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GRAPH THEORY & ITS APPLICATIONS AND KONIGSBERG BRIDGE PROBLEM

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ABSTRACT: In different fields the field of mathematics plays key role. In mathematics, graph theory is one of the important fields used in structural models. The field graph theory began in 1735 with the Konigsberg Bridge problem. This paper provides a description of implementations of graphical theory in a number on information science, electrical engineering, physics and chemistry, computer network science, biotechnology and graphical theoretical applications. Several articles focused on graph theory have been studied concerning scheduling principles, engineering technology implementations and an outline. A graph is an abstraction of relationships that emerge in nature.

Keywords: Graph, Computer Science, Mathematics Physics, Chemistry, Konigsberg Bridge Problem

1. INTRODUCTION

A diagram consisting of many points and lines that unite several pairs of these points can be easily represented for several real-world contexts. The points might, for example, show individuals with lines who join couples with friends; or the points could be contact centres with lines showing connection connections. The definition of a graph is a statistical abstraction of conditions of this kind. Graph theory principles are commonly used in various fields to research and model different applications. This includes studying molecules, building chemical bonds and studying atoms. In sociology, for instance, graph theory is used to calculate the popularity of actors or to investigate processes of diffusion. The theory of graphs is used for biodiversity and conservation, where a vertex represents areas in which some species live and where edges represent migratory or moving paths between areas. This data is important for examining the breeding habits of disease, parasites and for investigating the effect of migration on other animals. This knowledge is important. In the field of computer science, graph theory concepts are widely used [1]

2. HISTORY OF GRAPH THEORY

The root of the graphic principle began with the Konigsberg bridge dilemma in 1735. This dilemma leads to the Euler graph principle. Euler analyzed the Konigsberg Bridge problem and created a structure to solve the problem known as the Aurelian graph. A.F Mobius offered the concept of a total graph and a bi-partisan graph in 1840, and Kuratowski showed that they were planar of leisure problems. Gustav

Kirchhoff introduced a linked graph without cycles in 1845 and used graphical technical concepts for the measurement of current in electrical networks or circuits. In 1852, the popular four-color issue was discovered by Thomas Guthrie. Then in 1856, Thomas, P. Kirkman and William Hamilton, researched polyhydric cycles and developed, by observing trips which visited a number of locations exactly once, the idea called the Hamiltonian graph. In 1913, H. Dudeney spoke about an issue of puzzles. Eventually, Kenneth Apple and Wolfgang Haken addressed the four-color dilemma only after a century. This period the birth of graph theory is considered [2]. To research the trees Caley learned specific analytical forms from the differential calculus. And has several consequences for theoretical chemistry. This leads to enumerative graph theory being invented. Anyway, in 1878, Sylvester introduced "Graph," where he drew an analogy from "quantum invariants" to algebra and molecular-diagram covariant [3]. In 1941 Ramsey experimented on the colours, leading to the identification of a subset of graphic science named severe graphic theory. The analysis of asymptotic graph connectivity has led to a random principle of graphics.

3. APPLICATIONS OF GRAPH THEORY

Graph theory principles are commonly utilized in diverse fields to research and model different applications. This includes studying compounds, building bonds in chemistry and studying atoms. In sociology, graph theory is similarly used for example to calculate the popularity of performers or to investigate processes of diffusion. Graphic theory is used in biology and conservation where the vertex describes the areas in which animals occur and the edges reflect the direction of migration or travel through regions. This knowledge is critical for examining breeding trends or monitoring the propagation of diseases and parasites and for investigating the effect of migration on other animals [4,5]. Theoretical graphic principles are commonly utilized in research operations. For example the dilemma of the tour sales person, the shortest stretch in a weighted graph, obtains optimal work and men match and finds the shortest route from two vertices in a diagram. It is also used for modelling transport networks, net- works of operation and game theory [6]. A digraph is used to describe the finite game method. The vertices here mark the locations and the edges represent the movements. Graph theory is widely employed in research and technology. Any of the following are given:

(a) Computer Science

For the analysis of algorithms such as: Dijkstra Algorithm, Prims Algorithm, Kruskal Algorithm theory is used in computer graphics. Graphs are used to portray contact networks. Graphs reflect the organization of results.. Graph theory is used for finding the shortest route or network direction. Google Maps shows different places as vertices or points, and the roads are seen as corners and the idea of the chart is used to find the shortest path between two nodes.

(b)Electrical Engineering

Graph theory is used in electrical engineering in the construction of circuit links. This relations are referred to as topologies. Certain topologies include sequence, bridge, star and parallel topologies.

(c)Physics and Chemistry

Chemistry graphs are used to model chemical compounds. This is modelled as a graph in which the vertices reflect the sample sequences. An edge is drawn between two vertices where there is a conflict between the sequences. The goal is to delete potential vertices (sequences) in order to remove all disputes. Chart theory is used in physics and chemistry to analyse molecules. The 3D layout of complex artificial atomic systems can be quantitatively analyzed by collecting statistics on graph-theoretical features in relation to atom topology. In this area, diagrams may describe local relations between the interacting sections of a system and the physical process dynamics on those structures. Graphs also express porous media micro channels in which the vertices reflect the pores and the borders represent the smaller pores. Graph is also useful in building both the molecular structure and the molecular grid. It also allows us to demonstrate the connection between atoms and molecules and helps us to compare the structure of a molecule with another.

(d)Biology

Nodes in biological networks are bimolecular such as chromosomes, proteins or metabolites and edges that link the nodes signify interactive, physical or chemical interactions between the bimolecular concerned. In transcriptional regulatory networks, graph theory is used. It is seen in metabolic networks as well. Graph theory is also useful in PPI (protein interaction) net- works. Characterizing drug goal partnerships. Drug target interactions.

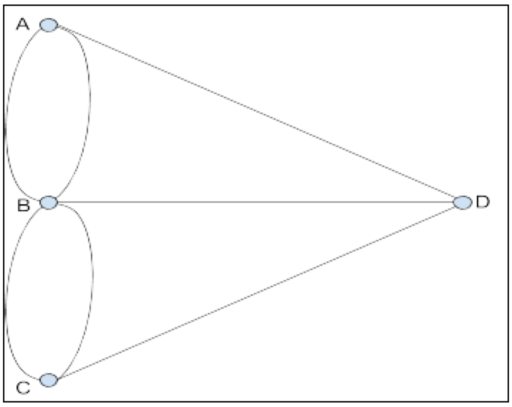
(e)Mathematics

Operational analysis is the essential area of mathematics. Graph theory offers numerous practical organizational analysis uses. Like Minimum route expenses, A issue with the schedule. Graphs reflect the roads between the towns. We may construct hierarchically organized details such as a family tree with the aid of a sort of graph.

Konigsberg Bridge Problem

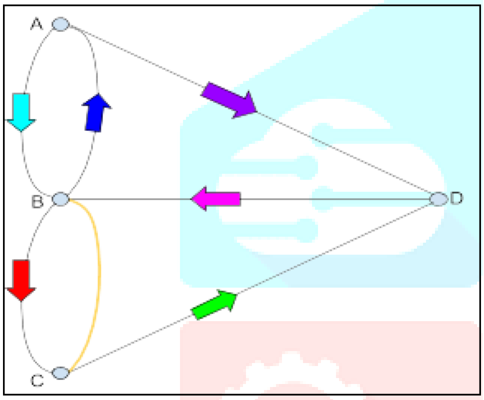
The criteria for the original problem was to find a path cross all seven bridges without crossing any bridge twice. Euler realized that trying to find a path by drawing the layout of the bridges and connecting them various ways would take a lot of time and would not necessarily result in a path that fulfilled the criteria. Instead, he made the problem into a graph problem. He made each bridge an edge, and each landmass became a node or vertex labelled by an uppercase letter

(A to D).. The graph he made to represent the Konigsberg bridges was similar to graph (1.1) pictured below.



Graph 1.1

In this graph, each of the four nodes have an odd degree which means they have an odd number of edges connected to them. Vertices A, C, and D all have a degree of three while only vertex B has a degree of five. If you start the path at one of the vertices with odd degree, you can visit each node but one of the bridges is left out of the path. In the example labelled graph (1.2),

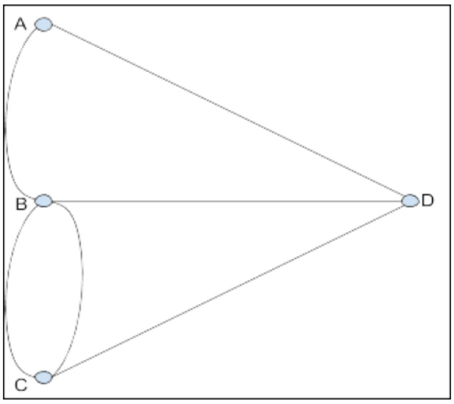


Graph 1.2

the path starts at node A and then continues to node B before returning to A, then on to D, then B, then C, before finally going back to D. One edge is left out between nodes B and C (see highlighted edge) and so the criteria for the problem is not meet because each bridge is not crossed. The path can be started at a different node or travel in a different order between the nodes, but there will always be at least one edge not left out. This is because if an odd vertex is the starting point, you have to leave, then come back, then leave again for another node. If that other node is also an odd degree, then you have to leave after coming in and then come back again and there is no other edge to allow you to leave again for the other vertices. Thus if there are odd vertices, then the path must start and end at an odd vertex. This graph has four vertices of odd degree so it is not possible to have a Eulerian path. Using this reasoning, Euler said that there is absolutely no path over the Konigsberg bridges.

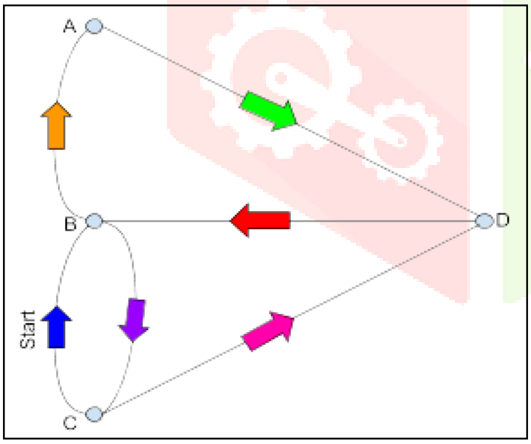
Alternate Versions of the Konigsberg Bridge Problem

After researching and thinking about the solution and conclusion for the original Konigsberg bridge problem, it came to my attention that the graph can be slightly changed so it is possible to solve. One of the changes would be to take away one bridge. Then there would be six edges between four nodes , if one edge was removed would look like the graph featured below labeled graph (1.3)



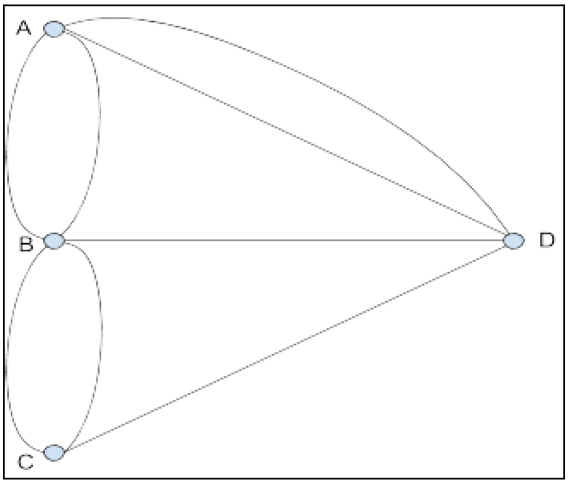
Graph 1.3

Here, one of the edges between nodes A and B has been removed. As a result, nodes A and B have an even degree of two and four respectively while nodes C and D each have an odd degree of three. Therefore there are only two vertices with an odd degree so a Eulerian path ispossible for this network. However, for the path to cross each and every edge once and only once, the path must start at a node of odd degree and end at a node of odd degree. One of thepossible paths for this graph is shown on graph (1.4) with arrows showing the direction of the path.



Graph 1.4

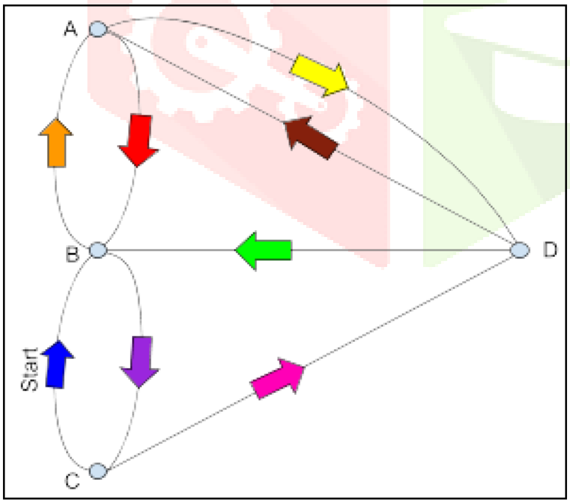
This specific path starts at node C then continues to node B, then A, then D, back to B, then C, before ending at node D. While this layout has proven to have a Eulerian path, any layout with one bridge removed would also have a Eulerian path as there would always be two nodes with an even degree and two nodes with an odd degree because of the layout of the original graph. There are also other ways than removing one edge to alter the original graph so that there is a solution. One of them is to add an edge between nodes to the graph. This would result in a graph with eight edges and four nodes like the possible example in graph



(1.5)

Graph 1.5

In this example, one edge has been added between nodes A and D. this causes both nodes A and D to have a degree of four while nodes C and B remain unaltered with a degree of three and five respectively. Thus this graph has only two nodes of odd degree with two nodes of even degree so according to the theorems of graph theory, there is a Eulerian path possible for this graph. However, like with the graph shown previously, the path must start at a node of odd degree and end at a node of odd degree. In the graph shown below and labeled graph (1.6), a possible Eulerian path is depicted with arrows showing the direction of the path.



Graph 1.6

The path begins at node C before going to B then back to C, then to node D, then A, before node B, then back to A, then D, before ending at node B. This is only one of the many possible paths for this graph and this is only one of the many possible graphs with one edge added to the original layout. The extra edge could have been added between any of the two nodes and still there would have been an Eulerian path because there would be two nodes with even degree and two with odd degree. If the eighth edge was instead added between nodes C and D, A path that crosses each edge once and only once and that starts and ends at the same point is called a

CONCLUSION

Programmers and designers, graph theory is an extraordinarily rich field. Graphs can help solve some very complicated issues, such as lower costs, visualization, program analysis, etc. To calculate an optimum traffic routing, network devices, such as routers and switches use graphics. This paper focuses mainly on presenting the recent developments in the field of graph theory and its various applications in the field of engineering.. There is a wide discussion of each domain application, which is very beneficial to any research. With the original layout of the seven bridges of Konigsberg, it is impossible to find a path that crosses each and every bridge once as both the people of Konigsberg discovered by trial and error and as Euler discovered using proofs based in the branch of mathematics known as graph theory. However, be adding or removing one or more bridges a path can be found and can depending on the number and choice of bridge result in a circuit being possible.

Reflection

I enjoyed investigating the Konigsberg bridge problem as it gave me the chance to learn more about a branch of mathematics I had previously had very little exposure to and about a simple solution to a problem I had encountered before. Learning about graph theory was interesting because I did not realize it was actually a viable branch of mathematics before this, and a very extensive and involved branch at that. I also found it interesting that a complicated logic problem like the Konigsberg bridge problem could be solved so easily and quickly also expand my knowledge of mathematics and the history associated with it.

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