IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

SOFT PARACONTINUOUS MAPS IN SOFT TOPOLOGICAL SPACES

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ABSTRACT: In this paper author introduced a new class of soft maps called soft paracontinuous maps in soft topological spaces. A map f: (X, τ, E) → (Y, μ, E) is called soft paracontinuous if $f^{-1}(F,E)$ is a soft open set in X for every soft paraopen set (F,E) in Y. Also has introduced soft $*$-paracontinuous, soft parairresolute, soft minimal paracontinuous and soft maximal paracontinuous maps in soft topological spaces. Some of their properties have been investigated.

2010 Mathematics classification: 54C05

Keywords: Soft paracontinuous, soft $*$-paracontinuous, soft parairresolute, soft minimal paracontinuous and soft maximal paracontinuous maps.

1. Introduction and Preliminaries

In the years 2001 and 2003, F.Nakaoka and N.Oda [1, 2, 3] presented and contemplated minimal open (resp. minimal closed) sets and maximal open (resp. maximal closed) sets, which are subclasses of open (resp. closed) sets. The complements of minimal open sets and maximal open sets are known as maximal closed sets and minimal closed sets respectively. In the year 1999, Russian specialist Molodtsov [4], started the idea on soft sets as another scientific instrument to manage vulnerabilities while demonstrating issues in building material science, software engineering, financial aspects, sociologies and restorative sciences. In Molodtsov[5], connected effectively in bearings, for example, smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability and hypothesis to estimation. The soft set is an accumulation of inexact portrayals of an article. Likewise he also indicated why soft set hypothesis is excluded from the parametrization deficiency disorder of fuzzy set hypothesis, rough set hypothesis, probability theory and game theory.

In 2002, Maji, Biswas and Roy [6], introduce few new statements on soft sets and displayed first practical use of soft sets in decision making problems that depends on the lessening of parameters to keep the ideal decision objects. In 2003, Maji, Biswas and Roy [7], examined the hypothesis of the soft sets started from Molodtsov. They characterized equity of two soft sets, subset and super set of the soft set, complement of the soft set, null soft set and absolute soft set along cases. The Soft binary operations like AND, OR furthermore the operations of union and the intersection were also characterized. In 2005, D. Chen [8], introduced another meaning of the soft set parametrization lessening and correlation with property decrease on soft set hypothesis. In 2005, D. Pie, D. Miao [9], examined the difference between soft sets and data frameworks. They demonstrated soft sets are a class of unique data frameworks. In 2008, Z. Kong, L. Gao, L. Wong, S. Li [10], presented the thought of ordinary parameter decrease of soft sets and its utilization to explore the issue of imperfect decision and included a parameter set in soft sets.

As of late, specialists have contributed a lot towards fuzzification of soft set theory. In 2001, Maji P. K., Biswas R and Roy A.R. [11], presented the idea of fuzzy soft Set and a few properties with respect to fuzzy soft union, intersection, supplement of a fuzzy soft set, De Morgan Law and so forth. In 2007, X. Yang, D. Yu, J. Yang, C. Wu [12], consolidated the interim esteemed fuzzy set and soft set models and presented the idea of interim esteemed fuzzy soft set. Topological of soft set and fuzzy soft set of topological structures are studied by a few creators as of late. In 2011, Muhammad Shabir and Munazza Naz and Naim Cagman et al, started the investigation of soft topology and soft topological spaces independtly. Muhammad Shabir and Munazza Naz [13], presented the thought of soft topological spaces which are characterized over an underlying universe with a settled arrangement of parameters and demonstrated that a soft topological space gives a parameterized group of topological spaces. They presented the meanings of soft open sets, soft closed sets, soft interior, soft closure and soft separation axioms. Additionally they got few fascinating results for soft separation axioms, which are truly profitable for exploration in this field. N. Cagman, S. Karatas and S. Enginoglu [14], characterized the soft topology on a soft set, and displayed its related properties and establishments of the hypothesis of soft topological spaces. The thought of soft topology by Naim Cagman et al. is broad than that by Shabir and Naz.

In the meantime, Abdul kadir Aygunoglu and Halis Aygun [15], presented soft topological spaces and soft continuity of soft mappings. They additionally explored starting soft topologies and soft compactness. In 2011, Sabir Hussain and Bashir Ahmad [16], examined the properties of soft open (closed), soft neighborhood and soft closure. Likewise characterized and examined the properties of soft interior, soft exterior and soft boundary which are essential for further research on soft topology and establishments of the hypothesis of soft topological spaces. In 2012, Bashir Ahmad and Sabir Hussain [17], characterized soft exterior and examined its essential properties and set up a few critical results relating soft interior, soft exterior, soft closure, and soft boundary in soft topological spaces. In addition, they described soft open sets, soft closed sets and soft clopen defines by means of soft boundary. In 2007, H. Hazra, P. Majumdar and S.K.Samanta [18], presented the thoughts of topology on soft subsets and soft topology. Some essential properties of these topologies are studied. In 2004, MetinAkdag and AlkanOzkan[19], presented and examined the idea of soft α-Open sets and soft α-constant functions. In 2015, A. Selvi and I. Arockiarani[20,21], presented and contemplated the idea of soft almost g-continuous functions. In 2014, MetinAkdag and AlkanOzkan[22], presented and considered the idea of soft β-Open Sets and soft β-Continuous functions .In 2010,Pinaki Majumdar and S.k. Samanta,[23] presented and examined the idea of soft mappings in soft topological spaces.In 2011, S. S. Benchalli, Basavaraj M.I and R. S. Wali[24] presented the idea On Minimal Open Sets and Maps in Topological Spaces.In 2015, Hai-Long Yang , Xiuwu Liao and Sheng-Gang Li[25] presented the idea On soft continuous mappings and soft connectedness of soft topological spaces. In 2015, Chetana C.and Naganagouda K.[26],presented the idea on Soft Minimal Continuous and Soft Maximal Continuous Maps in soft topological spaces. In 2016, Basavaraj M. Ittanagi and Shivanagappa S. Benchalli[27], introduced and studied the concept of paraopen sets and paraclosed sets in topological spaces. Also introduced and investigated a new class of maps are paracontinuous, ∗-paracontinuous, parairresolute, minimal paracontinuous and maximal paracontinuous maps and study their basic properties in topological spaces.

Review the accompanying statements, which are requirements for present study.

1.1 Definition [4]: Let U be an initial universe and E be the set of parameters. Let P(U) denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A → P(U).

1.2. Definition [5]: For two soft sets (F, A) and (G,B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if

i) $A \subseteq B$ and

ii) for all $e \in A$, F(e) and G(e) are identical approximations.

We write $(F, A) \tilde{\subset} (G, B)$. (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A) \tilde{\supset} (G, B)$.

1.3. Definition [5]: Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

1.4. Definition [5]: Let E = $\{e_1, e_2, e_3 . . . , e_n\}$ be a set of parameters. The NOT set of ⅂P denoted by ⅂E is defined by ⅂E=$\{e_1, e_2, e_3 . . . , e_n\}$, where ⅂$e_i$= not e_i for all i.

1.5. Definition [5]: The complement of a soft set (F, A) is denoted by $(F,A)^c=(F^c, ⅂A)$ where, F^c: ⅂A → P(U) is a mapping given by $F^c(\alpha) = U \setminus F(⅂\alpha)$, for all $\alpha \in ⅂A$

Let us call F^c to be the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c= (F , A)$.

1.6. Definition [5]: A soft set (F, A) over U is said to be a NULL soft set denoted by "ϕ" if $\forall \varepsilon \in A$, F (ε) = ϕ, (null-set).

1.7. Definition [5]: If (F, A) and (G, B) are two soft sets then (F, A) AND (G, B) denoted by (F, A)Λ(N, B) is defined by (F, A) Λ(G, B) = (H, A × B), where H((α, β)) = F(α) ∩ G(β), for all (α, β) ∩ A × B.

1.8. Definition [5]: If (F, A) and (G, B) are two soft sets then (F, A) OR (G, B) denoted by (F, A) V (G, B) is defined by (F, A) V (G, B) = (O, A × B) where, O((α, β)) = F(α) ∩G(β) for all (α, β) ∩A × B.

1.9. Definition [5]: The union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where C = A∪B and for all e ε C,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$ We write (F, A) ∪(G, B) = (H, C).

1.10. Definition [8]: The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted (F, A) ∩ (G, B), is defined as C=A∩B, and H(e) = M (e) ∩ N(e) for all e ∈ C

1.11. Definition [13]: Let τ be the collection of soft sets over X; then τ is called a soft topology on X if τ satisfies the following axioms:

i) Φ, X belong to τ.

ii) The union of any number of soft sets in τ belongs to τ.

iii) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. The members of τ are said to be soft open in X. A soft set (F, E) over X is said to be soft closed in X if its relative complement $(F, E)^c$ belongs to τ

1.12. Definition [13]: Let (F, E) be a soft set over X and x ∈X. We say that x ∈(F, E) read as 'x' belongs to the soft set (F, E), whenever x∈F(α) for all α ∈E.

Note that for x ∈X, x ∉(F, E) if x ∉F(α) for some α ∈E.

1.13. Definition [13]: Let x ∈X; then (x, E) denotes the soft set over X for which x(α)={a}, for all α ∈E.

1.14. Definition [2]: Let(X, τ, E)be a soft topological space over X, (G, E) be a soft set over X and x∈X. Then (G, E) is said to be a soft neighborhood of a if there exists a soft open set (F, E) such that x ∈(F, E) $\tilde{\subseteq}$ (G, E).

1.15. Definition [2]: Let(X, τ, E) be a soft topological space and (A, E) be a soft set over X.

i) The soft interior of (A, E) is the soft set sint(A, E) = ∪{(O, E): (O, E)is soft openand(O, E) $\tilde{\subseteq}$ (A, E)}.

ii) The soft closure of (A, E)is the soft set scl(A, E) = ∩{(F, E) : (F, E)is soft closed and (A, E) $\tilde{\subseteq}$ (F, E)}.

1.16 Definition [1]: A proper nonempty open subset U of a topological space X is said to be a minimal open set if any open set which is contained in U is ϕ or U.

1.17 Definition [2]: A proper nonempty open subset U of a topological space X is said to be maximal open set if any open set which contains U is X or U.

1.18 Definition [3]: A proper nonempty closed subset F of a topological space X is said to be a minimal closed set if any closed set which is contained in F is ϕ or F.

1.19 Definition [3]: A proper nonempty closed subset F of a topological space X is said to be maximal closed set if any closed set which contains F is X or F.

1.20 Definition [26]: A proper nonempty soft open subset (F,E) of (X,τ,E) is said to be a soft minimal open set if and only if any soft open set which is contained in (F,E) is either ϕ or (F,E) itself.

1.21 Definition [26]: A proper nonempty soft open subset (F,E) of (X,τ,E) is said to be a soft maximal open set if and only if any soft open set which is contains in (F,E) is either X or (F,E) itself.

1.22 Definition [22]: A soft set (F, E) of a soft topological space (X, τ, E) is called soft α-open set if (F, E)$\tilde{\subset}$ int(cl(int(F,E))). The complement of soft α-open set is called soft α-closed set.

1.23 Definition [22]: A soft set (F, E) is called soft preopen set (resp., soft semiopen) in a soft topological space X if (F, E)$\tilde{\subset}$ int(cl(F, E)) (resp., (F, E)$\tilde{\subset}$ cl(int(F, E))).

1.24 Definition [22]: A soft mapping f:X $\rightarrow$ Y is said to be soft α- continuous if the inverse image of each soft open subset of Y is a soft α-open set in X.

1.25 Definition[22]:A soft mapping f:X $\rightarrow$ Y is called soft precontinuous(resp., soft semicontinuous) if the inverse image of each soft open set in Y is soft preopen (resp., soft semiopen) in X.

1.26 Definition [20]: A function f:X $\rightarrow$ Y is called soft almost open(soft almost closed), if the image of every soft regular open subset of X is soft open(soft regular closed) subset of Y.

1.27 Definition [22]: A subset (F, E) of a topological space X is called soft generalized-closed (soft g -closed), if cl(F,E) $\subset$ (G,E) whenever (F,E) $\subset$ (G,E) and (G,E) is soft open in X.

1.28 Definition [19]: A subset (F, E) of a soft topological spaceX is called soft regular closed, if cl(int(F,E)) = (F,E). The complement of soft regular closed set is soft regular open set.

1.29 Definition [19]:A soft set (F, E) of a soft topological space (X, τ, E) is said to be soft β-open if (F,E)$\tilde{\subset}$cl(int(cl(F,E))).

1.30 Definition [19]: Let X and Y are two nonempty sets and E' be a parameter set.Then the mappingf:$X^1$$\rightarrow$U($Y^1$) is called a soft mapping from X to Y under E',where Y' is the collection of all mappings from X to Y.

1.31 Definition[17]: A soft mapping f:X $\rightarrow$ Y is called soft β-continuous (resp., soft α-continuous, soft precontinuous , and soft semicontinuous) if the inverse image of each soft open set in Y is soft β-open (resp., soft α-open, soft preopen, and soft semiopen) set in X.

1.32Definition[24]: Consider X and Y are topological spaces. The mapping f:X $\rightarrow$ Y is known as

i) minimal continuous (min-continuous) if f^{-1}(F,E) is an open set in X for each minimal open set (F,E)in Y.

ii) maximal continuous (max-continuous) if f^{-1}(F,E) is anopen set in X for each maximal open set (F,E) in Y.

1.33 Definition[25]: Let(X, τ_1, E) and (Y,τ_2 , E) be two soft topological spaces over X and Y respectively, and f be a mapping from X to Y. If $\forall (G,E)\in\tau_2$ we have mapping $\bar{f}\,(G,E)\in\tau_1$ then f is called a soft continuous mapping from(X, τ_1, E) to (Y,τ_2 , E).

1.34 Definition [27]: Consider (X, τ, E) and (Y, μ, E) are soft topological spaces.The mapping f: $(X, \tau, E) \rightarrow (Y, \mu, E)$ is said to be

i) Soft minimal continuous (soft min-continuous) if f^{-1}((F,E)) is asoft open set in X for each soft minimal open set (F,E) in Y.

ii) Soft maximal continuous (soft max-continuous) if f^{-1}((F,E)) is a soft open set in X for each soft maximal open set (F,E) in Y.

2. Paracontinuous, soft ∗-paracontinuous, soft parairresolute, soft minimal paracontinuous and soft maximal paracontinuous maps

2.1 Definition: Let (X, τ, E) and (Y, μ, E) be soft topological spaces. A map f: X→Y is called

i) soft paracontinuous (briefly soft p_a-continuous) if $f^{-1}(F,E)$ is a soft open set in X for every soft paraopen set (U,E) in Y.

ii) soft∗-paracontinuous(briefly soft∗-p_a-continuous) if $f^{-1}(F,E)$ is soft paraopen set in X for every soft open set (F,E) in Y.

iii) soft parairresolute(briefly p_a-irresolote) if $f^{-1}(F,E)$ is soft paraopen set in X for every soft paraopen set (F,E) in Y.

iv) soft minimal paracontinuous(briefly soft min-p_a-continuous) if $f^{-1}(F,E)$ is soft paraopen set in X for every soft minimal open set (F,E) in Y.

v)soft maximal paracontinuous(briefly soft max-p_a-continuous) if $f^{-1}(F,E)$ is soft paraopen set in X for every soft maximal open set (F,E) in Y.

2.2 Theorem: Every soft continuous map is soft paracontinuous but not conversely.

Proof: Let f: X→Y be a soft continuous map. To prove f is soft paracontinuous. Let (F,E) be any soft paraopen set in Y. Since every soft paraopen set is soft open set. Therefore (F,E) is soft open set in Y.Since f is soft continuous,$f^{-1}(F,E)$is soft open set in X. Hence f is a soft paracontinuous.

2.3Example: Let $X = Y = \{a,b,c,d\}$, $E = \{e_1,e_2\}$, $\tau = \{\phi,(F_1,E),(F_2,E),(F_3,E),X\},$ $where$ $F_1(e_1) = \{a\},$ $F_1(e_2) = \{b\}; F_2(e_1) = \{a,b\},$ $F_2(e_2) = \{c,d\}; F_3(e_1) = \{a,b,c\},$ $F_3(e_2) = \{b,c,d\}$ and $\mu = \{\phi,(F_1,E),(F_2,E),(F_3,E),(F_4,E),Y\},$ $where$ $F_1(e_1) = \{a\},$ $F_1(e_2) = \{b\}; F_2(e_1) = \{b\},$ $F_2(e_2) = \{c\}; F_3(e_1) = \{a,b\},$ $F_3(e_2) = \{c,d\}; F_4(e_1) = \{a,b,c\},$ $F_4(e_2) = \{b,c,d\}$.Let f: X→Y be soft identity map.Then f is soft paracontinuous but it is not a soft continuous map,since for the soft open set (F_2,E) in Y,$f^{-1}(F_2,E)$=(F_2,E) which is not soft open set in X.

2.4 Theorem: Every soft ∗-paracontinuous map is soft continuous map but not conversely.

Proof: Let$f$: X→Y be a soft ∗-paracontinuous map. To prove f is soft continuous map. Let (F,E) be a soft open set in Y.Since f is soft ∗-paracontinuous,$f^{-1}(F,E)$is a soft paraopen set in X. Since every soft paraopen set is soft open set,$f^{-1}(F,E)$ is soft open set in X. Hence f is a soft continuous map.

2.5Example: Let $X = Y = \{a,b,c,d\}$, $E = \{e_1,e_2\}$, $\tau = \{\phi,(F_1,E),(F_2,E),(F_3,E),X\},$ $where$ $F_1(e_1) = \{a\},$ $F_1(e_2) = \{b\}; F_2(e_1) = \{a,b\},$ $F_2(e_2) = \{b,c\}; F_3(e_1) = \{a,b,c\},$ $F_3(e_2) = \{b,c,d\}$ and $\mu = \{\phi,(F_1,E),(F_2,E),(F_3,E),Y\},$ $where$ $F_1(e_1) = \{a\},$ $F_1(e_2) = \{b\}; F_2(e_1) = \{a,b\},$ $F_2(e_2) = \{c,d\}; F_3(e_1) = \{a,b,c\},$ $F_3(e_2) = \{b,c,d\}$. Let$f$: X→Y be soft identity map. Then f is soft paracontinuous but it is not a soft ∗-paracontinuous map, since for the soft open set (F_1,E) in Y,$f^{-1}(F_1,E)$=(F_1,E) which is not soft paraopen set in X.

2.6 Theorem: Every soft ∗-paracontinuous map is soft paracontinuous map but not conversely.

Proof: The proof follows from the Theorem 2.2 and 2.4.

2.7Example: In Example 2.5, f is a soft paracontinuous map but it is not a soft ∗-paracontinuous map.

2.8 Theorem: Every soft parairresolute map is soft paracontinuous map but not conversely.

Proof: Let$f$: X→Y be a soft parairresolute map.To prove f is soft paracontinuous map. Let (F,E) be any soft paraopen set in Y. Since f is soft parairresolute,$f^{-1}(F,E)$ is a soft paraopen set in X. Since every soft paraopen set is soft open set,$f^{-1}(F,E)$ is soft open set in X. Hence f is a soft paracontinuous map.

2.9Example: Let$X = Y = \{a,b,c,d\}, E = \{e_1,e_2\}, \tau = \{\phi,(F_1,E),(F_2,E),X\}, where\ F_1(e_1) = \{a,b\}, F(e_2) = \{c,d\}; F_2(e_1) = \{a,b,c\} F_2(e_2) = \{b,c,d\};\ and\ \mu = \{\phi,(F_1,E),(F_2,E),(F_3,E),Y\}, where\ F_1(e_1) = \{a\},\ F_1(e_2) = \{b\}; F_2(e_1) = \{a,b\}, F_2(e_2) = \{c,d\}; F_3(e_1) = \{a,b,c\},\ F_3(e_2) = \{b,c,d\}$. Let f: X→Y be soft identity map. Then f is soft paracontinuous map but it is not a soft parairresolute map, since for the soft paraopen set (F_2,E) in Y,$f^{-1}(F_2,E)$=(F_2,E) which is not a soft paraopen set in X.

2.10 Theorem: Every soft ∗-paracontinuous map is soft parairresolute map but not conversely.

Proof: Let$f$: X→Y be a soft ∗-paracontinuous map.To prove f is soft parairresolute map.Let (F,E) be any soft paraopen set in Y.Since every soft paraopen set is a soft open set, (F,E) is a soft open set in Y. Since f is soft ∗-paracontinuous,$f^{-1}(F,E)$ is soft paraopen set in X. Hence f is a soft parairresolute map.

2.11Example: In Example 2.5, f is a soft parairresolute map but it is not a soft ∗-paracontinuous map.

2.12 Remark: Soft parairresolute and soft continuous maps are independent of each other.

2.13Example: In Example 2.3, f is a soft parairresolute map but it is not a softcontinuous map.

2.14 Theorem: Every soft minimal paracontinuous map is soft minimal continuous but not conversely.

Proof: Let$f$: X→Y be a soft minimal paracontinuous map. To prove f is minimal continuous. Let (F,E) be any soft minimal open set in Y. Since f is soft minimal paracontinuous,$f^{-1}(F,E)$ is soft paraopen set in X. Since every soft paraopen set is an open set,$f^{-1}(F,E)$ is soft open set in X Hence f is a soft minimal continuous.

2.15Example: In Example 2.5, f is a soft minimal continuous but it is not a soft minimal paracontinuous, since for the soft minimalopen set(F_1,E) in Y,$f^{-1}(F_1,E)$=(F_1,E) which is not a soft paraopen set in X.

2.16 Remark: Soft minimal paracontinuous and soft paracontinuous (resp. continuous) maps are independent of each other.

2.17Example: Let $X = Y = \{a,b,c,d,e\}, E = \{e_1,e_2\}, \tau = \{\phi,(F_1,E),(F_2,E),(F_3,E),X\}, where\ F_1(e_1) = \{a\},\ F_1(e_2) = \{b\}; F_2(e_1) = \{a,b\},\ F_2(e_2) = \{b,c\}; F_3(e_1) = \{a,b,c\},\ F_3(e_2) = \{b,c,d\}\ and\ \mu = \{\phi,(F_1,E),(F_2,E),(F_3,E),Y\}, where\ F_1(e_1) = \{a,b\},\ F_1(e_2) = \{c,d\}; F_2(e_1) = \{a,b,d\},\ F_2(e_2) = \{c,d,e\}; F_3(e_1) = \{a,b,c,d\},\ F_3(e_2) = \{b,c,d,e\}$. Let f: X→Y be a soft identity map. Then f is soft minimal paracontinuous but it is not a soft paracontinuous(resp. continuous), since for the soft paraopen (resp. soft open)set (F_2,E) in Y,$f^{-1}(F_2,E)$=(F_2,E) which is not a soft open set in X. In Example 2.5, f is a soft paracontinuous (resp. soft continuous) but it is not soft minimal paracontinuous.

2.18 Theorem: Every soft maximal paracontinuous map is soft maximal continuous but not conversely.

Proof: Let$f$: X→Y be a softmaximal paracontinuous map.To prove f is soft maximal continuous.Let (F,E) be any soft maximal open set in Y.Since f is soft maximal paracontinuous,$f^{-1}(F,E)$ is a soft paraopen set in X. Since every soft paraopen set is soft open set,$f^{-1}(F,E)$ is a soft open set in X. Hence f is a soft maximal continuous.

2.19Example: In Example 2.5, f is a soft maximal continuous but it is not soft maximal paracontinuous, since for the soft maximal open set (F_3,E) in Y,$f^{-1}(F_3,E)$=(F_3,E) which is not a soft paraopen set in X.

2.20 Remark: Soft maximal paracontinuous and soft paracontinuous (resp. continuous) maps are independent of each other.

2.21Example: Let $X = Y = \{a, b, c, d, e\}, E = \{e_1, e_2\}, \tau = \{\phi, (F_1, E), (F_2, E), (F_3, E), X\}$, where $F_1(e_1) = \{a, b\},\ F_1(e_2) = \{c, d\}; F_2(e_1) = \{a, b, c\},\ F_2(e_2) = \{c, d, e\}; F_3(e_1) = \{a, b, c, d\},\ F_3(e_2) = \{b, c, d, e\}$ and $\mu = \{\phi, (F_1, E), (F_2, E), (F_3, E), Y\}$, where $F_1(e_1) = \{a\},\ F_1(e_2) = \{b\}; F_2(e_1) = \{a, c\},\ F_2(e_2) = \{b, d\}; F_3(e_1) = \{a, b, c\},\ F_3(e_2) = \{c, d, e\}$.Let f: X→Y be a soft identity map. Then f is soft maximal paracontinuous map but it is not a soft paracontinuous(resp. continuous), since for the soft paraopen (resp. soft open)set (F_2, E) in Y,$f^{-1}(F_2, E)$=(F_2, E) which is not a soft open set in X. In Example 2.5, f is a soft paracontinuous(resp. soft continuous) but it is not soft maximal paracontinuous.

2.22 Remark: Soft minimal paracontinuous and soft maximal paracontinuous maps are independent of each other.

2.23Example: In Example 2.17, f is a soft minimal paracontinuous but it is not a soft maximal paracontinuous.In Example 2.21, f is a soft maximal paracontinuous but it is not a soft maximal paracontinuous.

2.25 Theorem: Let X and Y be soft topological spaces and (A, E) be a nonempty soft subset of X. If f: X→Y is soft paracontinuous then the restriction map f_A: A→Y is a paracontinuous.

Proof: Let f: X→Y be a soft paracontinuous map and (A, E)⊂X. To prove f_A is a paracontinuous. Let (F, E) be a paraopen set in Y. Since f is paracontinuous, $f^{-1}((F, E))$ is an open set in X. By definition of relative topology $f_A^{-1}((F, E))$= $(A, E)\cap f^{-1}((F, E))$. Therefore $(A, E)\cap f^{-1}((F, E))$ is a soft open set in A. Hence f_A is a soft paracontinuous.

2.26 Remark: From the above discussion and known results we have the following implications.

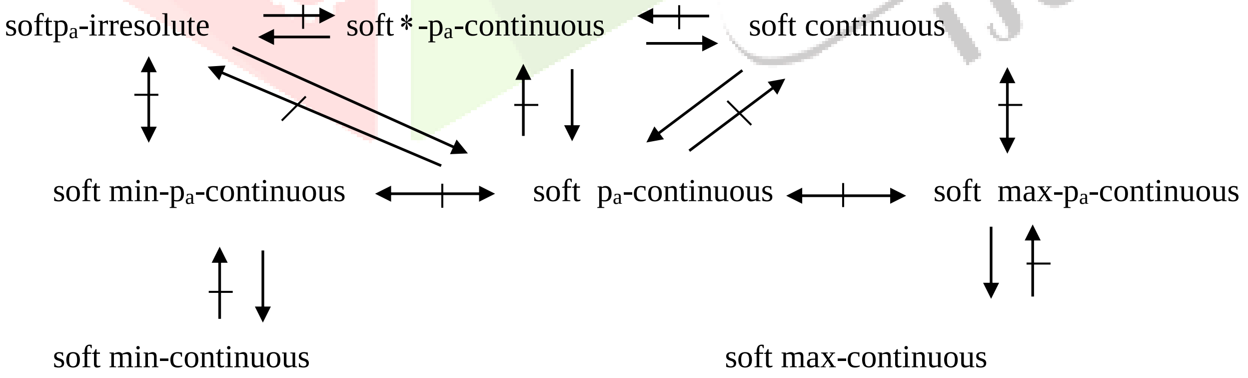

Diagram 2.1

2.27 Theorem: Let X and Y be soft topological spaces and (A, E) be a nonempty soft subset of X. If f: X→Y is soft paracontinuous then the restriction map f_A: A→Y is a paracontinuous.

Proof: Let f: X→Y be a soft paracontinuous map and (A, E)⊂X. To prove f_A is a paracontinuous. Let (F, E) be a paraopen set in Y. Since f is paracontinuous, $f^{-1}((F, E))$ is an open set in X. By definition of relative topology $f_A^{-1}((F, E))$= $(A, E)\cap f^{-1}((F, E))$. Therefore $(A, E)\cap f^{-1}((F, E))$ is a soft open set in A. Hence f_A is a soft paracontinuous.

2.28 Remark: The composition of soft paracontinuous maps need not be soft paracontinuous.

2.29 Example: Let $X = Y = \{a, b, c, d, e\}$, $E = \{e_1, e_2\}$, $\tau = \{\phi, (F_1, E), X\}, where\ F_1(e_1) = \{a, b, c\}$, $F_1(e_2) = \{c, d, e\}\ and\ \mu = \{\phi, (F_1, E), (F_2, E), (F_3, E), Y\}, where\ F_1(e_1) = \{a, b\},\ F_1(e_2) = \{c, d\}; F_2(e_1) = \{a, b, c\}, F(e_2) = \{c, d, e\}; F_3(e_1) = \{a, b, c, d\} F_3(e_2) = \{b, c, d, e\}\ and$ $\eta = \{\phi, (F_1, E), (F_2, E), (F_3, E), Z\}, where\ F_1(e_1) = \{a\},\ F_1(e_2) = \{b\}; F_2(e_1) = \{a, b\},\ F_2(e_2) = \{c, d\}; F_3(e_1) = \{a, b, c\},\ F_3(e_2) = \{c, d, e\}$. Let f: X→Y and g: Y→Z are soft identity maps. Then f and g are soft paracontinuous maps but gof: X→Z is not a soft paracontinuous, since for the soft paraopen set (F_1, E) in Z, $(gof)^{-1}((F_1, E))=(F_1, E)$ which is not a soft open set in X.

2.30 Theorem: If f: X→Y is soft continuous and g: Y→Z is soft paracontinuous maps. Then gof: X→Z is a soft paracontinuous.

Proof: Let (U, E)be any soft paraopen set in Z. Since g is soft paracontinuous, $g^{-1}(F, E)$ is soft open set in Y. Again since f is soft continuous, $f^{-1}(g^{-1}(F, E))=(gof)^{-1}(F, E)$ is a soft open set in X. Hence gof is a soft paracontinuous.

2.31 Theorem: Let X and Y be soft topological spaces. A map$f$: X→Y is a soft $*$-paracontinuous if and only if the inverse image of each closed set in Y is a soft paraclosed set in X.

Proof: The proof follows from the definition and fact that the complement of soft paraopen set is soft paraclosed set.

2.32 Remark: Let X and Y be soft topological spaces. If f: X→Y is soft $*$-paracontinuous then f_A: A→Y is need not be a soft $*$-paracontinuous.

2.33 Example: Let $X = Y = \{a, b, c, d\}$, $E = \{e_1, e_2\}$, $\tau = \{\phi, (F_1, E), (F_2, E), (F_3, E), X\}, where\ F_1(e_1) = \{a\}, F_1(e_2) = \{b\};\ , F_2(e_1) = \{a, b\}, F_2(e_2) = \{c, d\}; F_3(e_1) = \{a, b, c\},\ F_3(e_2) = \{b, c, d\} and\ \mu = \{\phi, (F_1, E), Y\}, where\ F_1(e_1) = \{a, b\},\ F_1(e_2) = \{c, d\}$. Let $Z = \{a, c, d\}$, $E = \{e_1, e_2\}$, $A = \{\phi, (F_1, E), (F_2, E), Z\}, where\ F_1(e_1) = \{a\},\ F_1(e_2) = \{b\}; F_2(e_1) = \{a, c\},\ F_2(e_2) = \{c, d\}$.Let$f$: X→Y be soft identity map. Then f is a soft$*$-paracontinuous map but f_A: A→Y is not a soft$*$-paracontinuous, since for the soft open set (F_1, E) in Y, $f_A^{-1}(F_1, E)$=A∩(F_1, E)=(F_1, E) which is not a soft paraopen set in A.

2.34 Theorem: Iff: X→Y and g: Y→Z are soft $*$-paracontinuous maps. Then gof: X→Z is a soft $*$-paracontinuous.

Proof: Let (F, E) be ansoft open set in Z. Since g is soft $*$-paracontinuous, $g^{-1}(F, E)$ is a soft paraopen set in Y. Since every soft paraopen set is a soft open, $g^{-1}(F, E)$ is a soft open set in Y. Again since f is soft $*$-paracontinuous, $f^{-1}(g^{-1}(F, E))=(gof)^{-1}(F, E)$ is a soft paraopen set in X. Hence gof is a soft $*$-paracontinuous.

2.35 Theorem: If f: X→Y is soft paracontinuous and g: Y→Z is soft$*$-paracontinuous maps. Then gof: X→Z is a soft paracontinuous (resp. soft continuous) map.

Proof: Let (F, E) be any soft paraopen (resp. soft open) set in Z. Since every soft paraopen set is a soft open, (F, E) is a soft open set in Z. Since g is soft$*$-paracontinuous, $g^{-1}(F, E)$ is a soft paraopen set in Y. Again since f is soft paracontinuous, $f^{-1}(g^{-1}(F, E))=(gof)^{-1}(F, E)$ is a soft open set in X. Hence gof is a soft paracontinuous (resp. soft continuous) map.

2.36 Theorem: Let X and Y be soft topological spaces. A map f: X→Y is a soft parairresolute if and only if the inverse image of each soft paraclosed set in Y is soft paraclosed set in X.

Proof: The proof follows from the definition and fact that the complement of soft paraopen set is soft paraclosed set.

2.37 Remark: Let X and Y be soft topological spaces. If f: X→Y is a soft parairresolute then f_A: A→Y is need not be a soft parairresolute.

2.38 Example: In Example 2.32, Let$f$: X→Y be soft identity map. Then f is a soft parairresolute map but f_A: A→Y is not a soft parairresolute, since for the soft paraopen set (F_1, E)in Y,$f_A^{-1}(F_1, E)$=A∩(F_1, E)=(F_1, E) which is not a soft paraopen set in A.

2.39 Theorem: If f: X→Y is soft parairresolute and g: Y→Z is soft paracontinuous maps. Then gof: X→Z is a soft paracontinuous map.

Proof: Let (F, E) be any soft paraopen set in Z. Since g is soft parairresolute, $g^{-1}(F, E)$ is a soft paraopen set in Y. Again since f is soft paracontinuous, $f^{-1}(g^{-1}(F, E)) = (gof)^{-1}(F, E)$ is a soft open set in X. Hence gof is a soft paracontinuous map.

2.40 Theorem: If f: X→Y and g: Y→Z are soft parairresolute maps. Then gof: X→Z is a soft parairresolute map.

Proof: Let (F, E) be any soft paraopen set in Z. Since g is soft parairresolute, $g^{-1}(F, E)$ is a soft paraopen set in Y. Again since f is soft parairresolute, $f^{-1}(g^{-1}(F, E)) = (gof)^{-1}(F, E)$ is a soft paraopen set in X. Hence gof is a soft parairresolute map.

2.41 Theorem: If f: X→Y is soft $*$-paracontinuous and g: Y→Z is soft parairresolute maps. Then gof: X→Z is a soft parairresolute map.

Proof: Let (F, E) be any soft paraopen set in Z. Since g is soft parairresolute, $g^{-1}(F, E)$ is a soft paraopen set in Y. Since every soft paraopen set is a softopen, we have $g^{-1}(F, E)$ is a soft open set in Y. Again since f is soft $*$-paracontinuous, $f^{-1}(g^{-1}(F, E)) = (gof)^{-1}(F, E)$ is a soft paraopen set in X. Hence gof is a soft parairresolute map.

2.42 Theorem: If f: X→Y is soft parairresolute and g: Y→Z is soft $*$-paracontinuous maps. Then gof: X→Z is a soft parairresolute map.

Proof: Let (F, E) be any soft paraopen set in Z. Since every soft paraopen set is a soft open set, (F, E) is a soft open set in Z. Since g is soft $*$-paracontinuous, $g^{-1}(F, E)$is a soft paraopen set in Y. Again since f is soft parairresolute, $f^{-1}(g^{-1}(F, E)) = (gof)^{-1}(F, E)$is a soft paraopen set in X. Hence gof is a soft parairresolute map.

2.43 Theorem: Let X and Y be soft topological spaces. A map$f$: X→Y is a soft minimal paracontinuous if and only if the inverse image of each soft maximal closed set in Y is a soft paraclosed set in X.

Proof: The proof follows from the definition and fact that the complement of soft minimal open set is soft maximal closed set and the complement of soft paraopen set is soft paraclosed set.

2.44 Theorem: The composition of soft minimal paracontinuous maps need not be a soft minimal paracontinuous.

2.45 Example: Let $X = Y = \{a,b,c,d,e\}$, $E = \{e_1,e_2\}$, $\tau = \{\phi,(F_1,E),(F_2,E),(F_3,E),X\}$, $where\ F_1(e_1) = \{a\},\ F_1(e_2) = \{b\};\ F_2(e_1) = \{a,b\},, F_2(e_2) = \{c,d\}; F_3(e_1) = \{a,b,c\} F_3(e_2) = \{c,d,e\} and\ \mu = \{\phi,(F_1,E),(F_2,E),(F_3,E),Y\}, where\ F_1(e_1) = \{a,b\},\ F_1(e_2) = \{c,d\}; F_2(e_1) = \{a,b,c\},, F_2(e_2) = \{c,d,e\}; F_3(e_1) = \{a,b,c,d\} F_3(e_2) = \{b,c,d,e\}\ and\ \eta = \{\phi,(F_1,E),(F_2,E),Z\}, where\ F_1(e_1) = \{a,b,c\},\ F_1(e_2) = \{c,d,e\}; F_2(e_1) = \{a,b,c,d\},\ F_2(e_2) = \{b,c,d,e\}$.Let f: X→Y and g: Y→Z are soft identity maps. Then f and g are soft minimal paracontinuous maps but gof: X→Z is not a soft minimal paracontinuous, since for the soft minimal open set (F_1,E) in Z, $(gof)^{-1}(F_1,E)$=(F_1,E)} which is not a soft paraopen set in X.

2.46 Theorem: If f: X→Y is soft parairresolute and g: Y→Z is soft minimal paracontinuous maps, then gof: X→Z is a soft minimal paracontinuous.

Proof: Let (F,E)be any soft minimal open set in Z. Since g is soft minimal paracontinuous, $g^{-1}(F,E)$ is a soft paraopen set in Y. Again since f is soft parairresolute, $f^{-1}(g^{-1}(F,E))$ = $(gof)^{-1}(F,E)$ is a soft paraopen set in X. Hence gof is a soft minimal paracontinuous.

2.47 Theorem: If f: X→Y is soft paracontinuous and g: Y→Z is soft minimal paracontinuous maps, then gof: X→Z is a soft minimal continuous.

Proof: Let (F,E) be any soft minimal open set in Z. Since g is soft minimal paracontinuous, $g^{-1}(F,E)$ is a soft paraopen set in Y. Again since f is soft paracontinuous, $f^{-1}(g^{-1}(F,E))$ = $(gof)^{-1}(F,E)$ is a soft open set in X. Hence gof is a soft minimal continuous.

2.48 Theorem: Iff: X→Y is soft parairresolute and g: Y→Z is soft $*$-paracontinuous maps, then gof: X→Z is a soft minimal paracontinuous.

Proof: Let (F,E) be any soft minimal open set in Z. Since every soft minimal open set is a soft open set, (F,E) is a soft open set in Z. Since g is soft $*$-paracontinuous, $g^{-1}(F,E)$ is a soft paraopen set in Y. Again since f is soft parairresolute, $f^{-1}(g^{-1}$(F,E)) = $(gof)^{-1}$(F,E) is a soft paraopen set in X. Hence gof is a soft minimal paracontinuous.

2.49 Theorem: Let X and Y be soft topological spaces. A map f: X→Y is a soft maximal paracontinuous if and only if the inverse image of each soft minimal closed set in Y is soft paraclosed set in X.

Proof: The proof follows from the definition and fact that the complement of soft maximal open set is soft minimal closed set and the complement of soft paraopen set is soft paraclosed set.

2.50 Remark: The composition of soft maximal paracontinuous maps need not be a soft maximal paracontinuous.

2.51 Example: Let $X = Y = \{a,b,c,d,e\}$, $E = \{e_1,e_2\}$, $\tau = \{\phi,(F_1,E),(F_2,E),(F_3,E),X\}$, $where\ F_1(e_1) = \{a,c\},\ F_1(e_2) = \{b,d\};\ F_2(e_1) = \{a,b,c\},, F_2(e_2) = \{c,d,e\}; F_3(e_1) = \{a,b,c,d\}, F_3(e_2) = \{b,c,d,e\} and\ \mu = \{\phi,(F_1,E),(F_2,E),(F_3,E),Y\}, where\ F_1(e_1) = \{a\},\ F_1(e_2) = \{b\}; F_2(e_1) = \{a,b\},, F_2(e_2) = \{c,d\}; F_3(e_1) = \{a,b,c\} F_3(e_2) = \{c,d,e\}\ and\ \eta =$

$\{\phi, (F_1, E), (F_2, E), Z\}$ where $F_1(e_1) = \{a\}$, $F_1(e_2) = \{b\}$; $F_2(e_1) = \{a, b\}$, $F_2(e_2) = \{c, d\}$. Let f: X→Y and g: Y→Z are soft identity maps. Then f and g are soft maximal paracontinuous maps but gof: X→Z is not soft maximal paracontinuous, since for the soft maximal open set (F_2, E) in Z, $(gof)^{-1}((F_2, E))=(F_2, E)$ which is not a soft paraopen set in X.

2.52 Theorem: If f: X→Y is soft parairresolute and g: Y→Z is soft maximal paracontinuous maps. Then gof: X→Z is a soft maximal paracontinuous.

Proof: Let (F, E) be any soft maximal open set in Z. Since g is soft maximal paracontinuous, $g^{-1}(F, E)$ is a soft paraopen set in Y. Again since f is soft parairresolute, $f^{-1}(g^{-1}(F, E))=(gof)^{-1}(F, E)$ is a soft paraopen set in X. Hence gof is a soft maximal paracontinuous.

2.53 Theorem: If f: X→Y is soft paracontinuous and g: Y→Z is soft maximal paracontinuous maps, then gof: X→Z is a soft maximal continuous.

Proof: Let (F, E) be any soft maximal open set in Z. Since g is soft maximal paracontinuous, $g^{-1}(F, E)$ is a soft paraopen set in Y. Again since f is soft paracontinuous, $f^{-1}(g^{-1}(F, E))=(gof)^{-1}(F, E)$ is a soft open set in X. Hence gof is a soft maximal continuous.

2.54 Theorem: If f: X→Y is soft parairresolute and g: Y→Z is soft $*$-paracontinuous maps, then gof: X→Z is a soft maximal paracontinuous.

Proof: Let (F, E) be any soft maximal open set in Z. Since every soft maximal open set is a soft open set, (M, E)s a soft open set in Z. Since g is soft $*$-paracontinuous, $g^{-1}(F, E)$ is a soft paraopen set in Y. Again since f is soft parairresolute, $f^{-1}(g^{-1}(M, E))=(gof)^{-1}(M, E)$ is a soft paraopen set in X. Hence gof is a soft maximal paracontinuous.

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