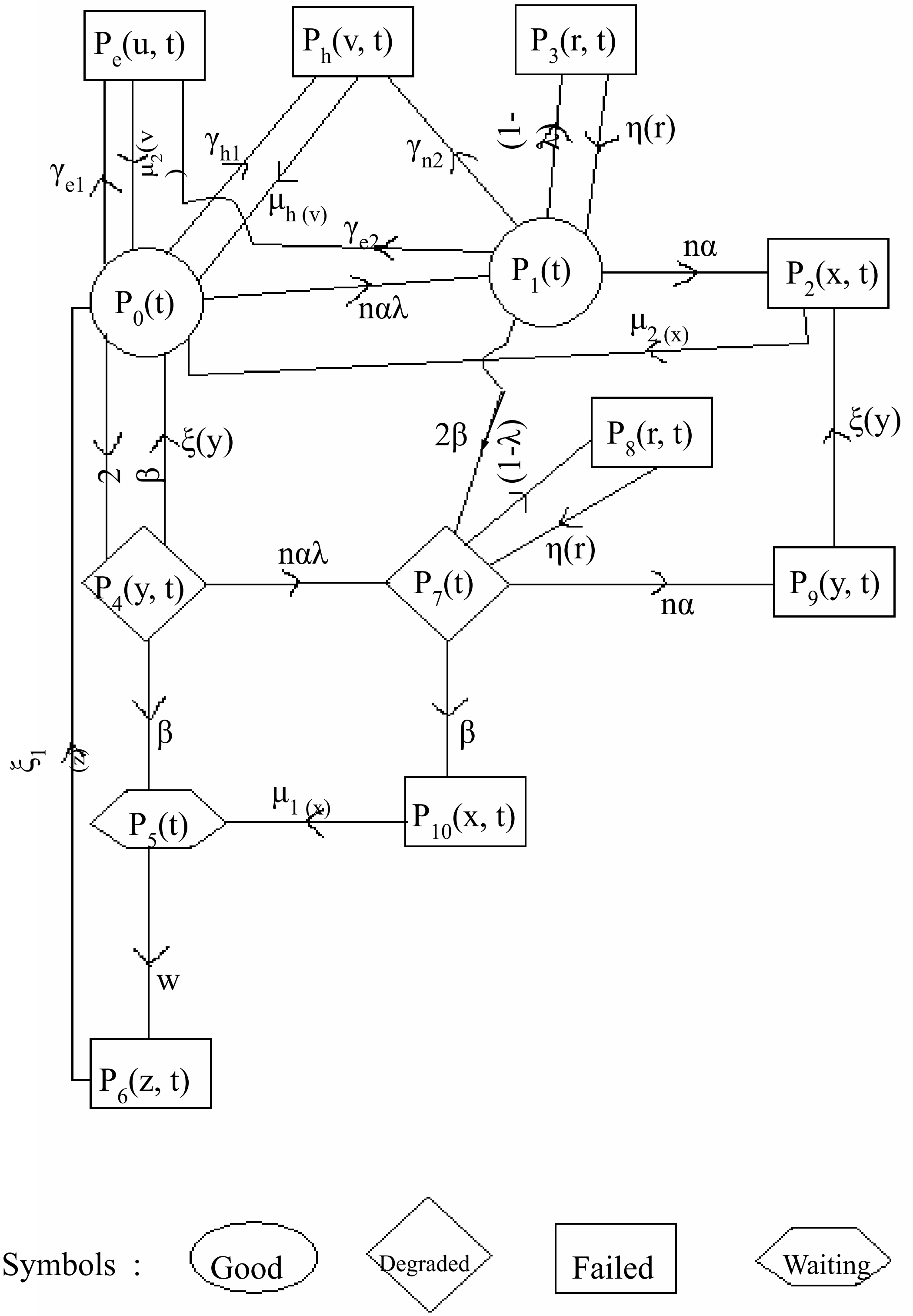
Using continuity argument, we obtain the following set of difference-differential equations governing the behaviour of the model under consideration:-

$$\left[\frac{d}{dt} + n\alpha\,\lambda + 2\beta + \gamma_{e_1} + \gamma_{h_1}\right] P_0(t) = \int_0^\infty P_e(u,t)\,\mu_C(u)\,du + \int_0^\infty P_4(y,t)\,\xi(y)\,dy + \int_0^\infty P_h(v,t)\,\mu_h(v)\,dv + \int_0^\infty P_2(x,t)\,\mu_2(x)\,dx + \int_0^\infty P_6(z,t)\,\xi_1(z)\,dz \qquad ----(1)$$

$$\left[\frac{d}{dt} + n\alpha + 2\beta + \gamma_{e_2} + \gamma_{h_2} + (1-\lambda)\right] P_1(t) = n\alpha\,\lambda\,P_0(t) + \int_0^\infty P_3(r,t)\,\eta(r)\,dr \qquad ----(2)$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_2(x)\right] P_2(x,t) = 0 \qquad ----(3)$$

$$\left[\frac{\partial}{\partial r} + \frac{\partial}{\partial t} + \eta(r)\right] P_3(r,t) = 0 \qquad ----(4)$$



TRANSITION - STATE DIAGRAM

Fig

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\xi(y)+n\alpha\lambda+\beta\right]P_4(y,t)=0 \qquad ----(5)$$

$$\left[\frac{\partial}{\partial t} + w\right] P_5(t) = \beta\, P_4(t) + \int_0^{\infty} P_{10}(x,t)\,\mu_1(x)\,dx \qquad ----(6)$$

$$\left[\frac{\partial}{\partial z} + \frac{\partial}{\partial t} + \xi_1(z)\right] P_6(z,t) = 0 \qquad ----(7)$$

$$\left[\frac{\partial}{\partial t} + (1-\lambda) + n\alpha + \beta\right] P_7(t) = 2\beta\, P_1(t) + n\alpha\lambda\, P_4(t) + \int_0^{\infty} P_8(r,t)\,\eta(r)\,dr \qquad ----(8)$$

$$\left[\frac{\partial}{\partial r} + \frac{\partial}{\partial t} + \eta(r)\right] P_8(r,t) = 0 \qquad ----(9)$$

$$\left[\frac{\partial}{\partial y} + \frac{\partial}{\partial t} + \xi(y)\right] P_9(y,t) = 0 \qquad ----(10)$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_1(x)\right] P_{10}(x,t) = 0 \qquad ----(11)$$

$$\left[\frac{\partial}{\partial u} + \frac{\partial}{\partial t} + \mu_e(u)\right] P_e(u,t) = 0 \qquad ----(12)$$

$$\left[\frac{\partial}{\partial v} + \frac{\partial}{\partial t} + \mu_h(v)\right] P_h(v,t) = 0 \qquad ----(13)$$

Boundary conditions are:

$$P_2(0,t) = n\alpha\, P_1(t) + \int_0^{\infty} P_9(y,t)\cdot\xi(y)\,dy \qquad ----(14)$$

$$P_3(0, t) = (1-\lambda)\; P_1(t) \qquad --------(15)$$

$$P_4(0, t) = 2\beta\; P_0(t) \qquad -------(16)$$

$$P_6(0, t) = w\; P_5(t) \qquad -------(17)$$

$$P_8(0, t) = (1-\lambda)\; P_7(t) \qquad -------(18)$$

$P_9\,(0, t) = n\alpha\; P_7(t)$ -------(19)

$P_{10}\,(0, t) = \beta\; P_7(t)$ -------(20)

$P_e\,(0, t) = \gamma_{e1}P_0(t) + \gamma_{e2}\,P_1(t)$ -------(21)

$P_h\,(0, t) = \gamma_{h1}P_0(t) + \gamma_{h2}\,P_1(t)$ -------(22)

Initial conditions are:

$P_0\,(0) = 1$ and other state probabilities at t = 0 are zero --------(23)

6.5 SOLUTION OF THE MODEL:

Taking Laplace transforms of equations (1) through (22) by making use of initial conditions (23), we may obtain:

$$\left[s + n\alpha\,\lambda + 2\beta + \gamma_{e_1} + \gamma_{h_1}\right]\overline{P}_0(s) = 1 + \int_0^\infty \overline{P}_e(u,s)\,\mu_e(u)\,du + \int_0^\infty \overline{P}_4(y,s)\,\xi(y)\,dy + \int_0^\infty \overline{P}_h(v,s)\,\mu_h(v)\,dv + \int_0^\infty \overline{P}_2(x,s)\,\mu_2(x)\,dx + \int_0^\infty \overline{P}_6(z,s)\,\xi_1(z)\,dz \quad -----(24)$$

$$\left[s + n\alpha + 2\beta + \gamma_{e_2} + \gamma_{h_2} + 1 - \lambda\right]\overline{P}_1(s) = n\alpha\,\lambda\,\overline{P}_0(s) + \int_0^\infty \overline{P}_3(r,s)\,\eta(r)\,dr \quad -----(25)$$

$$\left[\frac{\partial}{\partial x} + s + \mu_2(x)\right]\overline{P}_2\,(x,s) = 0 \quad ----(26)$$

$$\left[\frac{\partial}{\partial r} + s + n(r)\right]\overline{P_4}(r,s) = 0 \quad -----(27)$$

$$\left[\frac{\partial}{\partial y} + s + \xi(y) + n\alpha\,\lambda + \beta\right]\overline{P}_4\,(y,\,s) = 0 \quad ----(28)$$

$$\left[s + w\right] \overline{P}_5(s) = \beta \overline{P}_4(s) + \int_0^\infty \overline{P}_{10}(x,s)\, \mu_1(x)\, dx \qquad ------(29)$$

$$\left[\frac{\partial}{\partial z} + s + \xi_1(z)\right] \overline{P}_6(z,s) = 0 \qquad -------(30)$$

$$\left[s + 1 - \lambda + n\alpha + \beta\right] \overline{P}_7(s) = 2\beta \overline{P}_1(s) + n\alpha\lambda \overline{P}_4(s) + \int_0^\infty \overline{P}_8(r,s)\eta(r)\, dr \qquad ------(31)$$

$$\left[\frac{\partial}{\partial r} + s + \eta(r)\right] \overline{P}_8(r,s) = 0 \qquad ----(32)$$

$$\left[\frac{\partial}{\partial y} + s + \xi(y)\right] \overline{P}_9(y,s) = 0 \qquad ----(33)$$

$$\left[\frac{\partial}{\partial x} + s + \mu_1(x)\right] \overline{P}_{10}(x,s) = 0 \qquad ----(34)$$

$$\left[\frac{\partial}{\partial u} + s + \mu_e(u)\right] \overline{P}_e(u,s) = 0 \qquad ----(35)$$

$$\left[\frac{\partial}{\partial v} + s + \mu_h(v)\right] \overline{P}_h(v,s) = 0 \qquad ------(36)$$

$$\overline{P}_2(0, s) = n\alpha \overline{P}_1(s) + \int_0^\infty \overline{P}_9(y,s)\xi(y)\, dy \qquad ------(37)$$

$$\overline{P}_3(0, s) = (1-\lambda) \overline{P}_1(s) \qquad ------(38)$$

$$\overline{P}_4(0, s) = 2\beta \overline{P}_0(s) \qquad ------(39)$$

$$\overline{P}_6(0, s) = w \overline{P}_5(s) \qquad ------(40)$$

$$\overline{P}_8(0, s) = (1-\lambda) \overline{P}_7(s) \qquad ------(41)$$

$$\overline{P}_9(0, s) = n\alpha\ \overline{P}_7(s) \qquad -----(42)$$

$$\overline{P}_{10}(0, s) = \beta\ \overline{P}_7(s) \qquad -----(43)$$

$$\overline{P}_c\ (0, s) = \gamma_{e1}\ \overline{P}_0(s) + \gamma_{e2}\ \overline{P}_1(s) \qquad ---------(44)$$

$$P_h\ (0, s) = \gamma_{h1}\ \overline{P}_0(s) + \gamma_{h2}\ \overline{P}_2(s) \qquad ----------(45)$$

Equation (28) gives on integration, using (39):

$$\overline{P}_4(y,s) = 2\beta\ \overline{P}_0(s)\cdot e^{-(s+n\alpha\lambda+\beta)y-\int\xi(y)\,dy}$$

$$\Rightarrow \overline{P}_4(s) = 2\beta\ \overline{P}_0(s)\cdot D_4(s+n\alpha\lambda+\beta) \qquad ----(46)$$

Integration equation (28) by using (38), we get

$$\overline{P}_3(s) = (1-\lambda)\ \overline{P}_1(s)\ D_3(s) \qquad -----(47)$$

Equation (25) gives on simplification

$$\overline{P}_1(s) = A(s)\ \overline{P}_0(s) \qquad -----(48)$$

where

$$A(s) = \frac{n\alpha\lambda}{s+n\lambda+2\beta+\gamma_{e_2}+\gamma_{h_2}+(1-\lambda)\ s\ D_3(s)} \qquad -----(49)$$

Integration equation (32) by using (41), we get

$$\overline{P}_8(s) = (1-\lambda)\ \overline{P}_7(s)\ \overline{D}_8(s) \qquad -----(50)$$

Equation (31) gives on simplification

$$\overline{P}_7(s) = B(s)\ \overline{P}_0(s) \qquad -----(51)$$

where

$$B(s) = \frac{2\beta[\ A(s)+n\alpha\lambda\cdot D_4(s+n\alpha\lambda+\beta)]}{s+n\alpha+\beta+(1-\lambda)\ s\ \overline{D}_8(s)} \qquad -----(52)$$

Integrating equation (33) by using (42), we get

$$\overline{P}_9(s) = n\alpha\, B(s) \cdot \overline{P}_0(s) \cdot D_9(s) \qquad ------(53)$$

On integrating equation (34) by making use of (43), we get

$$\overline{P}_{10}(s) = \beta\, B(s) \cdot \overline{P}_0(s) \cdot D_{10}(s) \qquad ------(54)$$

On integrating equation (35) by making use of (44), we get

$$\overline{P}_e(s) = \left[\gamma_{e_1} + \gamma_{e_2} A(s)\right] \cdot \overline{P}_0(s) \cdot D_e(s) \qquad ------(55)$$

Integrating equation (36) by making use of (45), we get

$$\overline{P}_h(s) = \left[\gamma_{h_1} + \gamma_{h_2} A(s)\right] \cdot \overline{P}_0(s) \cdot D_h(s) \qquad ------(56)$$

Integrating equation (26) by making use of (37), we get

$$\overline{P}_2(s) = n\alpha\left[A(s) + B(s)\, \overline{S}_9(s)\right] \cdot \overline{P}_0(s) \cdot D_2(s) \qquad ------(57)$$

Equation (29) gives on simplification

$$\overline{P}_5(s) = C(s)\ \overline{P}_0(s) \qquad ------(58)$$

where

$$C(s) = \frac{\beta}{s+w}\left[2\beta\, D_4(s + n\alpha\lambda + \beta) + B(s)\ \overline{S}_{10}(s)\right] \qquad ------(59)$$

Integrating equation (30) by using (40), we get

$$\overline{P}_6(s) = w\, C(s) \cdot \overline{P}_0(s) \cdot D_6(s) \qquad ------(60)$$

Finally equation (24) gives on simplification by making use of relevant relations:

$$\overline{P}_0(s) = \frac{1}{E(s)} \qquad ------(61)$$

where,

$$\begin{aligned} E(s) &= s+n\alpha\lambda+2\beta+\gamma_{e_1}+\gamma_{h_1}-\left[\gamma_{e_1}+\gamma_{e_2}A(s)\right]\overline{S}_e(s)-2\beta\,\overline{S}_4(s+n\alpha\lambda+\beta) \\ &-\left[\gamma_{h_1}+\gamma_{h_2}A(s)\right]\overline{S}_h(s)-n\alpha\left[A(s)+B(s)\cdot\overline{S}_9(s)\right]\overline{S}_2(s) \\ &-\ w\,C(s)\cdot\overline{S}_6(s) \end{aligned} \quad ------(62)$$

Thus, we have finally the Laplace transforms of various state probabilities as below:

$$\overline{P}_0(s) = \frac{1}{E(s)} \quad ------(63)$$

$$\overline{P}_1(s) = \frac{A(s)}{E(s)} \quad ------(64)$$

$$\overline{P}_2(s) = \frac{n\alpha[\ A(s)+B(s)\cdot\overline{S}_9(s)]}{E(s)}\ D_2(s) \quad ------(65)$$

$$\overline{P}_3(s) = \frac{(1-\lambda)\ D_3(s)\ A(s)}{E(s)} \quad ------(66)$$

$$\overline{P}_4(s) = \frac{2\beta D_4(s+n\alpha\lambda+\beta)}{E(s)} \quad ------(67)$$

$$\overline{P}_5(s) = \frac{C(s)}{E(s)} \quad ------(68)$$

$$\overline{P}_6(s) = \frac{w\,C(s)\cdot D_6(s)}{E(s)} \quad ------(69)$$

$$\overline{P}_7(s) = \frac{B(s)}{E(s)} \quad ------(70)$$

$$\overline{P}_8(s) = \frac{(1-\lambda)B(s)\cdot\overline{D}_8(s)}{E(s)} \quad ------(71)$$

$$\overline{P}_9(s) = \frac{n\alpha \cdot B(s) \cdot D_9(s)}{E(s)} \qquad -----(72)$$

$$\overline{P}_{10}(s) = \frac{\beta \cdot B(s) \cdot D_{10}(s)}{E(s)} \qquad -----(73)$$

$$\overline{P}_e(s) = \frac{\left[\gamma_{e_1} + \gamma_{e_2} A(s)\right] D_e(s)}{E(s)} \qquad -----(74)$$

$$\overline{P}_h(s) = \frac{\left[\gamma_{h_1} + \gamma_{h_2} A(s)\right] D_h(s)}{E(s)} \qquad -----(75)$$

where,

$$A(s) = \frac{n\alpha\lambda}{s + n\lambda + 2\beta + \gamma_{e_2} + \gamma_{h_2} + (1-\lambda)\ s\ D_3(s)} \qquad -----(76)$$

$$B(s) = \frac{2\beta\ [A(s) + n\alpha\lambda\ \ D_4(s + n\alpha\lambda + \beta)}{s + n\alpha + \beta + (1-\lambda)\ s\ \overline{D}_8(s)} \qquad -----(77)$$

$$C(s) = \frac{\beta}{s+w}\left[2\beta\ D_4(s + n\alpha\lambda + \beta) + B(s) \cdot \overline{S}_{10}(s)\right] \qquad -----(78)$$

and

$$E(s) = s + n\alpha\lambda + 2\beta + \gamma_{e_1} + \gamma_{h_1} - \left[\gamma_{e_1} + \gamma_{e_2} A(s)\right] \overline{S}_e(s) - 2\beta\,\overline{S}_4(s + n\alpha\lambda + \beta)$$

$$-\left[\gamma_{h_1} + \gamma_{h_2} A(s)\right] \overline{S}_h(s) - n\alpha\left[A(s) + B(s) \cdot \overline{S}_9(s)\right] \overline{S}_2(s)$$

$$-\ w\ C(s) \cdot \overline{S}_6(s) \qquad -----(79)$$

6.6 ERGODIC BEHAVIOUR OF THE SYSTEM:

By using Abel's Lemma in probabilities; viz.

$\lim_{s\to 0} s\overline{F}(s) = \lim_{t\to\infty} F(t) = F \ (Say)$, provided the limit on R.H.S. exists; one can obtain the following time independent state probabilities from equations (63) through (75):

$$P_0 = \frac{1}{E'(0)} \qquad \text{------}(80)$$

$$P_1 = \frac{A_1}{E'(0)} \qquad \text{------}(81)$$

$$P_2 = \frac{n\alpha[A_1 + B_1]}{E'(0)} M_2 \qquad \text{------}(82)$$

$$P_3 = \frac{(1-\lambda) \ A_1}{E'(0)} M_3 \qquad \text{------}(83)$$

$$P_4 = \frac{2\beta}{E'(0)} \cdot D_4(n\alpha\lambda + \beta) \qquad \text{------}(84)$$

$$P_5 = \frac{C_1}{E'(0)} \qquad \text{------}(85)$$

$$P_6 = \frac{w \ C_1}{E'(0)} \cdot M_6 \qquad \text{------}(86)$$

$$P_7 = \frac{B_1}{E'(0)} \qquad \text{------}(87)$$

$$P_8 = \frac{(1-\lambda) B_1}{E'(0)} \cdot M_8 \qquad \text{------}(88)$$

$$P_9 = \frac{n\alpha \cdot B_1}{E'(0)} \cdot M_9 \qquad \text{------}(89)$$

$$P_{10} = \frac{\beta \cdot B_1}{E'(0)} \cdot M_{10} \qquad \text{------}(90)$$

$$P_e = \frac{\lfloor \gamma_{e_1} + \gamma_{e_2} \rfloor}{E'(0)} \cdot M_h \qquad -----(91)$$

$$P_h = \frac{\lfloor \gamma_{h_1} + \gamma_{h_2} A_1 \rfloor}{E'(0)} \cdot M_h \qquad -----(92)$$

where,

$$A_1 = \frac{n\alpha\lambda}{n\lambda + 2\beta + \gamma_{e_2} + \gamma_{h_2}} \qquad -----(93)$$

$$B_1 = \frac{2\beta\,[A_1 + n\alpha\lambda \ D_4(n\alpha\lambda + \beta)}{n\alpha + \beta} \qquad -----(94)$$

$$C_1 = \frac{\beta}{w}\left[2\beta \ D_4(n\alpha\lambda + \beta) + B_1\right] \qquad -----(95)$$

$$M_i = -\overline{S}'_i(0) \qquad \forall \quad i \qquad ------(96)$$

and

$$E'(0) = \left\lfloor \frac{d}{ds} \ E(s) \right\rfloor_{s=0} \qquad ------(97)$$

6.7 PARTICULAR CASES:

(a)*When all repairs follow exponential time distribution*

Setting $\overline{S}_i(k) = \frac{\mu_i}{(k+\mu_i)} \quad \forall \ k \ and \ i, \ D_i(k) = \frac{1}{(k+\mu_i)} \quad \forall \ i \ \& \ k \ etc.,$ in equations (63) through (75), one can obtain

$$\overline{P}_0(s) = \frac{1}{F(s)} \qquad ----(98)$$

$$\overline{P}_1(s) = \frac{G(s)}{F(s)} \quad ----(99)$$

$$\overline{P}_2(s) = \frac{n\alpha\left[\,G(s)+H(s)\cdot\left(\frac{\xi}{s+\xi}\right)\right]\cdot}{F(s)}\cdot\left(\frac{1}{s+\mu_2}\right) \quad -----(100)$$

$$\overline{P}_3(s) = \frac{(1-\lambda)\,G(s)}{F(s)}\cdot\left(\frac{1}{s+\eta}\right) \quad -----(101)$$

$$\overline{P}_4(s) = \frac{2\beta}{F(s)}\cdot\frac{1}{(s+n\alpha\lambda+\beta+\xi)} \quad -----(102)$$

$$\overline{P}_5(s) = \frac{I(s)}{F(s)} \quad -----(103)$$

$$\overline{P}_6(s) = \frac{w\cdot I(s)}{F(s)}\cdot\left(\frac{1}{s+\xi_1}\right) \quad -----(104)$$

$$\overline{P}_7(s) = \frac{H(s)}{F(s)} \quad -----(105)$$

$$\overline{P}_8(s) = \frac{(1-\lambda)\,H(s)}{F(s)}\cdot\left(\frac{1}{s+\eta}\right) \quad -----(106)$$

$$\overline{P}_9(s) = \frac{n\alpha\cdot H(s)}{F(s)}\cdot\left(\frac{1}{s+\xi}\right) \quad -----(107)$$

$$\overline{P}_{10}(s) = \frac{\beta\cdot H(s)}{F(s)}\cdot\left(\frac{1}{s+\mu_1}\right) \quad -----(108)$$

$$\overline{P}_e(s) = \frac{\gamma_{e_1}+\gamma_{e_2}G(s)}{F(s)}\cdot\left(\frac{1}{s+\mu_e}\right) \quad -----(109)$$

$$\overline{P}_h(s) = \frac{\gamma_{h_1} + \gamma_{h_2} G(s)}{F(s)} \cdot \left(\frac{1}{s+\mu_h}\right) \qquad -----(110)$$

where,

$$G(s) = \frac{n\alpha\lambda}{s + n\lambda + 2\beta + \gamma_{e_2} + \gamma_{h_2} + (1-\lambda)\left(\frac{s}{s+\eta}\right)} \qquad -----(111)$$

$$H(s) = \frac{2\beta\left[G(s) + n\alpha\lambda \cdot \frac{1}{(s+n\alpha\lambda+\beta+\xi)}\right]}{s + n\alpha + \beta + (1-\lambda)\left(\frac{s}{s+\eta}\right)} \qquad -----(112)$$

$$I(s) = \frac{\beta}{s+w}\left[2\beta \cdot \frac{1}{(s+n\alpha\lambda+\beta+\xi)} + H(s)\left(\frac{\mu_1}{s+\mu_1}\right)\right] \qquad -----(113)$$

and

$$\begin{aligned} F(s) = {} & s + n\alpha\lambda + 2\beta + \gamma_{e_1} + \gamma_{h_1} - \left[\gamma_{e_1} + \gamma_{e_2} G(s)\right]\left(\frac{\mu_e}{s+\mu_e}\right) - 2\beta\frac{\xi}{(s+n\alpha\lambda+\beta+\xi)} \\ & - \left[\gamma_{h_1} + \gamma_{h_2} G(s)\right]\left(\frac{\mu_h}{s+\mu_h}\right) - n\alpha\left[G(s) + H(s)\cdot\left(\frac{\xi}{s+\xi}\right)\right]\left(\frac{\mu_2}{s+\mu_2}\right) \\ & - w\, I(s)\cdot\left(\frac{\xi_1}{s+\xi_1}\right) \end{aligned} \qquad -----(114)$$

it is interesting to note that

Sum of equations (98) through (110) $= \frac{1}{s}$ --------(115)

(b) ***Evaluation of up and down state probabilities*:**

We have,

$$\overline{P}_{up}(s) = \frac{1}{(s+n\alpha\lambda+2\beta+\gamma_{e_1}+\gamma_{h_1})}\left[1+\frac{n\alpha\lambda}{[(s+n\lambda+2\beta+\gamma_{e_2}+\gamma_{h_2}+(1-\lambda)]} + \frac{2\beta}{(s+n\alpha\lambda+2\beta)}\right] \quad ------(116)$$

On inverting this, we get

$$P_{up}(t) = (1+L+N)e^{-(n\alpha\lambda+2\beta+\gamma_{e1}+\gamma_{h1})t} + M e^{-(n\lambda+2\beta+\gamma_{e2}+\gamma_{h2}+(1-\lambda))t} + Q e^{-(n\alpha\lambda+\beta)t} \quad -------(117)$$

where,

$$L = \frac{n\alpha\lambda}{n\lambda(1-\alpha)-\gamma_{e_1}-\gamma_{h_1}+\gamma_{e_2}+\gamma_{h_2}+(1-\lambda)} = -M \quad -----(118)$$

$$N = \frac{2\beta}{\gamma_{e_1}+\gamma_{h_1}+\beta} = -Q \quad -----(119)$$

Note that $P_{up}(0) = 1$ ----------(120)

Also, $P_{down}(t) = 1 - P_{up}(t)$ ----------(121)

(c) *Reliability evaluation*:

we have,

$$\overline{R}(s) = \frac{1}{s+n\alpha\lambda+2\beta+\gamma_{e_1}+\gamma_{h_1}} \quad ------(122)$$

On inverting this, we get

$$R(t) = e^{-(n\alpha\lambda+2\beta+\gamma_{e1}+\gamma_{h1})t} \quad ------(123)$$

(d) *Mean time to system failure (M.T.S.F.):*

$$M.T.S.F. = \lim_{s \to 0} \overline{R}(s) \qquad ------(124)$$

$$\Rightarrow M.T.S.F. = \lim_{s \to 0} \left[\frac{1}{s + n\alpha\lambda + 2\beta + \gamma_{e_1} + \gamma_{h_1}} \right]$$

$$\Rightarrow M.T.S.F. = \frac{1}{(n\alpha\lambda + 2\beta + \gamma_{e_1} + \gamma_{h_1})} \qquad ------(125)$$

(e) ***Numerical computation:***

For a numerical example, let us consider the values

n = 5, $\alpha = 0.01$, $\lambda = 0.06$, $\beta = 0.02$, $\eta = 0.01$, $\gamma_{e1} = \gamma_{h1} = 0.001$, $\gamma_{e2} = \gamma_{h2} = 0.002$, w = 0.005, $\xi = \xi_1 = 0.004$, $\mu_1 = 0.003$, $\mu_2 = 0.007$, $\mu_e = 0.04$, $\mu_h = 0.03$ and t = 0,1,2-------,10.

6.8 CONCLUSION OF THE PAPER:

When we plot various graphs, shown in the figs (2) through (6), we observer that:

(i) Availability of the considered system decreases slowly and for t = *7* and *8* it remains nearly same, after this it again decreases approximately in the constant manner.

(ii) Reliability of the considered system decreases rapidly up to t = 5 and thereafter it decreases smoothly.

(iii) When we make increase in the value of α the M.T.S.F. decreases at the constant rate and for α = 0.07 and 0.08, it remains nearly the same.

(iv) When we make increase in the value of β the M.T.S.F. decreases rapidly initially up to β = 0.07 and thereafter it decreases solely in constant way.

t	P_{up} (t)
0	1
1	0.992998
2	0.986628
3	0.980012

4	0.972939
5	0.965384
6	0.957365
7	0.948918
8	0.940078
9	0.930878
10	0.921351

Table – 1

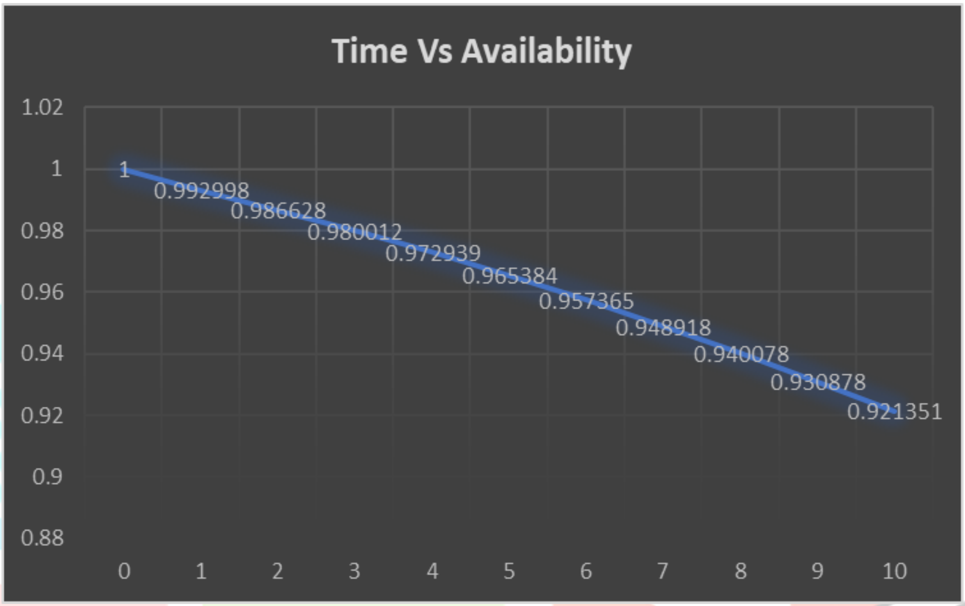


Fig- 2

t	P_{down} (t)
0	0
1	0.007002
2	0.013372
3	0.019988
4	0.027061
5	0.034616
6	0.042635
7	0.051082
8	0.059922
9	0.069122
10	0.078649

Table – 2

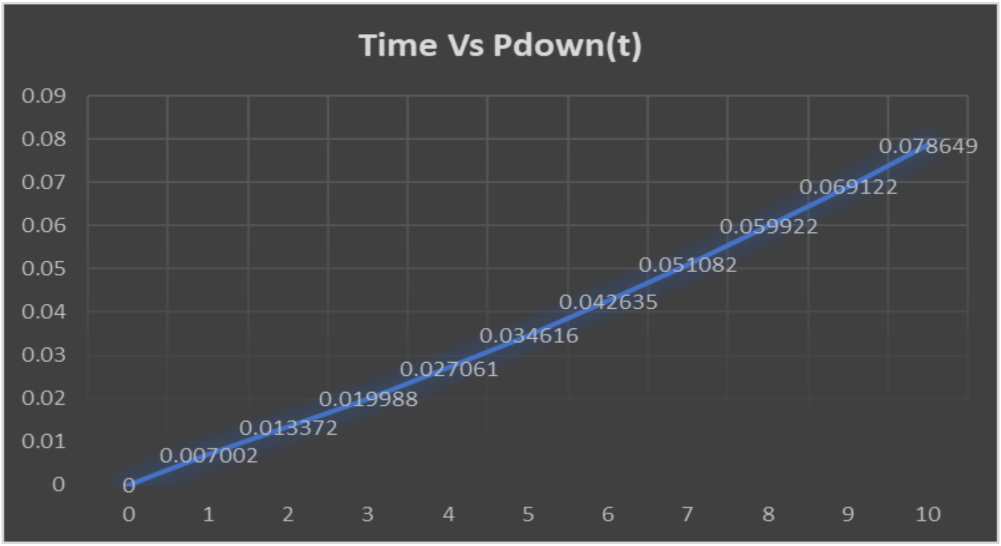


Fig-3

t	R (t)
0	1
1	0.95599748
2	0.91393118
3	0.87371591
4	0.83527021
5	0.79851621
6	0.76337949
7	0.72978887
8	0.69767632
9	0.66697681
10	0.63762815

Table- 3

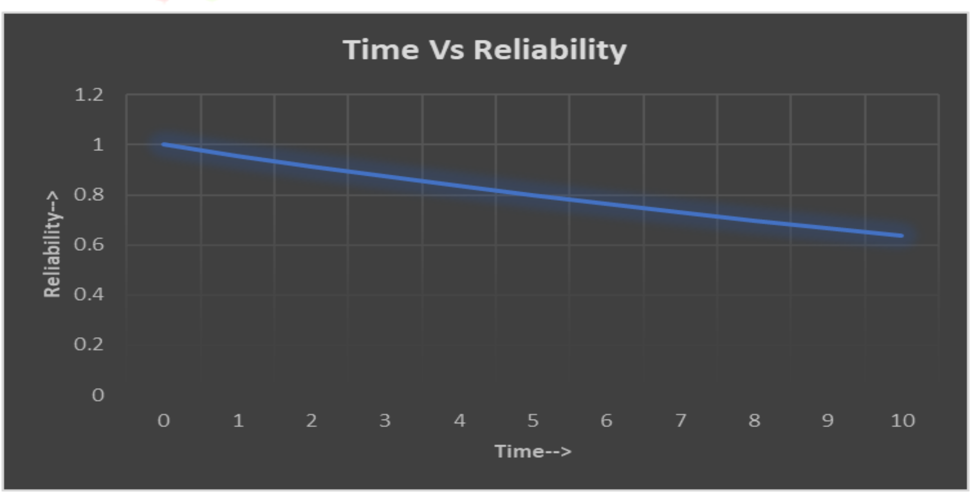


Fig-4

α	M.T.S.F.
0.01	22.222223
0.02	20.833334
0.03	19.607843
0.04	18.518518
0.05	17.543859
0.06	16.666667
0.07	15.873015
0.08	15.151515
0.09	14.492753
0.10	13.888889

Table-4

β	M.T.S.F.
0.01	40.000000
0.02	22.222223
0.03	15.384615
0.04	11.764705
0.05	9.523809
0.06	8.000000
0.07	6.896552
0.08	6.060606
0.09	5.405405
0.10	4.878049

Table-5

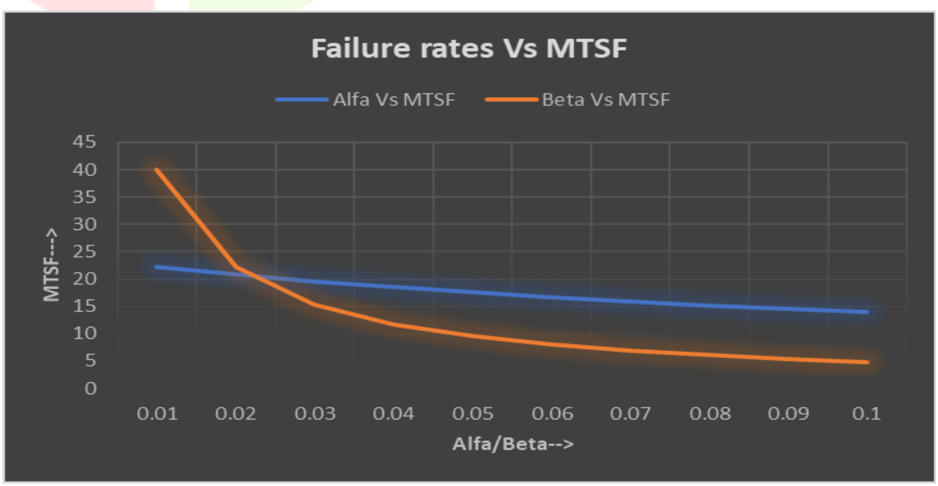


Fig-5