



# THE DEGREE OF EDGE IN CARTESIAN PRODUCT AND COMPOSITION OF THREE FUZZY GRAPHS.

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## INTRODUCTION

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. Though it is very young, it has been growing fast and has numerous applications in various fields. During the same time Yeg and Bang have also introduced various concepts in connectedness in fuzzy graphs. Mordeson. J. N. and Peng. C. S introduced the concept of operations on fuzzy graphs. Sunitha. M. Sand Vijaykumar . A discussed about the complement of the operations of the operations of union, join, Cartesian product and composition on two fuzzy graphs. The degree of a vertex in fuzzy graphs which are obtained from two given fuzzy graphs using these operations was discussed by Nagoorgani and A. Radha. K. In this paper we introduce the concept of a degree of an edge and total degree of an edge in fuzzy graphs. We study about the degree of an edge in fuzzy graphs which are obtained from three given fuzzy graphs using the operations, Cartesian product and Composition. In general, the degree of an edge in Cartesian product and Composition of three fuzzy graphs  $G_1, G_2$  and  $G_3$  cannot be expressed in terms of these in  $G_1, G_2$  and  $G_3$ . In this paper, we find the degree of an edge in Cartesian product and composition of three fuzzy graphs  $G_1, G_2$  and  $G_3$  in terms of the degree of some basic concepts.

A fuzzy subset of a set  $V$  is a mapping  $\sigma$  from  $V$  to  $[0,1]$ . A fuzzy graph  $G$  is a pair of function  $G : (\sigma, \mu)$  where  $\sigma$  is a fuzzy subset of a non-empty set  $V$  and  $\mu$  is symmetric fuzzy relation on  $\sigma$ , (i.e)  $\mu(x, y) \leq \mu(y, x)$  denoted by  $G^*(V, E)$  where  $E \subseteq V \times V$ . Through this paper,  $G_1: (\sigma_1, \mu_1), G_2: (\sigma_2, \mu_2)$  and  $G_3: (\sigma_3, \mu_3)$  denote three fuzzy graphs with underlying crisp graphs  $G_1(V_1, E_1), G_2(V_2, E_2)$  and  $G_3(V_3, E_3)$  with  $|V_i| = p_i, i = 1, 2, 3$ . Also  $d_{G_i^*}(u_i)$  denotes the degree of  $u_i$  in  $G_i^*$ .

Let  $G(\sigma, \mu)$  be a fuzzy graph on  $G^*: (V, E)$ . The degree of a vertex  $u$  is  $d_G(u) = \sum_{u \neq v} \mu(u, v)$ . Since  $\mu(u, v) > 0$  for  $uv \in E$ ,  $\mu(u, v) = 0$  for  $uv \notin E$ . This is equivalent to  $d_G(u) = \sum_{uv \in E} \mu(u, v)$ . The minimum degree of  $G$  is  $\delta(G) = \wedge \{d_G(v), \forall v \in V\}$  and the maximum degree of  $G$  is  $\Delta(G) = \vee \{d_G(v), \forall v \in V\}$ .

Let  $G: (\sigma, \mu)$  be a fuzzy graph on  $G^*(V, E)$ . The total degree of a vertex  $u \in V$  is denoted by  $td_G(u)$ . This is equivalent to  $td_G(u) = d_G(u) + \sigma(u)$ .

The order and size of a fuzzy graph  $G$  are defined by  $O(G) = \sum_{u \in V} \sigma(u)$

And  $S(G) = \sum_{uv \in E} \mu(u, v)$ .

Let  $G: (V, E)$  be a graph and let  $e = uv$  be an edge in  $G^*$ . Then the degree of an edge  $e = uv$  is defined by  $d_{G^*}(uv) = d_{G^*}(u) + d_{G^*}(v) - 2$ .

## DEFINITION:

Let  $G^* = G_1^* \times G_2^* \times G_3^* = (V, E)$  be the Cartesian product of three graphs  $G_1^*, G_2^*$  and  $G_3^*$  where  $V = V_1 \times V_2 \times V_3$  and  $E = \{(u, u_2), (u, v_2), (u, w_2): u \in V_1, u_2, v_2, w_2 \in E_2\} \cup \{(u_1, y), (v_1, y), (w_1, y): u_1, v_1, w_1 \in E_1, y \in V_1\}$ .

Then the Cartesian product of three fuzzy graphs  $G_1, G_2, G_3$  is a fuzzy graph.  $G = G_1 \times G_2 \times G_3 : (\sigma_1 \times \sigma_2 \times \sigma_3, \mu_1 \times \mu_2 \times \mu_3)$ .

Defined by,

$$(\mu_1 \times \mu_2 \times \mu_3)((u, u_2), (u, v_2), (u, w_2)) = \sigma_1(u) \wedge \mu_2(u_2, v_2, w_2),$$

$$\forall u \in V_1, u_2, v_2, w_2 \in E_2$$

$$(\mu_1 \times \mu_2 \times \mu_3)((u_1, y)(v_1, y)(w_1, y)) = \sigma_2(y) \wedge \mu_1(u_1 v_1 w_1),$$

$$\forall y \in V_2, u_1, v_1, w_1 \in E_1$$

## DEFINITION:

Let  $G^* = G_1^* \circ G_2^* \circ G_3^* = (V, E)$  be the Composition of three graphs  $G_1^*$ ,  $G_2^*$  and  $G_3^*$ . Where  $V = V_1 \times V_2 \times V_3$  and  $E = \{(u, u_2)(u, v_2)(u, w_2), u \in V_1, u_2, v_2, w_2 \in E_2\} \cup \{(u_1, y)(v_1, y)(w_1, y) : u_1, v_1, w_1 \in E_1, y \in V_2\} \cup \{(u_1, u_2)(v_1, v_2)(w_1, w_2) : u_1, v_1, w_1 \in E_1, (w_1, w_2) \in E_3\}$

Defined by,

$$(\sigma_1 \circ \sigma_2 \circ \sigma_3)(u_1, u_2, u_3) = \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_3(u_3), \forall (u_1, u_2, u_3) \in V$$

$$(\mu_1 \circ \mu_2 \circ \mu_3)((u, u_2)(u, v_2)(u, w_2)) = \sigma_1(y) \wedge \mu_1(u_2 v_2 w_2), \forall u \in V_1, u_2, v_2 \in E_2, w_2 \in E_3$$

$$(\mu_1 \circ \mu_2 \circ \mu_3)((u_1, y)(v_1, y)(w_1, y)) = \sigma_2(y) \wedge \mu_1(u_1 v_1 w_1), \forall y \in V_2, \forall u_1, v_1 \in E_1, w_1 \in E_3$$

$$(\mu_1 \circ \mu_2 \circ \mu_3)((u_1, u_2)(v_1, v_2)(w_1, w_2)) = \sigma_1(u_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1 v_1 w_1), \forall u_2 \neq v_2 \neq w_2, \forall u_1, v_1 \in E_1, w_1 \in E_3$$

If  $G_1 : (\sigma_1, \mu_1), G_2 : (\sigma_2, \mu_2), G_3 : (\sigma_3, \mu_3)$  are three fuzzy graphs such that  $\sigma_1 \leq \mu_2 \leq \sigma_3$  then  $\sigma_2 \geq \mu_1 \geq \mu_3$  and vice versa.

## EDGE DEGREE AND TOTAL EDGE DEGREE OF A FUZZY GRAPH

### Edge degree of a fuzzy graph:

Let  $G : (\sigma, \mu)$  be a fuzzy graph on  $G^* : (V, E)$ . The degree of an  $u v$  is

$$d_G(u, v) = d_G(u) + d_G(v) - 2\mu(u, v). \text{ Since } \mu(u, v) > 0 \text{ for } u v \in E, \mu(u, v) = 0 \text{ for } u v \notin E$$

This is equivalent to  $d_G(u, v)$

$$= \sum_{uv \in E, y \neq v} \mu(u, y) + \sum_{yv \in E, y \neq u} \mu(v, y) + \sum_{yw \in E, w \neq y} \mu(w, y).$$

The minimum degree of G are

$$\delta_E(G) = \wedge \{d(uvw), \forall uvw \in E\}$$

$$\Delta_E(G) = \vee \{d_G(uvw), \forall uvw \in E\}$$

**Total Edge degree of a fuzzy graph:**

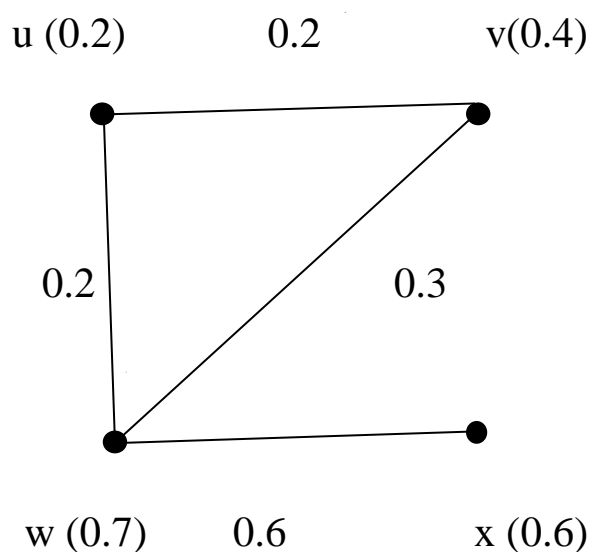
Let  $G : (\sigma, \mu)$  be a fuzzy graph on  $G^* : (V, E)$ . The total degree of a vertex  $u \in V$  is defined by

$$T d_G(u, v) = d_G(u) + d_G(v) - \mu(u, v).$$

Since  $\mu(u, v) > 0$  for  $uv \in E$ ,  $\mu(uv) = 0$  for  $uv \notin E$

$$\begin{aligned} \text{This is equivalent to } td_G(u, v) &= \sum_{uy \in E, y \neq v} \mu(uy) + \sum_{yv \in E, y \neq u} \mu(yv) + \\ &\sum_{wy \in E, y \neq w} \mu(yw) + \sum_{uvw \in E} \mu(uvw) \\ &= d_G(uvw) + \mu(uvw) \end{aligned}$$

**EXAMPLE:**



$$d_G(u)=0.4, td(u)=0.6, \delta(G)=0.4, \Delta(G)=1.1.$$

$$d(uv)=0.5, td(uv)=0.7, \delta_E(G)=0.5, \Delta_E(G)=1.1.$$

## DEGREE OF AN EDGE IN CARTESIAN PRODUCT

By definition, for any  $(u_1, u_2, u_3) \in V_1 \times V_2 \times V_3$  and  $((u_1, u_2), (v_1, v_2), (w_1, w_2)) \in E$  with  $u_1 \neq v_1, u_2 = v_2$  (or)  $u_1 = w_1, v_1 \neq w_1$  (or)  $w_2 = u_2, w_1 = v_1$ .

$$d_{G_1 \times G_2 \times G_3}((u_1, u_2), (v_1, v_2), (w_1, w_2)) =$$

$$\sum_{(u_1, u_2), (w_1, w_2), (y_1, y_2) \in E, (y_1, y_2) \neq (v_1, v_2)} (\mu_1 \times \mu_2 \times \mu_3)((u_1, u_2), (y_1, y_2), (w_1, w_2))$$

$$+ \sum_{(w_1, w_2), (y_1, y_2), (v_1, v_2) \in E, (y_1, y_2) \neq (v_1, v_2)} (\mu_1 \times \mu_2 \times \mu_3)((y_1, y_2), (w_1, w_2), (v_1, v_2))$$

$$+ \sum_{(u_1, u_2), (v_1, v_2), (y_1, y_2) \in E, (y_1, y_2) \neq (w_1, w_2)} ((\mu_1 \times \mu_2 \times \mu_3)(u_1, u_2), (v_1, v_2), (y_1, y_2))$$

If  $u_1 \neq v_1, u_2 = v_2$ ,

$$d_{G_1 \times G_2 \times G_3}((u_1, v_1), (v_1, v_2), (w_1, w_2))$$

$$= \sum_{(u_1, u_2), (w_1, w_2), (y_1, y_2) \in E, (y_1, y_2) \neq (v_1, v_2)} (\mu_1 \times \mu_2 \times \mu_3)((u_1, u_2), (y_1, y_2), (w_1, w_2))$$

$$+ \sum_{(w_1, w_2), (y_1, y_2), (v_1, v_2) \in E, (y_1, y_2) \neq (u_1, u_2)} (\mu_1 \times \mu_2 \times \mu_3)((y_1, y_2), (w_1, w_2), (v_1, v_2))$$

$$(y_1, y_2) \neq (u_1, u_2)$$

$$+ \sum_{(u_1, u_2), (v_1, v_2), (y_1, y_2) \in E, (y_1, y_2) \neq (w_1, w_2)} ((\mu_1 \times \mu_2 \times \mu_3)(u_1, u_2), (v_1, v_2), (y_1, y_2))$$

$$(y_1, y_2) \neq (w_1, w_2)$$

$$\begin{aligned}
& d_{G_1 \times G_2 \times G_3}((u_1, u_2)(v_1, u_2)(w_1, w_2)) \\
&= \sum_{y_2 \in V_2, y_2 \in V_2} \sigma_1 \wedge \mu_2(u_2, y_2) \wedge \mu_3(w_1, w_2) \\
&+ \sum_{y_1 \in V_1, y_1 \neq v_1} \mu_1(u_1, y_1) \wedge \sigma_2(u_2) \wedge \mu_3(w_1, w_2) \\
&+ \sum_{y_2 \in V_2} (\sigma_1(u_1) \wedge \mu_2(y_2, v_2) \wedge \mu_3(w_1, w_2)) \\
&+ \sum_{y_2 \in V_1, \mu_1(y_1, u_1) \wedge \sigma_2(v_2) \wedge \mu_3(w_1, w_2)} \\
&+ \sum_{w_2 \neq y_2, y_2 \in V_2} \mu_1(y_2, w_2) \wedge \mu_2(u_2, v_2) \wedge \sigma_3(v_3) \\
&+ \sum_{y_1 \in V_1, w_1 = y_1} \mu_3(w_1, y_1) \wedge \mu_1(u_1, v_1) \wedge \sigma_3(v_3)
\end{aligned}$$

If  $u_1 = w_2, u_1 \neq w_1,$

$$\begin{aligned}
& d_{G_1 \times G_2 \times G_3}((u_1, v_1)(v_1, v_2)(w_1, w_2)) \\
&= \sum_{(u_1, u_2)(w_1, w_2)(y_1, y_2) \in E} (\mu_1 \times \mu_2 \times \mu_3)((u_1, u_2)(y_1, y_2)(w_1, \\
&w_2))_{(y_1, y_2) \neq (v_1, v_2)} \\
&+ \sum_{(w_1, w_2)(y_1, y_2)(v_1, v_2) \in E} (\mu_1 \times \mu_2 \times \mu_3)((y_1, y_2)(w_1, w_2)(v_1, v_2)) \\
&_{(y_1, y_2) \neq (u_1, u_2)} \\
&+ \sum_{(u_1, u_2)(v_1, v_2)(y_1, y_2) \in E} ((\mu_1 \times \mu_2 \times \mu_3)(u_1, u_2)(v_1, v_2)(y_1, y_2)) \\
&_{(y_1, y_2) \neq (w_1, w_2)}
\end{aligned}$$

$$\begin{aligned}
& d_{G_1 \times G_2 \times G_3}(w_2, u_2)(v_1, v_2) = \sum_{y_2 \in V_2, y_2 \neq u_1} \sigma_1(u_1) \wedge \mu_2(y_2, u_2) \wedge \mu_3(w_1, w_2) \\
&+ \sum_{y_1 \in V_1, y_1 \neq u_1} \mu_1(u_1, y_1) \wedge \sigma_2(u_2) \wedge \mu_3(w_1, w_2) \\
&+ \sum_{y_2 \in V_1, y_2 \neq v_1} \sigma_1(u_1) \wedge \mu_2(y_2, u_2) \wedge \mu_3(w_1, w_2) \\
&+ \sum_{w_2 \neq y_2, y_2 \in V_1} \mu_1(y_2, w_2) \wedge \mu_2(u_2, v_2) \wedge \sigma_3(v_3) \\
&+ \sum_{y_1 \in V_1, y_2 = u_2} \mu_3(u_1, y_2) \wedge \mu_1(w_2, u_2) \wedge \sigma_3(v_3)
\end{aligned}$$

If  $w_2 = v_2, w_1 = v_1$

$$\begin{aligned}
& d_{G_1 \times G_2 \times G_3}((u_1, v_1)(v_1, v_2)(w_1, w_2)) \\
&= \sum_{(u_1, u_2)(w_1, w_2)(y_1, y_2) \in E} (\mu_1 \times \mu_2 \times \mu_3)((u_1, u_2)(y_1, y_2)(w_1, w_2)) \\
&_{(y_1, y_2) \neq (v_1, v_2)} \\
&+ \sum_{(w_1, w_2)(y_1, y_2)(v_1, v_2) \in E} (\mu_1 \times \mu_2 \times \mu_3)((y_1, y_2)(w_1, w_2)(v_1, v_2)) \\
&_{(y_1, y_2) \neq (u_1, u_2)}
\end{aligned}$$

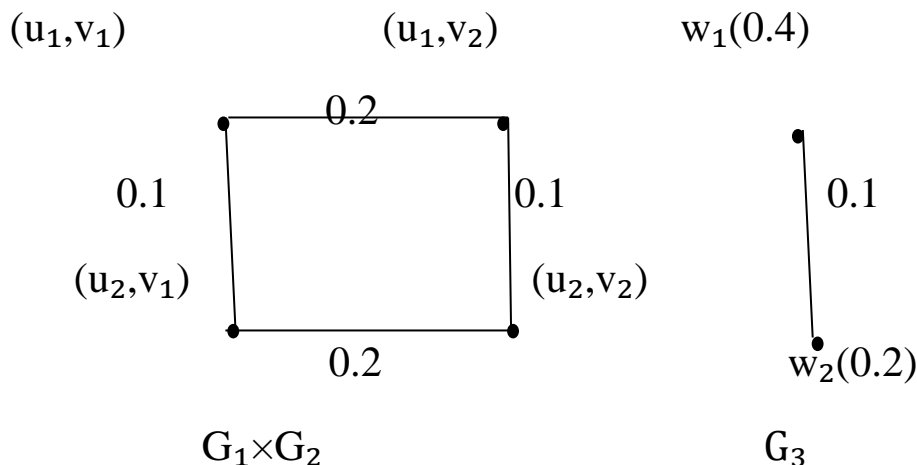
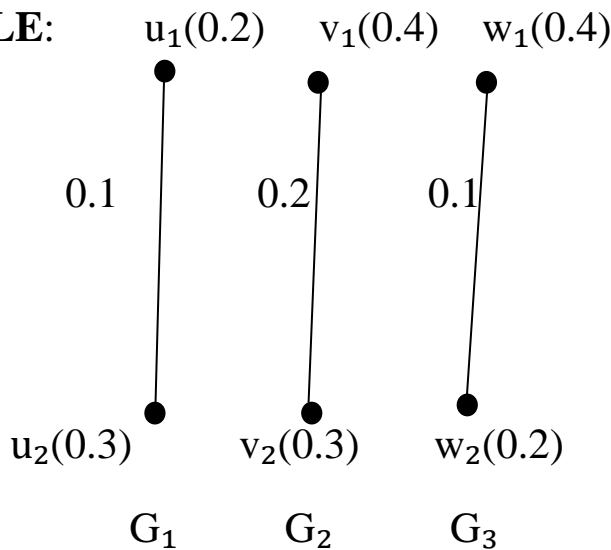
$$+\sum_{(u_1,u_2)(v_1,v_2)(y_1,y_2) \in E((\mu_1 \times \mu_2 \times \mu_3)(u_1,u_2)(v_1,v_2)(y_1,y_2))}$$

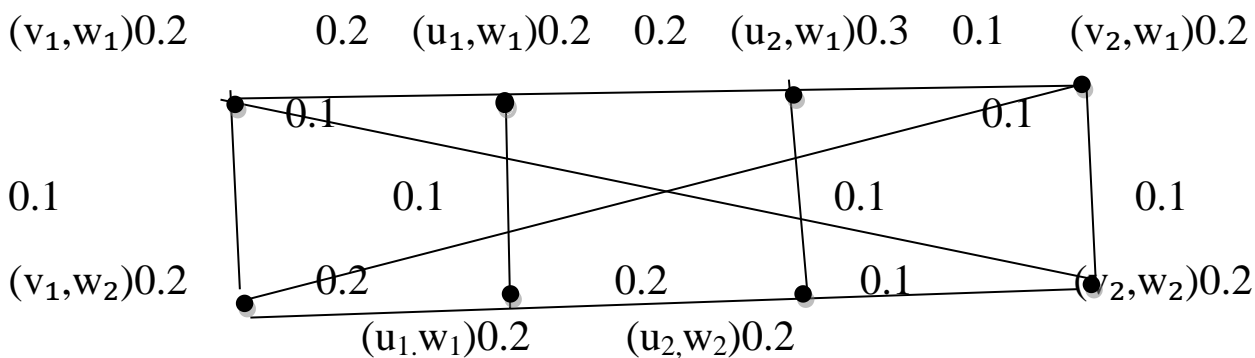
$$(y_1,y_2) \neq (w_1,w_2)$$

$$\begin{aligned} d_{G_1 \times G_2 \times G_3}((v_1,v_2)(w_1,u_2)) &= \sum_{y_1 \in V_1, y_1=w_1} \sigma_2(u_2) \wedge \mu_2(v_1,y_1) \wedge \mu_3(w_1,w_2) \\ &+ \sum_{y_2 \in V_2, y_2=u_2} \mu_1(u_1,y_1) \wedge \sigma_1(u_1) \wedge \mu_3(w_1,w_2) \\ &+ \sum_{y_2 \in V_2} \sigma_1(u_1) \wedge \mu_2(y_2,v_2) \wedge \mu_3(w_1,w_2) \\ &+ \sum_{y_1 \in V_1} \mu_1(y_1,u_1) \wedge \sigma_2(v_2) \wedge \mu_3(w_1,w_2) \\ &+ \sum_{y_1 \in V_1} \mu_3(w_1,y_1) \wedge \mu_1(u_1,v_1) \wedge \sigma_3(v_3) \\ &+ \sum_{y_2 \in V_2, y_2=w_1} \mu_1(y_2,w_2) \wedge \mu_2(u_2,v_2) \wedge \sigma_3(v_3) \end{aligned}$$

In the following theorems ,we find the degree of  $(u_1,v_2)(v_1,u_2), (w_2,u_2)(v_1,v_2)$  and  $(w_1,w_2)(v_1,u_2)$  in  $G_1 \times G_2 \times G_3$  in terms of those in  $G_1, G_2, G_3$  in some particular cases.

**EXAMPLE:**





$G_1 \times G_2 \times G_3$  Graph

$(u_1, v_1) = v_1 = 0.2$

$(u_1, v_2) = u_1 = 0.2$

$(u_2, v_1) = u_2 = 0.3$

$(u_2, v_2) = v_2 = 0.3$

$d(v_1, w_1) = 0.4, \delta(G_1 \times G_2 \times G_3) = 0.3, \Delta(G_1 \times G_2 \times G_3) = 0.5$

$td(v_1, w_1) = 0.6, t\delta(G_1 \times G_2 \times G_3) = 0.5, t\Delta(G_1 \times G_2 \times G_3) = 0.7$

**EDGE DEGREE:**

$d((v_1, w_1), (u_1, w_1)) = 0.5, \delta(G_1 \times G_2 \times G_3) = 0.4, \Delta(G_1 \times G_2 \times G_3) = 0.6.$

$td((v_1, w_1), (u_1, w_1)) = 0.7, t\delta(G_1 \times G_2 \times G_3) = 0.5, t\Delta(G_1 \times G_2 \times G_3) = 0.7.$

**THEOREM:**

Let  $G_1:(\sigma_1, \mu_1), G_2:(\sigma_2, \mu_2), G_3:(\sigma_3, \mu_3)$  be three fuzzy graphs.

If  $\sigma_1 \leq \mu_2 \geq \sigma_3, \sigma_2 \geq \mu_1 \geq \mu_3$  then,

$d_{G_1 \times G_2 \times G_3}((u_1, u_2)(u_1, v_2)(w_1, w_2)) = 2d_{G_1}(u_1) + d_{G_2}(u_2, v_2) + d_{G_3}(w_1, w_2),$  if  $(u_1, u_2)(u_2, v_2)(w_1, w_2) \in E.$

$d_{G_1 \times G_2 \times G_3}((u_1, u_2)(v_1, u_2)(w_1, w_2)) = d_{G_1}(u_1, v_1) + 2d_{G_2}(u_2) + d_{G_3}(w_1, w_2),$  if  $(u_1, u_2)(v_2, u_2)(w_1, w_2) \in E.$



**Proof:**

We have  $\sigma_1 \geq \mu_2 \geq \sigma_3, \sigma_2 \geq \mu_1 \geq \mu_3$ .

$(u_1, u_2)(u_1, v_2)(w_1, w_2) \in E$ .

$$\begin{aligned} d_{G_1 \times G_2 \times G_3}((u_1, u_2)(u_1, v_2)(w_1, w_2)) \\ = \sum_{y_2 \in V_2, y_2 \in V_2} \sigma_1 \wedge \mu_2(u_2, y_2) \wedge \mu_3(w_1, w_2) \\ + \sum_{y_1 \in V_1, y_1 \neq v_1} \mu_1(u_1, y_1) \wedge \sigma_2(u_2) \wedge \mu_3(w_1, w_2) \\ + \sum_{y_2 \in V_2} (\sigma_1(u_1) \wedge \mu_2(y_2, v_2) \wedge \mu_3(w_1, w_2)) \\ + \sum_{y_2 \in V_1} \mu_1(y_1, u_1) \wedge \sigma_2(v_2) \wedge \mu_3(w_1, w_2) \\ + \sum_{w_2 \neq y_2, y_2 \in V_2} \mu_1(y_2, w_2) \wedge \mu_2(u_2, v_2) \wedge \sigma_3(v_3) \\ + \sum_{y_1 \in V_1, w_1 = y_1} \mu_3(w_1, y_1) \wedge \mu_1(u_1, v_1) \wedge \sigma_3(v_3) \end{aligned}$$

$$d_{G_1 \times G_2 \times G_3}((u_1, u_2)(u_1, v_2)(w_1, w_2)) = 2d_{G_1}(u_1) + d_{G_2}(u_2, v_2) + d_{G_3}(w_1, w_2),$$

$$\begin{aligned} d_{G_1 \times G_2 \times G_3}((u_1, u_2)(v_1, u_2)(w_1, w_2)) \\ = \sum_{y_2 \in V_2} \mu_2(u_2, y_2) \\ + \sum_{y_1 \in V_1, y_1 \neq v_1} \mu_1(u_1, y_1) \\ + \sum_{y_2 \in V_2} \mu_2(y_2, v_2) \\ + \sum_{y_2 \in V_1} \mu_1(y_1, u_1) \\ + \sum_{w_2 \neq y_2, y_2 \in V_2} \mu_1(y_2, w_2) \\ + \sum_{y_1 \in V_1, w_1 = y_1} \mu_3(w_1, y_1) \end{aligned}$$

$$d_{G_1 \times G_2 \times G_3}((u_1, u_2)(v_1, u_2)(w_1, w_2)) = d_{G_1}(u_1, v_1) + 2d_{G_2}(u_2) + d_{G_3}(w_1, w_2).$$

## DEGREE OF AN EDGE IN COMPOSITION.

By definition, for any  $(u_1, u_2) \in V_1 \times V_2 \times V_3$  and

$$(u_1, u_2)(v_1, u_2)(w_1, w_2) \in E$$

$$d_{G_1 \circ G_2 \circ G_3}((u_1, v_1)(v_1, v_2)(w_1, w_2))$$

$$= \sum_{(u_1, u_2)(w_1, w_2)(y_1, y_2) \in E} (\mu_1 \circ \mu_2 \circ \mu_3)((u_1, u_2)(y_1, y_2)(w_1, w_2))$$

$$(y_1, y_2) \neq (v_1, v_2)$$

$$+ \sum_{(w_1, w_2)(y_1, y_2)(v_1, v_2) \in E} (\mu_1 \circ \mu_2 \circ \mu_3)((y_1, y_2)(w_1, w_2)(v_1, v_2))$$

$$(y_1, y_2) \neq (u_1, u_2)$$

$$+ \sum_{(u_1, u_2)(v_1, v_2)(y_1, y_2) \in E} ((\mu_1 \circ \mu_2 \circ \mu_3)(u_1, u_2)(v_1, v_2)(y_1, y_2))$$

$$(y_1, y_2) \neq (w_1, w_2)$$

By using cartesian product,

If  $u_1 = v_1, u_2 \neq v_2, w_1 = u_1$

$$d_{G_1 \circ G_2 \circ G_3}((u_1, u_2)(u_1, v_2)(u_1, w_2))$$

$$= \sum_{(u_1, u_2)(u_1, w_2)(y_1, y_2) \in E} (\mu_1 \circ \mu_2 \circ \mu_3)((u_1, u_2)(y_1, y_2)(u_1, w_2))$$

$$(y_1, y_2) \neq (u_1, v_2)$$

$$+ \sum_{(u_1, w_2)(y_1, y_2)(v_1, v_2) \in E} (\mu_1 \circ \mu_2 \circ \mu_3)((y_1, y_2)(u_1, w_2)(u_1, v_2))$$

$$(y_1, y_2) \neq (u_1, u_2)$$

$$+ \sum_{(u_1, u_2)(u_1, v_2)(y_1, y_2) \in E} ((\mu_1 \circ \mu_2 \circ \mu_3)(u_1, u_2)(u_1, v_2)(y_1, y_2))$$

$$(y_1, y_2) \neq (u_1, w_2)$$

$$= \sum_{(u_2, w_2) \in E_2, (u_1, w_2) \in E_3, u_1 = y_1 = w_1, y_2 \neq v_2} \sigma_1(u_1) \wedge \mu_2(u_2, y_2) \wedge \mu_3(u_1, w_2)$$

$$+ \sum_{(u_1, y_1) \in E_1, (u_1, w_2) \in E_3, u_2 = y_1} \mu_1(u_1, y_1) \wedge \sigma_2(u_2) \wedge \mu_3(u_1, w_2)$$

$$+ \sum_{u_1, y_1 \in E_1, u_2 \neq y_2, u_1, w_2 \in E_3} \mu_1(u_1, y_1) \wedge \sigma_2(u_2) \wedge \sigma_2(y_2) \wedge \mu_3(u_1, w_2)$$

$$+ \sum_{y_2, v_2 \in E_2, y_1 = u_1, y_2 \neq u_2} (\sigma_1(u_1) \wedge \mu_2(y_2, v_2) \wedge \mu_3(w_1, w_2))$$

$$\begin{aligned}
& + \sum_{y_1 u_1 \in E, y_2 = v_2} \mu_1(y_1, u_1) \wedge \sigma_2(v_2) \wedge \mu_3(u_1, w_2) \quad + \sum_{u_1 y_2 \in E_3, u_1 v_2 \in V_2, y_2 \neq w_2} \\
& \mu_3(u_1, y_2) \wedge \mu_2(u_1, v_2) \wedge \sigma_3(u_3) \\
& + \sum_{u_1 u_2 \in E_1, y_1 w_2 \in E_3, y_1 \neq u_1} \mu_3(y_1, w_2) \wedge \mu_1(u_1, u_2) \wedge \sigma_3(u_3) \\
& + \sum_{y_1 u_1 \in E_1, u_1 w_2 \in E_3, y_2 \neq v_2} \mu_1(y_1 u_1) \wedge \sigma_2(y_2) \wedge \sigma_2(u_2) \wedge \mu_3(u_1 w_2) \\
& + \sum_{y_1 w_1 \in E_1, v_1 w_2 \in E_2, y_2 \neq u_2} \mu_1(y_1 w_1) \wedge \sigma_3(y_2) \wedge \sigma_2(v_2) \wedge \mu_1(v_1 w_2)
\end{aligned}$$

If  $u_1 = w_2, w_2 = v_2, v_1 \neq w_1$

$$\begin{aligned}
d_{G_1 \circ G_2 \circ G_3}((u_1, u_2)(u_1, w_2)(v_1, v_2)) &= \\
&= \sum_{(u_1, u_2)(v_1, v_2)(y_1, y_2) \in E} (\mu_1 \circ \mu_2 \circ \mu_3)((u_1, u_2)(y_1, y_2)(v_1, v_2) \\
& \quad (y_1, y_2) \neq (u_1, w_2)) \\
& \quad + \sum_{(u_1, w_2)(y_1, y_2)(v_1, v_2) \in E} (\mu_1 \circ \mu_2 \circ \mu_3)((y_1, y_2)(u_1, w_2)(v_1, v_2)) \\
& \quad (y_1, y_2) \neq (u_1, u_2)) \\
& \quad + \sum_{(u_1, u_2)(u_1, v_2)(y_1, y_2) \in E} ((\mu_1 \circ \mu_2 \circ \mu_3)(u_1, u_2)(u_1, w_2)(y_1, y_2)) \\
& \quad (y_1, y_2) \neq (v_1, v_2)) \\
&= \sum_{(u_2, w_2) \in E_2, (v_1 v_2) \in E_3, u_1 \neq y_1} \sigma_1(u_1) \wedge \mu_2(u_1, y_1) \wedge \mu_2(v_1, v_2) \\
& + \sum_{(u_1 y_1) \in E_1, (v_1 v_2) \in E_2, u_2 = y_1} \mu_1(u_1, y_1) \wedge \sigma_2(u_2) \wedge \mu_3(v_1, v_2) \\
& + \sum_{u_1 y_1 \in E_1, u_2 \neq y_1, v_1 v_2 \in E_2} \mu_1(u_1 y_1) \wedge \sigma_2(u_2) \wedge \sigma_2(y_2) \wedge \mu_2(v_1 v_2) \\
& + \sum_{y_2 v_2 \in E_2, y_1 = v_1, y_2 \neq v_2} (\sigma_1(u_1) \wedge \mu_2(y_2, v_2) \wedge \mu_2(v_1, v_2)) \\
& + \sum_{y_1 u_1 \in E, y_2 = u_2} \mu_1(y_1, u_1) \wedge \sigma_2(v_2) \wedge \mu_2(v_1, v_2) \\
& + \sum_{u_1 y_2 \in E_3, v_1 v_2 \in E_2, y_2 = u_2} \mu_3(u_1, y_2) \wedge \mu_2(v_1, v_2) \wedge \sigma_3(u_3) \\
& + \sum_{u_1 u_2 \in E_1, y_1 v_2 \in E_2, y_1 \neq v_1} \mu_2(y_1, v_2) \wedge \mu_1(u_1, u_2) \wedge \sigma_3(u_3) \\
& + \sum_{y_1 u_1 \in E_1, v_1 v_2 \in E_2, y_2 \neq v_2} \mu_1(y_1 u_1) \wedge \sigma_2(y_2) \wedge \sigma_2(u_2) \wedge \mu_2(v_1 v_2) \\
& + \sum_{y_1 w_1 \in E_1, v_1 v_2 \in E_2, y_2 \neq u_2} \mu_1(y_1 w_1) \wedge \sigma_3(y_2) \wedge \sigma_2(v_2) \wedge \mu_1(u_1 v_2)
\end{aligned}$$

If  $u_1 \neq w_1, u_2 \neq v_2, v_1 \neq w_2$

$$d_{G_1 \circ G_2 \circ G_3}((u_1, u_2)(v_1, v_2)(w_1, w_2))$$

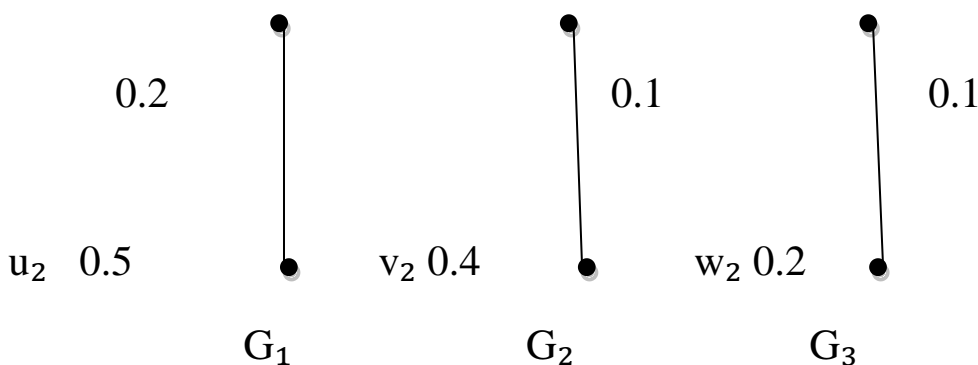
$$\begin{aligned}
 &= \sum_{(u_1, u_2)(v_1, v_2)(y_1, y_2) \in E} (\mu_1 \circ \mu_2 \circ \mu_3)((u_1, u_2)(y_1, y_2)(v_1, v_2)) \\
 &\quad (y_1, y_2) \neq (v_1, v_2) \\
 &+ \sum_{(v_1, v_2)(y_1, y_2)(w_1, w_2) \in E} (\mu_1 \circ \mu_2 \circ \mu_3)((y_1, y_2)(v_1, v_2)(w_1, w_2)) \\
 &\quad (y_1, y_2) \neq (u_1, u_2) + \sum_{(u_1, u_2)(v_1, v_2)(y_1, y_2) \in E} ((\mu_1 \circ \mu_2 \circ \mu_3)(u_1, u_2)(v_1, v_2)(y_1, y_2)) \\
 &\quad (y_1, y_2) \neq (w_1, w_2) \\
 &= \sum_{(u_2, y_2) \in E_2, (v_1, v_2) \in E_2, u_1 = y_1} \sigma_1(u_1) \wedge \mu_2(u_2, y_2) \wedge \mu_2(v_1, v_2) \\
 &+ \sum_{(u_1, y_1) \in E_1, (v_1, v_2) \in E_2, u_2 = y_1} \mu_1(u_1, y_1) \wedge \sigma_2(u_2) \wedge \mu_1(u_1, u_2) \\
 &+ \sum_{u_1, y_1 \in E_1, u_2 \neq y_2, w_1, w_2 \in E_3} \mu_1(u_1, y_1) \wedge \sigma_2(u_2) \wedge \sigma_2(y_2) \wedge \mu_3(w_1, w_2) \\
 &+ \sum_{y_2, v_2 \in E_2, y_1 = u_1, w_1, w_2 \in E_3} (\sigma_1(v_1) \wedge \mu_2(y_2, v_2) \wedge \mu_3(w_1, w_2)) \\
 &+ \sum_{y_1, v_1 \in E, y_2 = v_1} \mu_1(y_1, v_1) \wedge \sigma_2(v_2) \wedge \mu_3(w_1, w_2) \\
 &+ \sum_{y_1, v_1 \in E_1, w_1, w_2 \in E_3, y_2 = u_2} \mu_3(w_1, w_2) \wedge \mu_2(v_1, v_2) \wedge \sigma_1(w_1) \\
 &+ \sum_{u_1, u_2 \in E_1, w_2, y_2 \in E_2, y_1 = w_1} \mu_2(v_1, v_2) \wedge \mu_1(w_2, y_2) \wedge \sigma_1(w_1) \\
 &+ \sum_{y_1, w_1 \in E_1, u_1, u_2 \in E_1, y_2 = w_2} \mu_1(y_1, w_1) \wedge \sigma_2(y_2) \wedge \sigma_2(w_2) \wedge \mu_2(v_1, v_2) \\
 &+ \sum_{y_1, w_1 \in E_1, w_1, w_2 \in E_3, y_2 \neq w_2} \mu_1(y_1, w_1) \wedge \sigma_3(y_2) \wedge \sigma_2(w_2) \wedge \mu_3(w_1, w_2)
 \end{aligned}$$

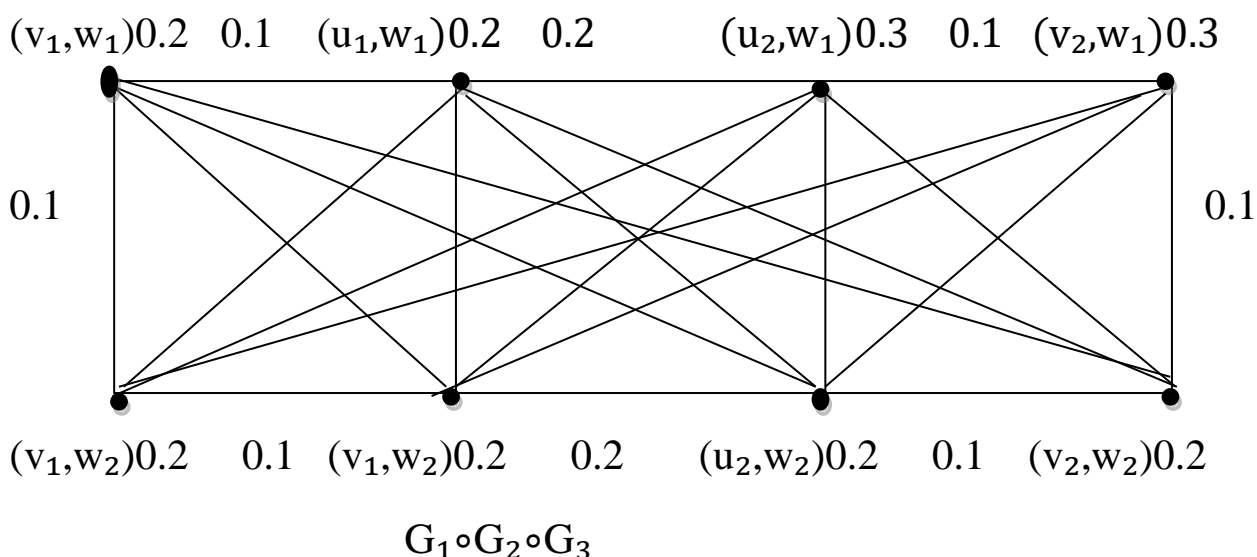
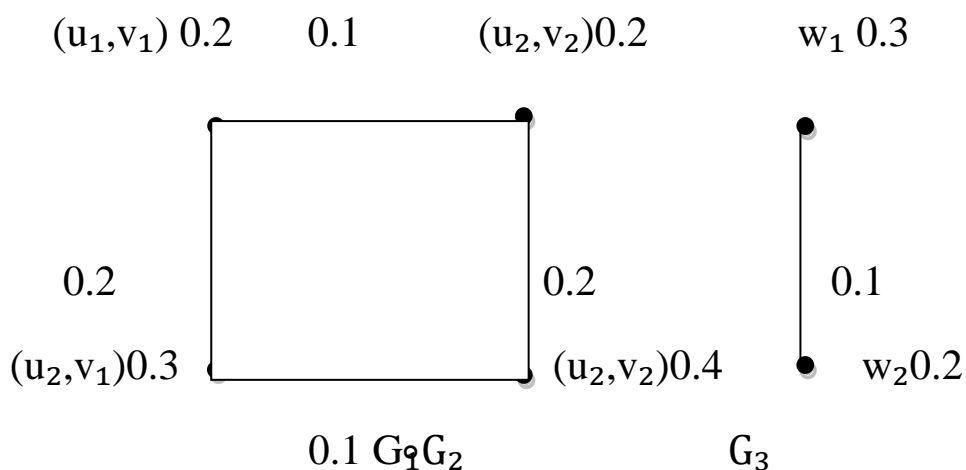
**EXAMPLE:**

$u_1 0.2$

$v_1 0.3$

$w_1 0.3$





Edge in inside of value in all is 0.1 Of  $G_1 \circ G_2 \circ G_3$  ;  $G_1 \circ G_2$  in 0.2

$$(u_1, v_1) = v_1$$

$$(u_1, v_2) = u_1$$

$$(u_2, v_1) = u_2$$

$$(u_2, v_2) = v_2$$

$$d(v_1, w_1) = 0.5, \delta(G_1 \circ G_2 \circ G_3) = 0.5, \Delta(G_1 \circ G_2 \circ G_3) = 0.7.$$

$$td(v_1, w_1) = 0.7, t\delta(G_1 \circ G_2 \circ G_3) = 0.7, t\Delta(G_1 \circ G_2 \circ G_3) = 1.$$

**EDGE DEGREE:**

$$d((v_1, w_1)(u_1, w_1)) = 1,$$

$$d\delta(G_1 \circ G_2 \circ G_3) = 0.8,$$

$$d\Delta(G_1 \circ G_2 \circ G_3) = 1.2.$$

$$t d((v_1, w_1)(u_1, w_1)) = 1.1,$$

$$t d\delta(G_1 \circ G_2 \circ G_3) = 0.8,$$

$$t d\Delta(G_1 \circ G_2 \circ G_3) = 1.3$$

**CONCLUSION:**

In this paper, have found the degree of edges in  $G_1 \times G_2 \times G_3$  and  $G_1[G_2]G_3$  in terms of the degree of vertices and edges in  $G_1, G_2$  and  $G_3$  and also in terms of the degree of vertices in  $G_1^*, G_2^*$  and  $G_3^*$  under some conditions.

They will be more helpful especially when the graphs are very large. Also they will be useful in studying various conditions, properties of Cartesian product and composition of three fuzzy graphs and used to further study for edge regular on some fuzzy graphs.

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