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Introduction To W_h Closed Sets In Hexa Topological Spaces

T. Kalaiselvi[1] and G. Sindhu[2]

[1]Research Scholar, Department of Mathematics,

Nirmala College for Women, Coimbatore(TN) India.

[2]Associate Professor, Department of Mathematics,

Nirmala College for Women, Coimbatore(TN) India.

Abstract

The aim of this paper is to introduce a new notion of set called W_h closed set in hexa topological space. Also, hexa interior and hexa closure of W_h sets were framed. Furthermore, some of the theorems and properties are verified with examples and the relationship between the W_h closed set and other existing sets were investigated.

Keywords: hexa W_h open, hexa W_h closed , hexa W_h interior set, hexa W_h closure.

1 Introduction

The single topology is extended to bi-topological space by Kelly[1], tri-topological space by Kovar[2], quad-topological space by Mukundan[3], penta-topological space by Muhammad Shankar Khan and Gulzar Ali Khan[4] and hexa topological space by R.V Chandra[5]. Also, he introduced the notation of h-open sets and h-closed sets in hexa topological spaces.

An α-open, pre-open, b-open, β-open sets have been introduced and investigated by O.Njasted who knows the α open set and studied α continuous and α irresolute[9,11,12], Mashhour [11] introduced and studied the concept pre open set, pre continuous and pre irresolute in topological space. While Andrjevic[13] presented b open set and studied its characteristics of b continuous and b irresoltute. El-Monsef [13] introduced the ideal of β open set and β continuous, so studied its characteristics β irresolute by Maheshwair and Thakur [9]. In hexa topological set

Semi-h-open, α_h-open, β_h-open,pre_h open, b_h-open sets are introduced by Asmaa S. Qaddoori[6]. In this paper, W_h closed set in hexa topological space were discussed. Also, hexa interior and hexa closure of W_h closed sets were framed. Also, some of its basic properties were studied and its relationship with other existing sets were investigated and the converse part is verified with the examples.

2 Preliminaries

Definition 2.1 [5]

Let X be a non- empty set and τ_1, τ_2, τ_3, τ_4, τ_5, τ_6 are general topology on X. Then a subset A of space X is said to be hexa-open(h-open) set if $A\in\tau_1\cup \tau_2 \cup \tau_3 \cup \tau_4 \cup \tau_5 \cup \tau_6$

Definition 2.2 [5]

Let X be a non- empty set and τ_1, τ_2, τ_3, τ_4, τ_5, τ_6 are general topology on X. Then a subset A of space X is said to be hexa-closed(h-closed) set if $A\in\tau_1 \cap \tau_2\cap\tau_3 \cap\tau_4 \cap \tau_5 \cap \tau_6$

Definition 2.3 [5]

The set X with six topologies called (X, τ_1, τ_2,τ_3, τ_4, τ_5, τ_6) Hexa topology (h-topology).

Definition 2.4 [5]

If (X,τ) is a topological space then a set $A\subseteq X$ is said to be open if $A\in\tau$

Definition 2.5 [5]

If (X,τ) is a topological space then a set $A\subseteq X$ is said to be closed if $A^c\in\tau$

Definition 2.6 [6]

If $(M, \tilde{y}_h)$ is a h_topological and $F \leq M$. Then

1. The h_ interior of F is the union of all h-open subset contained in F and is denoted by int $(F)_h$. So be int $(F)_h$is the largest h-open subset of F
2. The h_ closure of E is the intersection of all h −closed sets containing E and is denoted by $cl(E)_h$. So be $cl(E)_h$is the smallest h_closed set containing E.

Definition 2. 1 [6]

A subset P of space $(M, \tilde{y}_h)$ is said to be:

1- Hexa Semi-open set if A ⊆ τ-$cl_h(int_h(A))$

2- Hexa α₋ $open$ set if A⊆ int_h $(cl_h(int_h(A)))$. Hence A^c is called α_h _closed set.

3- Hexa pre_open set (p_{h-}) if A⊆$int_h(cl_h(A))$. Hence A^c is called p_{h-}closed set.

4- Hexa β_open set (β_h_$open$) if A⊆$cl_h(int_h(cl_h(A)))$. Hence A^c is called β_h_closed set.

5- Hexa b_open set (h _ open) if A⊆$(cl_h(int_h(A)) \cup int_h(cl_h(A))$. Hence A^c called b_h_closed set.

3 W_h-closed set in Hexa topological space

Definition: 3.1

Let (X, τ_h) be a hexa topological space. The subset, A of X is said to be W_h-closed set if $cl_h(A) \subseteq U$ and U is semi-h-open in X.

Theorem 3.2

In a hexa topological space (X, τ_h), every W_h closed is Semi-h-closed .

Proof:

Let (X, τ_h) be the hexa topological space. Let A be the subset of X. Let A be the W_h-closed set in X. Then establish A is Semi-h-closed. Since A is W_h-closed, for A⊆ U and U is semi-h-open in (X, τ_h), we have $cl_h(A) \subseteq U$. A⊆ U, U is semi-h-open, $cl_h(A) \subseteq U$. A⊆ $cl_h(A)$, A⊆ $cl_h(A) \subseteq U$, A⊆ $cl_h(A)$, $int_h(A) \subseteq int_h(cl_h(A))$, $int_h(A) \subseteq A$, $int_h(cl_h(A)) \subseteq A$. Then A is Semi-h-closed.

Example 3.3: X={1,2,3,4,5}, τ_1={φ ,X,{1}}, τ_2={φ ,X,{2}}, τ_3={φ ,X,{3}}, τ_4={φ ,X,{1,2}}, τ_5={φ ,X,{2,3}}, τ_6={φ ,X,{1,3}}

Semi-h-closed sets are {{2,3,4,5},{1,3,4,5},{1,2,4,5},{3,4,5},{2,4,5}, {2,3,5}, {2,3,4},{1,4,5}, {1,3,5}, {1,3,4}, {1,2,5}, {1,2,4}, {4,5}, {2,5}, {2,3}, {3,4}, {2,4}, {1,5}, {1,3},{1,4}, {1,2},{3,5}, {5}, {1}, {2}, {3}, {4}}

W_h closed sets are {{1,4,5}, {2,4,5}, {3,4,5}, {2,3,4,5}, {1,3,4,5},{1,2,4,5}}

Theorem 3.4

Every α_h closed is W_h closed

Proof:

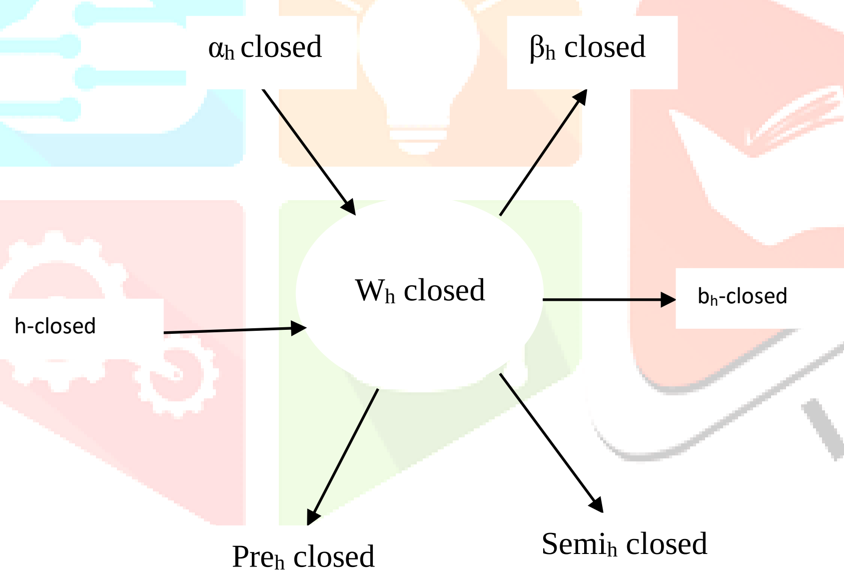
Suppose A is α_h closed, let $A \subseteq U$ and U is Semi-h-open, then $\alpha\text{-}cl_h(A) = A \subseteq U$, $\alpha\text{-}cl_h(A) \subseteq U$, Since every α_h closed is closed $\alpha\text{-}cl_h(A) \subseteq cl_h(A)$ then $cl_h(A) \subseteq U$ so every α_h closed is W_h closed.

Example 3.5: X={1,2,3,4,5}, τ_1={φ ,X,{1}}, τ_2={φ ,X,{2}}, τ_3={φ ,X,{3}}, τ_4={φ ,X,{4}}, τ_5={φ ,X,{1,2}}, τ_6={φ ,X,{3,4}}

α_h closed sets are {{2,3,4,5},{1,3,4,5},{1,2,4,5},{1,2,3,5},{3,4,5},{2,4,5},{2,3,5}, {1,4,5}, {1,3,5}, {1,2,5},{4,5},{1,5}, {3,5},{5}, {1}}

W_h closed sets are {{1}, {5}, {1,5}, {2,5}, {3,5}, {4,5}, {1,4,5}, {1,2,5}, {1,3,5}, {2,4,5}, {2,3,5},{3,4,5}, {2,3,4,5}, {1,3,4,5},{1,2,4,5},{1,2,3,5}}

Remark: Similarly Every W_h closed set contains in h-closed, β_h closed, Pre_h closed and b_h closed. Their representation is in the following figure



4 W_h-Closure and Interior Operators

Definition 4.1

Let (X, τ_h) be a hexa topological space and $A \subseteq X$. The W_h int(A) is defined by the union of all W_h open sets contained in A. That is int W_h (A) = U {G: G⊆A and G is W_h open} then W_h interior of the subset A is denoted by int W_h (A).

Definition 4.2

Let (X, τ_h) be a hexa topological space and A ⊆ X. The W_h cl(A) is defined by the intersection of all W_h closed sets contained in A. That is cl W_h (A)= n {G: A⊆G and G is W_h closed} then W_h closed of the subset A is denoted by cl W_h (A)

Theorem 4.3

Let (X,τh) be the hexa topological space and let A⊆ B⊆ X, then

i) Int W_h (φ) = φ

ii) Int W_h (X) = X

iii) Int W_h (A) ⊆ A

iv) Int W_h (A) ⊆ Int W_h (B)

v) Int W_h (Int W_h (A)) = Int W_h (A)

Theorem 4.4

Let (X,τh) be the hexa topological space and let A⊆ B⊆ X, then

i) cl W_h (φ) = φ

ii) cl W_h (X) = X

iii) A ⊆ cl W_h (A)

iv) cl W_h (A) ⊆ cl W_h (B)

v) cl W_h (cl W_h (A)) = cl W_h(A)

Theorem 4.5

For hexa topological space (X,τh)

i) Int W_h (A) ∪ Int W_h (B) ⊆ Int W_h (A ∪ B)

ii) Int W_h (A) ∩ Int W_h (B) ⊇ Int W_h (A ∩ B)

Theorem 4.6

For hexa topological space (X,τh)

i) cl W_h (A) ∪ cl W_h (B) ⊇ cl W_h (A ∪ B)

ii) cl W_h (A) ∩ cl W_h (B) ⊆ Int W_h (A ∩ B)

5 Characteristics of W_h-closed sets

Theorem 5.1

Union of two W_h closed set is hexa W_h closed set.

Proof:

Let A and B be two W_h closed set in hexa topological space (X, τ_h), Since A is W_h closed, cl_h(A) ⊆ U, Whenever A⊆ U and U is Semi-h-open in X. Since B is W_h closed cl_h(B) ⊆ U, Whenever B⊆ U and U is Semi-h-open in X. We have cl_h(A∪B) = cl_h(A) ∪ cl_h(B), cl_h(A∪B) ⊆ U therefore (A∪B) is W_h closed in (X, τ_h). Therefore union of two W_h closed set is hexa W_h closed set.

Example 5.2 X={1,2,3,4,5} τ_1={φ ,X,{1},{2}}, τ_2={φ ,X,{1}}, τ_3={φ ,X,{2}}, τ_4 ={φ ,X,{4}}, τ_5={φ ,X,{1},{4}}, τ_6={φ ,X}.

h-open sets are {φ ,X,{1},{2},{4}}, h-closed sets are {X, φ , {2,3,4,5},{1,3,4,5},{1,2,3,5}}

W_h closed sets are {{1,3,5},{2,3,5},{3,4,5}, {2,3,4,5},{1,3,4,5}, {1,2,3,5}} Here, the intersection of W_h closed sets {1,3,5} n {2,3,5} is {3,5} which is not in W_h closed set.

Remark

The Intersection of two W_h closed set is need not be W_h closed set in (X, τ_h).

Theorem 5.3

Let (X, τ_h) be a hexa topological space, let A be the W_h closed set in (X, τ_h) and A⊆B⊆ cl_h(A), then B is also W_h closed set in (X, τ_h).

Proof:

Let A be W_h closed set in (X, τ_h) this implies, cl_h(A) ⊆ U, Whenever U is Semi-h-open in (X, τ_h). We have A⊆B⊆ cl_h(A), Since cl_h(A) ⊆ U then B⊆ cl_h(A) ⊆ U, then B ⊆ U Now B⊆ cl_h(A), cl_h(B) ⊆ cl_h(cl_h(A)), cl_h(B) ⊆ U. Therefore cl_h(B) ⊆ U whenever B ⊆ U and U is Semi-h-open in X. This implies B is W_h closed set in X.

Theorem 5.4

If a subset A of X is W_h closed in X then cl_h(A) \ A contain a Semi-h-open set in X.

Example 5.5: If $cl_h(A) \setminus A$ contains empty in semi-h-open subsets in X, then consider X= {1,2,3,4,5} with τ_1={φ ,X,{1}}, τ_2={φ ,X,{2}}, τ_3={φ ,X,{3}}, τ_4={φ ,X,{1,2}}, τ_5={φ ,X,{2,3}}, τ_6={φ ,X,{1,3}} then Subset of X, A={1,4,5} then $cl_h(A) \setminus A$ = {1,4,5} \ {1,4,5} = φ contained in every sets in Semi-h-open sets.

Theorem 5.6

If x϶X, the set X\{x} is W_h closed or Semi-h-open.

Proof:

Suppose X\{x} is not Semi-h-open then X is the only Semi-h-open set containing X\{x}. This implies cl_h\{X\{x}}⊆X. Hence X\{x} is a Semi-h-closed set in X.

Example 5.7: If X={6,7,8,9,10}, τ_1={φ ,X,{6}}, τ_2={φ ,X,{7}}, τ_3={φ ,X,{8}}, τ_4={φ ,X,{6,7}}, τ_5={φ ,X,{7,8}}, τ_6={φ ,X,{6,8}}

Semi-h-closed sets are {{7,8,9,10},{6,8,9,10},{6,7,9,10},{8,9,10},{7,9,10}, {7,8,10}, {7,8,9},{6,9,10}, {6,8,10}, {6,8,9}, {6,7,10}, {6,7,9}, {9,10}, {7,10}, {7,8}, {8,9}, {7,9}, {6,10}, {6,8},{6,9}, {6,7},{8,10}, {10}, {6}, {7}, {8}, {9}}

W_h closed sets are {{6,9,10}, {7,9,10}, {8,9,10}, {7,8,9,10}, {6,8,9,10},{6,7,9,10}}

Theorem 5.8

If A is Semi-h-open and W_h closed, then A is W_h closed in X.

Proof:

Consider a subset A be Semi-h-open and W_h closed in X then, A is W_h closed set in X. Let U be any Semi-h-open set in X then A ⊂ U. Since A is Semi-h-open and W_h closed. So we have $cl_h(A) \subset A$. Then $cl_h(A) \subset A \subset U$. Hence A is W_h closed in X.

Example 5.9: Let X={11,12,13,14,15}, τ_1={φ ,X,{11}}, τ_2={φ ,X,{12}}, τ_3={φ ,X,{13}}, τ_4={φ ,X,{11,12}}, τ_5={φ ,X,{12,13}}, τ_6={φ ,X,{11,13}}.

Semi-h-closed sets are {{12,13,14,15},{11,13,14,15},{11,12,14,15},{13,14,15},{12,14,15}, {12,13,15}, {12,13,14},{11,14,15}, {11,13,15}, {11,13,14}, {11,12,15}, {11,12,14}, {14,15}, {12,15}, {12,13}, {13,14}, {12,14}, {11,15}, {11,13},{11,14}, {11,12},{13,15}, {15}, {11}, {12}, {13}, {14}}

W_h closed sets are {{11,14,15}, {12,14,15}, {13,14,15}, {12,13,14,15}, {11,13,14,15}, {11,12,14,15}}.

Theorem 5.10

Let A be W_h closed in (X,τh). Then A is closed if and only if $cl_h(A) \setminus A$ is Semi-h-open.

Proof:

Let A is closed in X. Then $cl_h(A) = A$ and $cl_h(A) \setminus A = \varphi$. Which is Semi-h-open in X.

Conclusion

Here, we discussed the properties of hexa open, hexa closed, hexa interior, hexa closure sets. Further we decide to investigate the relation between general and hexa topology.

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