IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)
An International Open Access, Peer-reviewed, Refereed Journal

MHD Generalized Plane Couette Flow Through Porous Medium of Different Permeabilities

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ABSTRACT

The problem of MHD generalized plane Couette flow of a viscous incompressible fluid in a channel filled with porous medium of different permeabilities has been studied, when the free surface is exposed to atmospheric pressure. To discuss the solution,the flow region is divided into three zones. The flow region in the free fluid is called zone- I and the flow is governed by Navie r–Stokes equations. The flow regions in the high permeability and low permeability mediums are named as zone- II and zone – III and the flows are governed by Brinkman equations and Darcy's law respectively. The effects of magnetic parameter and permeability parameters are investigated on velocity and skin-friction. It is being found that the increase in Hartmann number decreases the flow in the channel.

Keywords- MHD, Couette flow, Navie r–Stokes equations, Brinkman equations, skin-friction

INTRUOCTION

As described in many papers flows through porous media are very much prevalent in nature and hence their study is of principal interest in many scientific and engineering problems. To understand the seepage of water in river beds and underground water flow one needs to investigate the flows of fluids through porous media. When the velocity of the flow in porous media is low, the flow is governed by Darcy's law. In case of flow past a porous media, Beavers and Joseph(1) have shown that usual no slip conditions at the boundary of the porous material is no longer valid and they postulated new boundary conditions in which transfer of momentum was taken into account. These boundary conditions are known as BJ slip condition in literature. Rajasekhara et.al. (2) have investigated couette flow over a permeable bed with an impermeable moving plate using BJ conditions at the lower permeable bed.

Darcy's law which is based on experiments is an imperical relation valid at low Reynolds numbers [maskat (3), Scheidegger (4)] however in many cases the flow velocity is not always in a porous medium. In order to formulate the flow in such a medium Brinkman (5) suggested a new model of boundary layer type equation. Bharqava and Sacheti (6) obtained heat transfer in generalized couette flow using Brinkman type equations of motion and energy.

A survey of recent literature shows that the Brinkman equations is also used as a generalization of Darcy's law which allow the matching of velocities and tractions at the boundary between fluid and porous media. Koplic et.al.(7) and kim and (8) have investigated some problems using such boundary conditions at the interface.

Flow in a porous medium with variable permeability has not attracted the attention of may research workers. Kumar (9) considered the flows between two permeable beds. He used Dacy's law in one permeable bed while in other Brinkman equation was used. Venkatramana and Bathaih (10) evaluated the velocity and temperature distributions of a conducting fluid between two permeable beds. Gupta and Sharma (11) discussed the flow in a channel with permeable bottom and impermeable moving plate.

In the present paper, we consider a generalized plane couette flow through a porous medium of different permeabilites in a channel in presence of transverse magnetic field. To discuss the solution, the flow region is divided into three zones.
Zone - I – free fluid region (where navier-stokes equations hold).
Zone - II – High permeability region (where Brinkman equations hold).
Zone – III – Low permeability region (where Darcy's law hold).

We have evaluated the velocity and skin-friction. The effects of M (magnetic parameters , $k_1 > k_2$ (permeabilities parameters with $k_1 > k_2$) on the velocity and skin- friction are investigate.

FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

We consider a physical model whose Geometry is given in fig.1 illustrating the problem consisting of three zones of height h each. Zone-I is free fluid flow bounded by a plate moving with uniform velocity U'_0 , zone-II is porous Region of high permeability in which Brinkman equation hold and zone-III is consists of a porous material of low permeability in which Darcy's law hold. The fluid is electrically conducting and a magnetic field of intensity B_0 is introduced in the flow field.

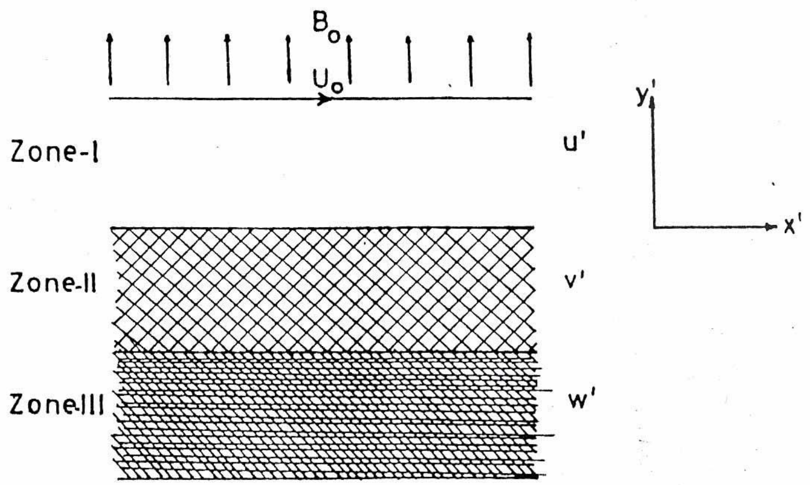

Fig. 1. Geometry of the Problem

If we take x′-axis along the flow and axis of y′ perpendicular to it and a constant pressure gradient ($-\frac{\partial p'}{\partial x'}$=C) acting at the mouth of the channel, the basic equations of the motion in three zones can be written as

Zone-I

$$\nu \frac{\partial^2 u'}{\partial y'^2} - \frac{1}{\rho}\frac{\partial p'}{\partial x'} - \frac{\sigma B^2_{o} u'}{\rho} = 0 \tag{1}$$

Zone-II

$$\nu_1 \frac{\partial^2 v'}{\partial y'^2} - \frac{1}{\rho}\frac{\partial p'}{\partial x'} - \frac{\nu_1}{} v^1 - \frac{\sigma B^2_{o} v'}{\rho} = 0 \tag{2}$$

Zone-III

$$w' = \frac{k_2'}{\rho \nu_2}\frac{\partial p'}{\partial x'} \tag{3}$$

Here we assume $k'_2 = k_o$ ee$^{-\beta y}$

The boundary conditions are

$$\begin{aligned} y' &= h \quad : \quad u' = U_0' \\ y' &= o \quad : \quad u' = \vartheta' \text{ and } \nu \frac{\partial u'}{\partial y'} = \nu_1 \frac{\partial v'}{\partial y'} \\ y' &= -h \quad : \quad u' = w' \end{aligned} \tag{4}$$

In writing the above equations, following assumptions are made

(i) Electrical conductivity of the fluid is large so that displacement current is neglected.

(ii) No external electric field is applied.

(iii) The secondary effects of magnetic induction are neglected.

The equations of motion and boundary conditions, after introducing following non-dimensional quantities

(u,v,w) = $\frac{h}{\nu}$ (u′, v′, w′) ,

$U_o = \frac{h}{\nu} u_0'$,

$y = \frac{y'}{h}$,

$p = \frac{p'}{\rho(\nu/h)^2}$, (5)

$k_1 = \frac{K_1'}{h^2}$,

$k_2 = \frac{K_2'}{h^2}$,

$M^2 = \frac{\sigma B_0^2 h^2}{\rho \nu}$

$\beta = \beta' h$

Reduce to

Zone –I

$$\frac{d^2u}{dy^2} - M^2u = -C \tag{6}$$

Zone-II

$$\frac{d^2v}{dy^2} - N^2v = -\emptyset_1 C \tag{7}$$

Where $N^2 = \frac{1}{k_1} + \emptyset_1 M^2$ and $\emptyset_1 = \frac{\nu}{\nu_1}$

Zone-III

$$w = \emptyset_2 k_2 C \; e^{\beta y} \tag{8}$$

Where $k_1 > k_2$ and $\emptyset_2 = \frac{\nu}{\nu_2}$

With boundary conditions

$y = 1 \;: u = U_o$

$y = 0 \;: u = v$ and $\emptyset_1 \frac{\partial u}{\partial y} = \frac{\partial v}{\partial y}$ (9)

$y = -1 : u = w$

SOLUTION OF THE PROBLEM

Solving the equations (6) to (8) using boundary conditions (9), we obtain the velocity the in the three zones as

Zone-I

$$U = C_1 \, e^{MY} + C_2 \, e^{-MY} + \frac{C}{M^2} \tag{10}$$

Zone-II

$$V = C_3 \, e^{Ny} + C_4 \, e^{-Ny} + \frac{\emptyset_1 C}{N^2} \tag{11}$$

Zone-III

$$W = k_2\, \emptyset_2\, C\, e^{-\beta y} \qquad (12)$$

Where C_1, $C_{2,}$ C_3 and C_4 are constants and are given as

$$C_1 = \frac{1}{A_1\emptyset_1 M}\left[U_o - C\left(\frac{\emptyset_1}{N^2} - \frac{1}{M^2}\right)\cosh M - \frac{C}{M^2} + \frac{C}{M}\left(\frac{1}{N^2} - \frac{K_{2\emptyset 2}}{\emptyset_1}\, e^{\beta}\right) e^{N}(\emptyset_1 M\cosh M + N\sinh M)\right](\emptyset_1 M\sinh N + N\cosh N) - \frac{(\emptyset_1 M + N}{2M}\, Ce^{N}\left(\frac{1}{N^2} - \frac{K_2\emptyset_2}{\emptyset_1}\, e^{\beta}\right) + \frac{C}{2}\left(\frac{\emptyset_1}{N^2} - \frac{1}{M^2}\right) \qquad (13)$$

$$C_2 = \frac{1}{A_1\emptyset_1 M}\left[U_o - C\left(\frac{\emptyset_1}{N^2} - \frac{1}{M^2}\right)\cosh M - \frac{C}{M^2} + \frac{C}{M}\left(\frac{1}{N^2} - \frac{K_{2\emptyset 2}}{\emptyset_1}\, e^{\beta}\right) e^{N}(\emptyset_2 M\cosh M + N\sinh M)\right](\emptyset_1 M\sinh N + N\cosh N) - \frac{(\emptyset_1 M + N)}{2M}\, Ce^{N}\left(\frac{1}{N^2} - \frac{K_2\emptyset_2}{\emptyset_1}\, e^{\beta}\right) + \frac{C}{2}\left(\frac{\emptyset_1}{N^2} - \frac{1}{M^2}\right) \qquad (14)$$

$$C_3 = e^{N}\left[\frac{1}{A_1}\left\{U_o - C\left(\frac{\emptyset_1}{N^2} - \frac{1}{M^2}\right)\cosh M - \frac{C}{M^2} + \frac{C}{M}\left(\frac{1}{N^2} - \frac{K_{2\emptyset 2}}{\emptyset_1}\, e^{\beta}\right)(\emptyset_1 M\cosh M + N\sinh M)e^{N}\right\} - \emptyset_1 C\left(\frac{1}{N^2} - \frac{K_2\emptyset_2}{\emptyset_1}\, e^{\beta}\right]\right. \qquad (15)$$

And

$$C_4 = e^{-N}\left[\frac{1}{A_1}\left\{-U_o + C\left(\frac{\emptyset_1}{N^2} - \frac{1}{M^2}\right)\cosh M + \frac{C}{M^2} - \frac{C}{M}\left(\frac{1}{N^2} - \frac{K_{2\emptyset 2}}{\emptyset_1}\, e^{\beta}\right)(\emptyset_1 M\cosh M + N\sinh M)e^{N}\right\} + \emptyset_1 C\left(\frac{1}{N^2} - \frac{K_2\emptyset_2}{\emptyset_1}\, e^{N}\right]\right. \qquad (16)$$

With

$$A_1 = \frac{2\emptyset_1 M\cosh M\sinh N + 2N\sinh M\cosh M}{\emptyset_1 M} \qquad (17)$$

SKIN FRICTION

The non-demensional form of skin-friction at the upper plate i.e. y=1 is given by

$$\tau_u = \left(\frac{du}{dy}\right) y = 1 \qquad (18)$$

$$\tau_u = M\, C_1\, e^{M} - M\, C_2\, e^{-M} \qquad (19)$$

Where C_1 and C_2 are given by equations (13) and (14).

Table – 1 : The dimensionless skin-friction coefficient for different values of M, k_1 and k_2 ($U_0 = 5.0$, $C = 1.0$, $\beta = 1.0$, $\phi_1 = 0.4$, $\phi_2 = 0.6$)

(a) $k_1 = 10.0$, $k_2 = 0.5$

M	1.0	2.0	3.0	4.0	5.0
τ_u	4.6760	9.4521	14.6350	19.7455	25.9870

(b) $k_1 = 10.0$, $k_2 = 1.0$

M	1.0	2.0	3.0	4.0	5.0
τ_u	4.2962	9.2535	14.6054	19.1245	25.4286

(c) $k_1 = 10.0$, $k_2 = 2.0$

M	1.0	2.0	3.0	4.0	5.0
τ_u	3.9974	8.5632	13.9710	18.7579	25.0023

d

(d) $k_1 = 25.0$, $k_2 = 0.5$

M	1.0	2.0	3.0	4.0	5.0
τ_u	4.6548	9.3552	14.6510	19.3278	25.6712

(e) $k_1 = 50.0$, $k_2 = 0.5$

M	1.0	2.0	3.0	4.0	5.0
τ_u	4.6460	9.2781	14.4976	19.1992	25.5223

NUMERICAL DISCUSSIONS

We have shown velocity profile in different zones graphically while skin-friction is given in tabular form for different values of magnetic parameter and porous parameters.

From fig. 2 it is being observed that the velocity increases with increase of k_2 and for increase of M and k_1 the process reverses in all the three zones.

From the tables for skin-friction it is observed that τ_u decreases with increases of both k_1 and k_2 and increases with M.

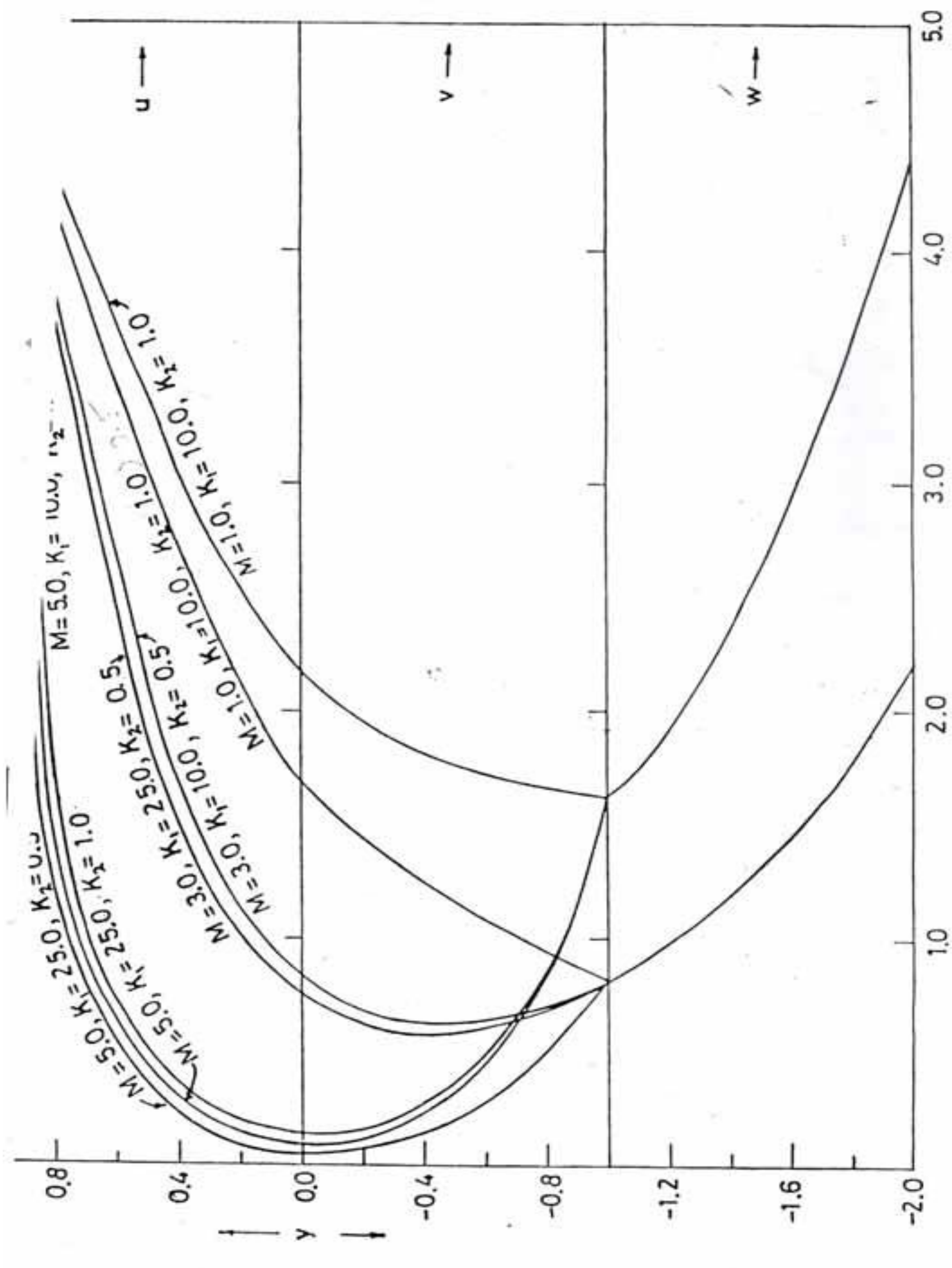

Fig. .2 Velocity profiles plotted against y for different values of M, K_1 and K_2.

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