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REGULAR AND IRREGULAR r-POLAR FUZZY GRAPHS

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Abstract

In this paper we define irregular r- polar fuzzy graphs and its various classifications. Size of Irregular r-polar fuzzy graph derived.The relation between highly and neighbourly irregular r- polar fuzzy are established. Some basic theorems related to the stated graphs have also been presented.

Key Words:

r- polar fuzzy graphs, regular r- polar fuzzy graphs, irregular and degree of r- polar graphs.

INTRODUCTION:

The Origin of graph theory started with Konigsberg bridge problem in 1735. This problem led to the concept of Euler graph. Euler studied the Konigsberg problem that is referred to as an Euler graph. We introduced this paper r- polar fuzzy set as a generalization of r- polar fuzzy graph.

Presently, graph theory concepts are highly utilize by a computer applications, especially in area of research, including data mining image segmentation, clustering and networking. The first basic definitions of r- polar fuzzy graph was proposed by G. Ghorai and M. Pal [1] from a study on r- polar fuzzy graphs. In 1999,Molodtsov [9] introduced the concept of soft set theory to solve imprecise problems in the field of engineering, social science, economics, medical science and environment. Molodtsov applied this theory to several directions such as smoothness of function, game theory, operation research, probability and measurement theory. In recent times, a number of research studies contributed into the Fuzzification of soft set theory. As a result many researchers were more active doing research on soft sets.

However H. Rashmantou, S.Samantha, M.Pal, R. A. Borzooel [4] introduced r-polar fuzzy graphs with categorical properties. Also S.Samantha, M. pal [5] defined basic definition of irregular r- polar fuzzy graphs. In 2015, G. Ghorai and M. pal [2,11,12] also introduced another group of explained definition of complement and isomorphism of r- polar fuzzy graph and some operations and density of r- polar fuzzy graphs. At this same time they elaborated faces and dual of r- polar fuzzy planar graphs. The complement of fuzzy graphs, Indian journal of pure and the applied mathematics was introduced byM.S. Sunitah, A. Vijayakumar [9].

In 2011, Akram [13] introduced the concept of r- polar fuzzy graphs and defined different operations on it. Now r- polar fuzzy graph theory is growing and expanding its application.

Also we refer basic definitions of fuzzy set theory and K. Kalaiarasi[10] defined optimization of fuzzy integrated Vendor- Buyer inventory models.

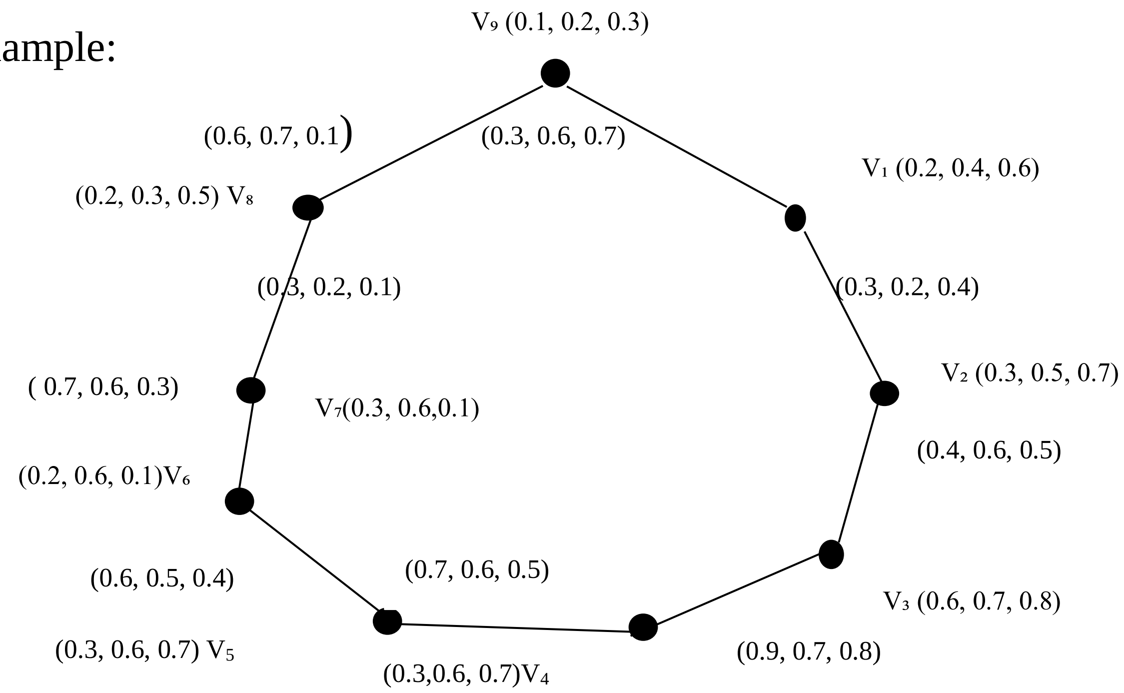
Some basic definitions and basic of regular and irregular theorems are discussed.

Definition: 1

r- polar fuzzy graph of a graph $G^*= (V, E)$ is a pair $G = (A, B, C)$ where $A: V \rightarrow [0, 1]^r$ is an r- polar fuzzy set in V and $B: \overline{V^2} \rightarrow [0, 1]^r$ is a polar fuzzy set in V^2 and $C: \overline{V^3} \rightarrow [0, 1]^r$ is a polar fuzzy set in V^3. Such that for each n =1, 2, 3, ...r. $k_i \circ [B, C (xyz)] \leq \min \{k_i \circ A(x), k_i \circ A(Y), k_i \circ A (z)\}$ for all $xyz \epsilon \overline{V^3}$ and $B, C (xyz) = 0$ for all $xyz \epsilon \overline{V^3} - E$, $\{0_= 0, 0, 0, \ldots\ldots .0\}$ is the smallest element in $[0, 1]^r$.A is called the r- polar fuzzy vertex and B, C is called the r- polar fuzzy edge.

This is called r- polar fuzzy graph.

Example:



Definition: 2

The Order of the r- polar fuzzy graph G = (V, A, B, C) is denoted by |v| or (O (G)) where

$$O(G) = |v| = \sum \frac{1+\sum_{i=0}^{r} qi \circ A(x)}{3}$$

Definition: 3

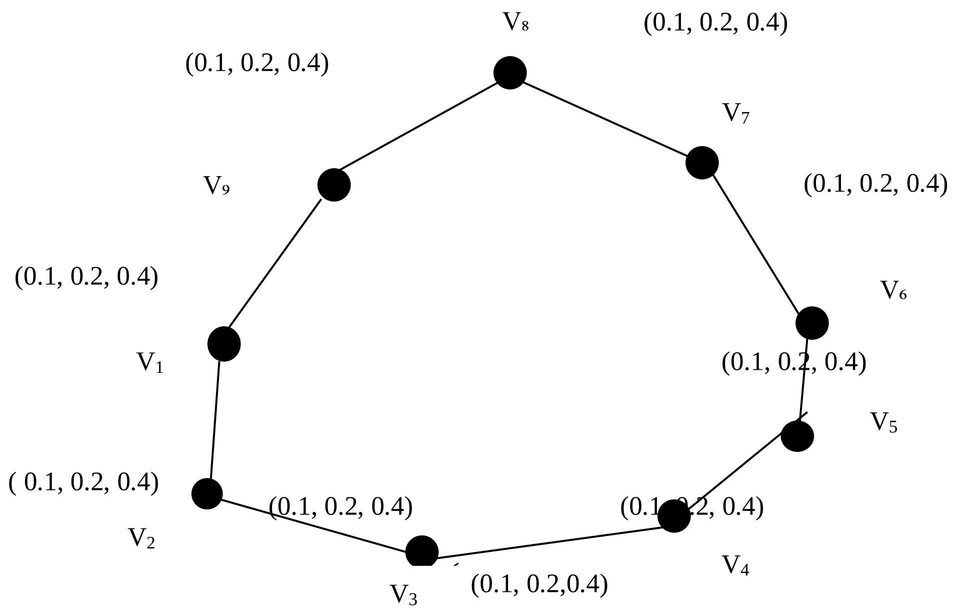
The Size of G = (A, B, C) is denoted by |E| or S (G) Where

$$S(G)=|E|=\sum_{xyz\in v} \frac{1+\sum_{i=1}^{r} qi \circ B, C(x, y, z)}{3}$$

Definition: **4**

Let G =(A,B,C) be a r-polar fuzzy graph where $A:V \rightarrow [0,1]^r$ is a r-polar fuzzy set in V and B: $\overline{V^2} \rightarrow [0,1]^r$ is a r- polar fuzzy set in $\overline{V}^2$ and C: $\overline{V}^3 \rightarrow [0,1]$ is a r-polar fuzzy sets on s non- empty finite set V and E $\subseteq V \times V \times V$ respectively. If $d(x) = k_1$, $d(y) = k_2$, $d(z) = k_3$ for all x, y, z $\epsilon X,Y,Z$. k_1, k_2, k_3 regular r-polar fuzzy graph. If ,there exists a vertices which is adjacent to a vertex with same.

Example:-



$d(V_1) = (0.2, 0.4, 0.8)$, $d(V_2) = (0.2, 0.4, 0.8)$, $d(V_3) =(0.2, 0.4, 0.8)$,

$d(V_4) = (0.2, 0.4, 0.8)$, $d(V_5) = (0.2, 0.4, 0.8)$, $d(V_6) = (0.2, 0.4, 0.8)$

$d (V_7) = (0.2, 0.4, 08)$, $d (V_8) = (0.2, 0.4, 0.8)$, $d (V_9) = (0.2, 0.4, 0.8)$

$d (V_1) = d (V_2) = d (V_3) = d (V_4) = d (V_5) = d (V_6) = d (V_7) = d (V_8) = d (V_9)$.

The graph is 3^3 polar fuzzy graph is said to regular.

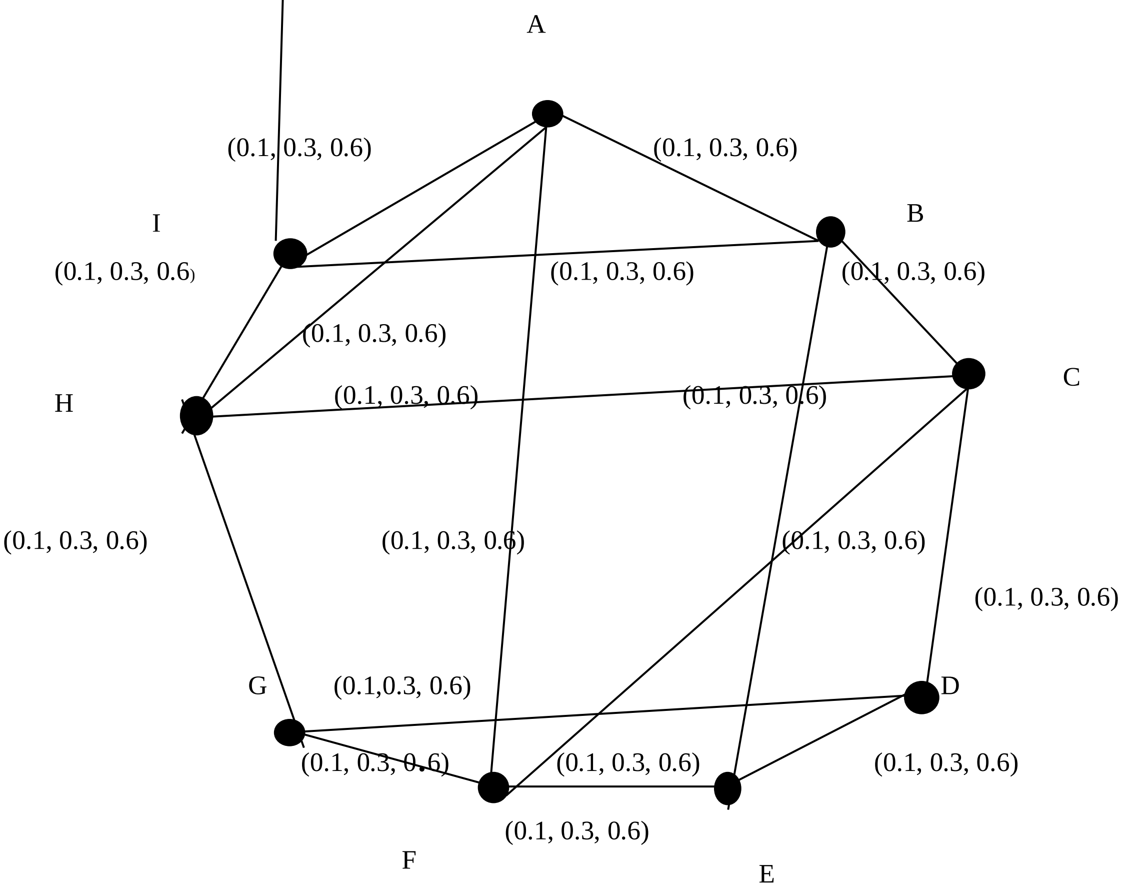
Remark:

If the graph is regular then odd graph or even graph of degree are same.

Example of odd and even regular graph:

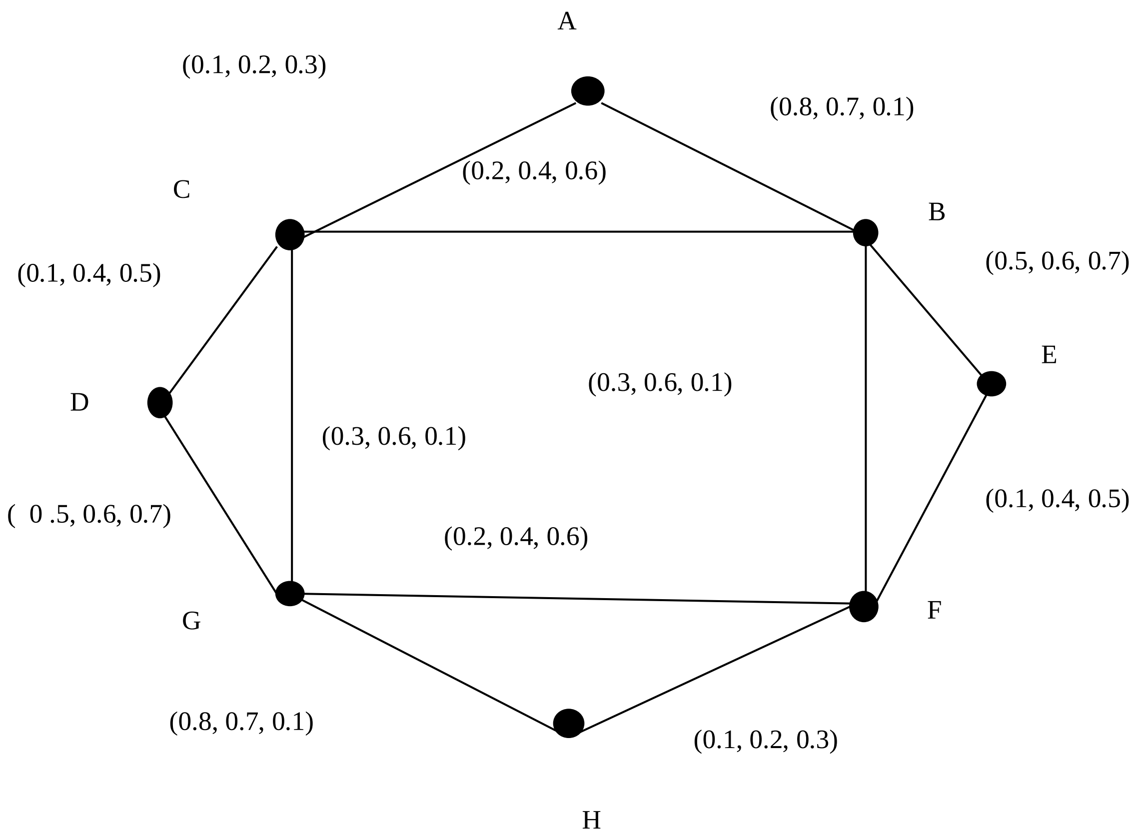
Odd regular graph.

In odd regular graph the all edges are same degree.



Even Regular Graph:-

In even regular graph the alternative edges are equal.

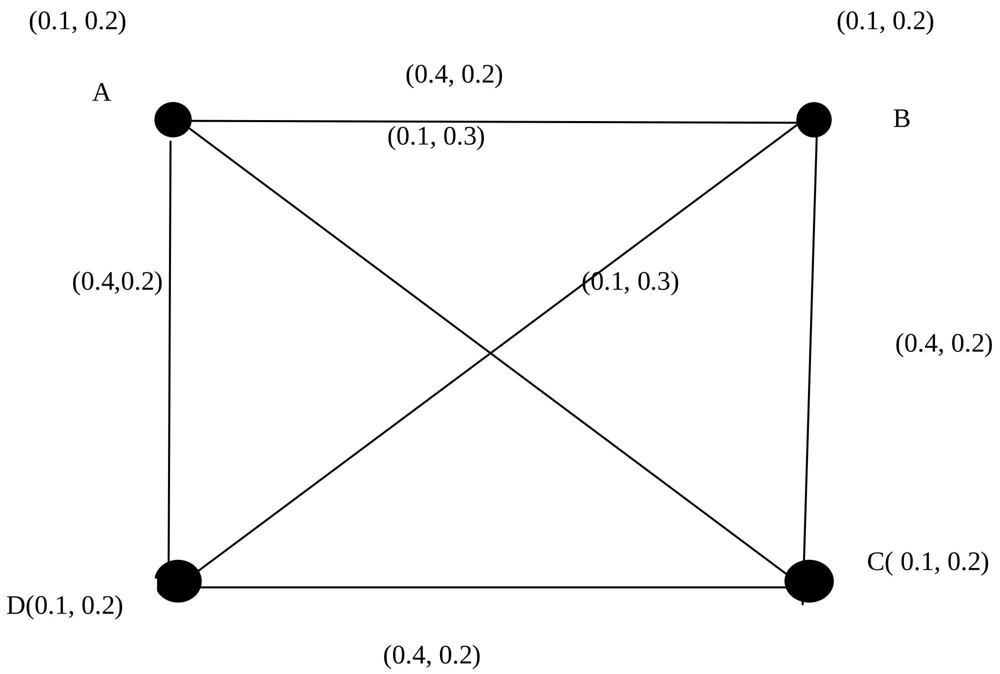


Definition: 5

Let $G = (A, B, C)$ be a r-polar fuzzy graph where $A:V\rightarrow[0,1]^r$ is a r- polar fuzzy set in V and $B:\overline{V}^2\rightarrow [0,1]^r$ is a r- polar fuzzy set in V^2 and $C:\overline{V}^3\rightarrow[0,1]^r$ be three r-polar fuzzy sets on a non- empty finite set V and $E\subseteq V\times V\times V$ respectively G is said to be totally regular r- polar fuzzy graph if there exists a vertex which is adjacent to a vertex with same total degree.

Example:-

Totally regular r-polar fuzzy graph.



Td (A) = (1.0, 0.9), Td (B) = (1.0, 0.9), Td (C) = (1.0, 0.9), Td (D) = (1.0,0.9)

Td (A) = Td (B) = Td C) = Td (D).

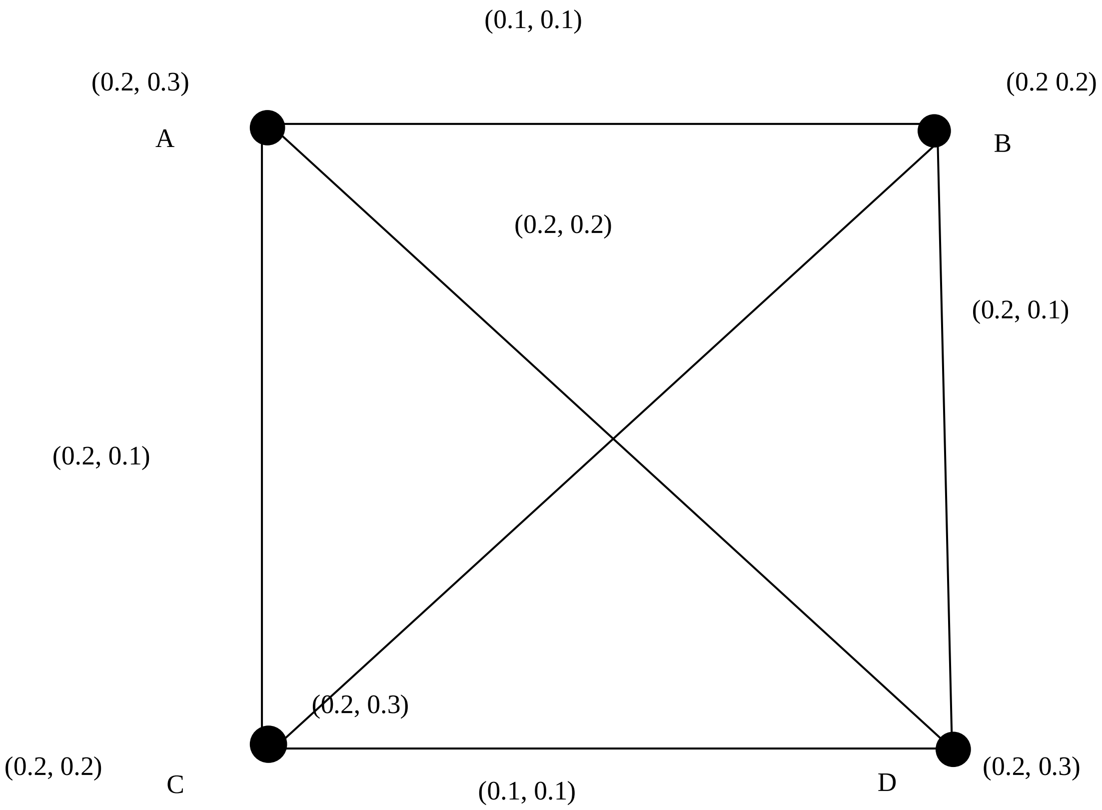
Example of 2^3 polar fuzzy graph.

Definition: **6**

Let G be connected r- polar fuzzy graph. Then G is called neighbourly to totally regular r- polar fuzzy graph if every three adjacent vertices of G have same total degrees.

Example:

Neighbourly totally regular r- polar fuzzy graph.



Td (A) = (0.7, 0.7), Td (B) = (0.7, 0.7), Td (C) = (0.7, 0.7), Td (D) = (0.7, 0.7)

Td (A) = Td (B) = Td (C) = Td (D).

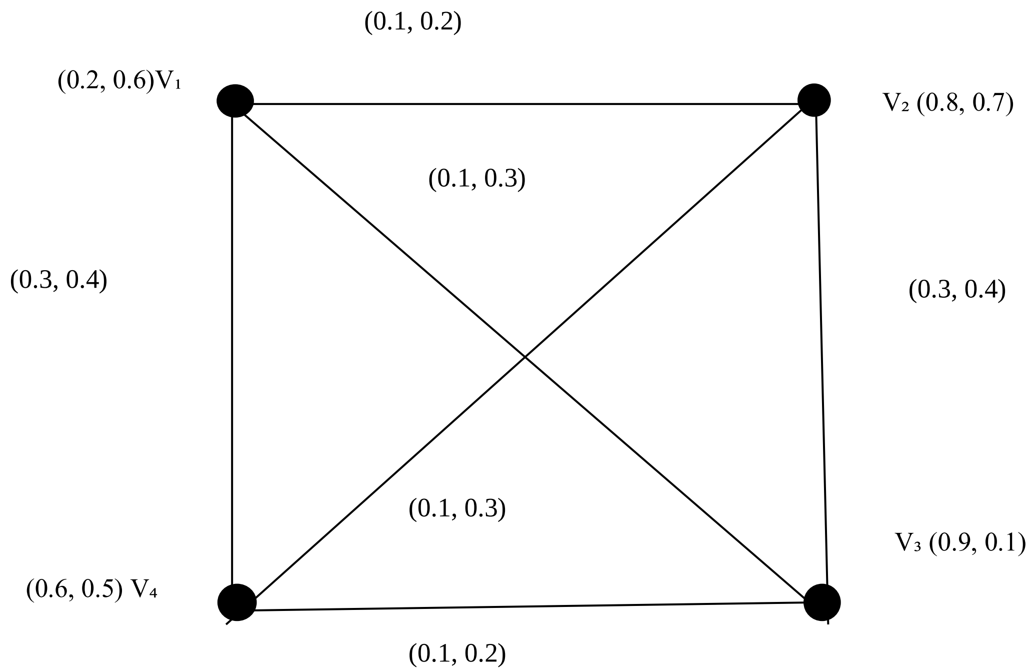
Example of 2^3 polar fuzzy graph.

Definition: 7

Let G be a connected r- polar fuzzy graph. The G is called neighbourly regular r- polar fuzzy graph if every three adjacent vertices of G have same degree.

Example

Neighbourly regular r- polar fuzzy graph



$d(V_1) = (0.5, 0.9)$, $d(V_2) = (0.5, 0.9)$, $d(V_3) = (0.5, 0.9)$, $d(V_4) = (0.5, 0.9)$

$d(V_1) = d(V_2) = d(V_3) = d(V_4)$.

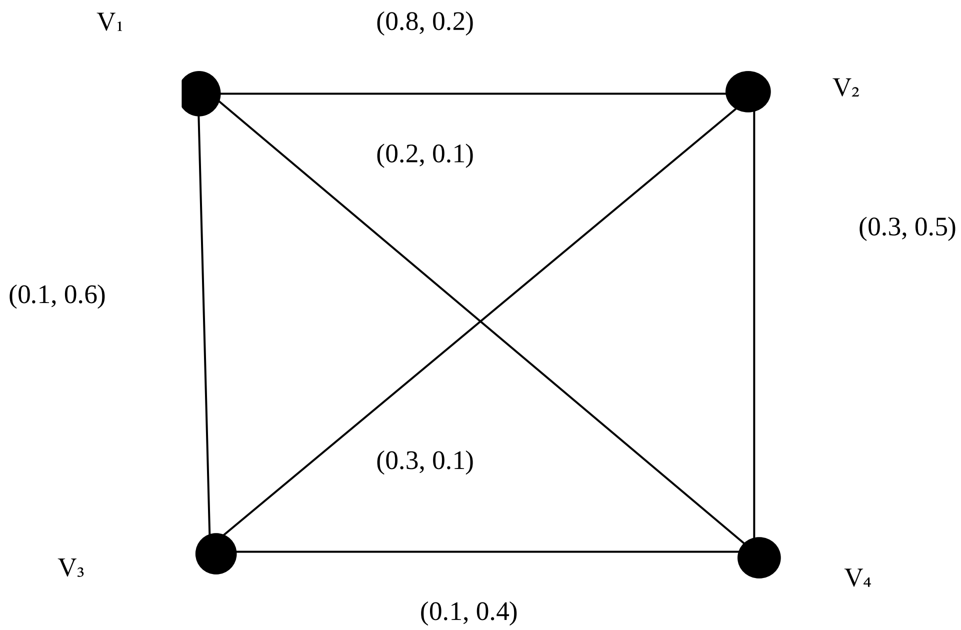
The example of 2^3 polar fuzzy graph.

Definition: 8

Let G be connected r- polar fuzzy graph. Then G is called neighbourly irregular r- polar fuzzy graph if for every three adjacent vertices of vertices of G have distinct degrees.

Example:

Neighbourly irregular r- polar fuzzy graph



$d(V_1) = (1.1, 0.9)$, $d(V_2) = (1.4, 0.8)$, $d(V_3) = (0.6, 1.0)$, $d(V_4) = (0.5, 1.1)$

$d(V_1) \neq d(V_2) \neq d(V_3) \neq d(V_4)$

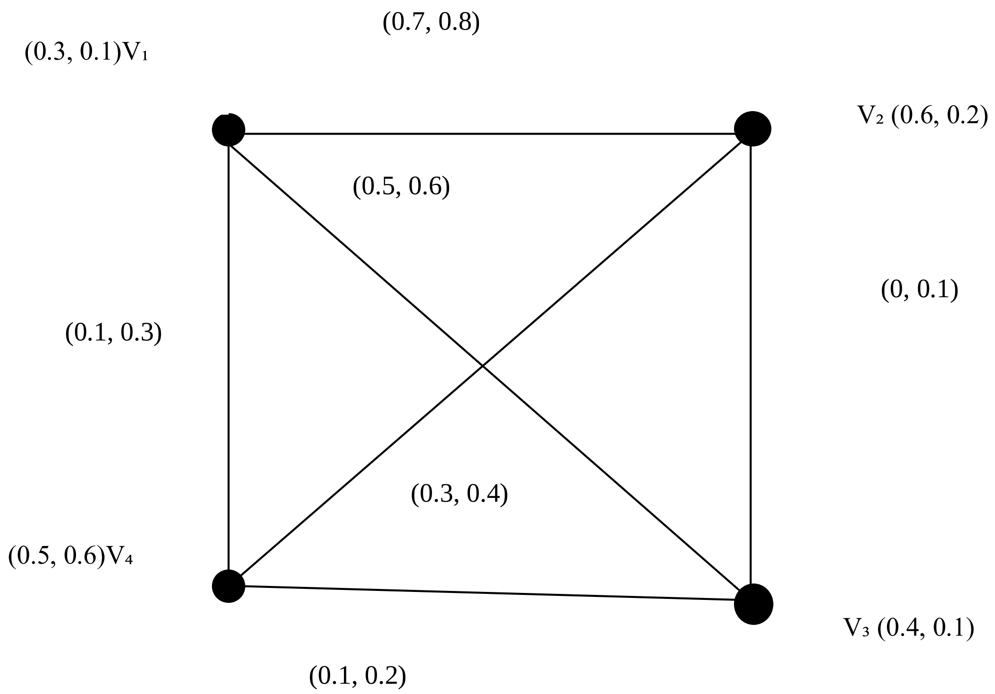
The example for 2^3 polar fuzzy graph.

Definition: 9

Let G =(A,B,C) be a r-polar fuzzy graph where $A:V\rightarrow[0,1]^r$ is a r- polar fuzzy set in V and $B:\overline{V}^2\rightarrow[0,1]^r$ is a r- polar fuzzy set in $\overline{V}^2$ and $C:\overline{V}^3\rightarrow[0,1]^r$ is a r- polar fuzzy set in $\overline{V}^3$ be the three r- polar fuzzy sets on a non- empty finite set V and $E\subseteq V\times V\times V$ respectively. G is said to be totally irregular r-polar fuzzy graph if there exists a vertex which is adjacent to a vertex with distinct total degrees.

Example:

Totally irregular r- polar fuzzy graph.



d $(V_1) = (1.3, 1.7)$, d $(V_2) = (1.0, 1.3)$, d $(V_3) = (0.6, 0.9)$, d $(V_4) = (0.5, 0.9)$

Td $(V_1) = (1.6, 1.8)$, Td $(V_2) = (1.6, 1.5)$, Td $(V_3) = (1.0, 1.0)$,

Td $(V_4) = (1.0, 1.5)$.

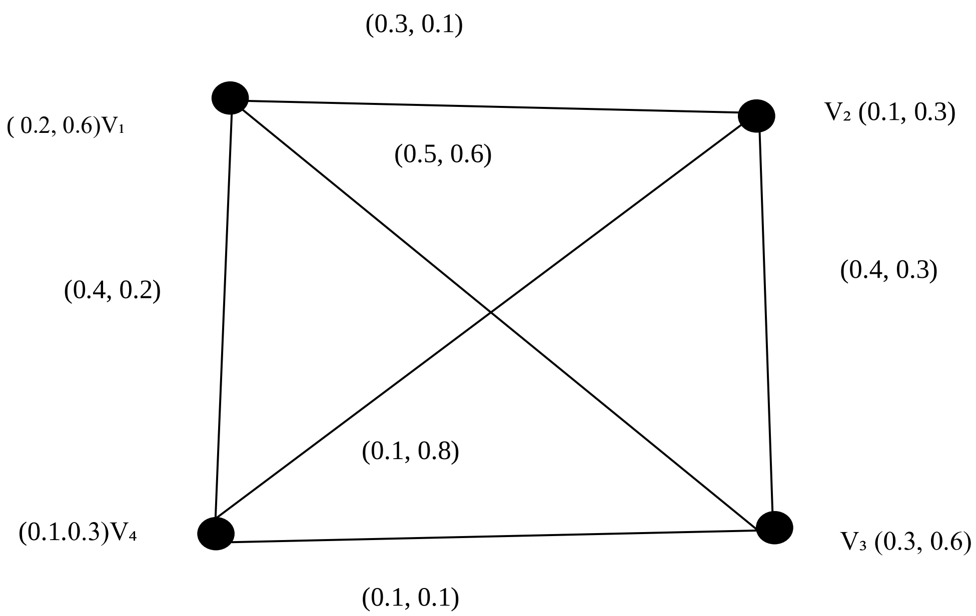
Td $(V_1) \neq$ Td $(V_2) \neq$ Td $(V_3) \neq$ Td (V_4)

This example for 2^3 polar fuzzy graph.

Definition: 10

Let G be a connected r- polar fuzzy graph. Then G is called neighbourly totally irregular r- polar fuzzy graph if for a every three adjacent vertices of G have distinct total degrees.

Example:



d $(V_1) = (1.2, 0.9)$, d $(V_2) = (1.2, 1.0)$, d $(V_3) = (0.1, 1.2)$ d $(V_4) = (1.2, 1.1)$

Td $(V_1) = (1.4, 1.5)$, Td $(V_2) = (1,3, 1.3)$, Td $(V_3) = (1.4, 1.8)$, Td $(V_4) = (1.3,1.9)$

$Td(V_1) \neq Td(V_2) \neq Td(V_3) \neq Td(V_4)$.

This example for 2^3 polar fuzzy graph .

Remark:

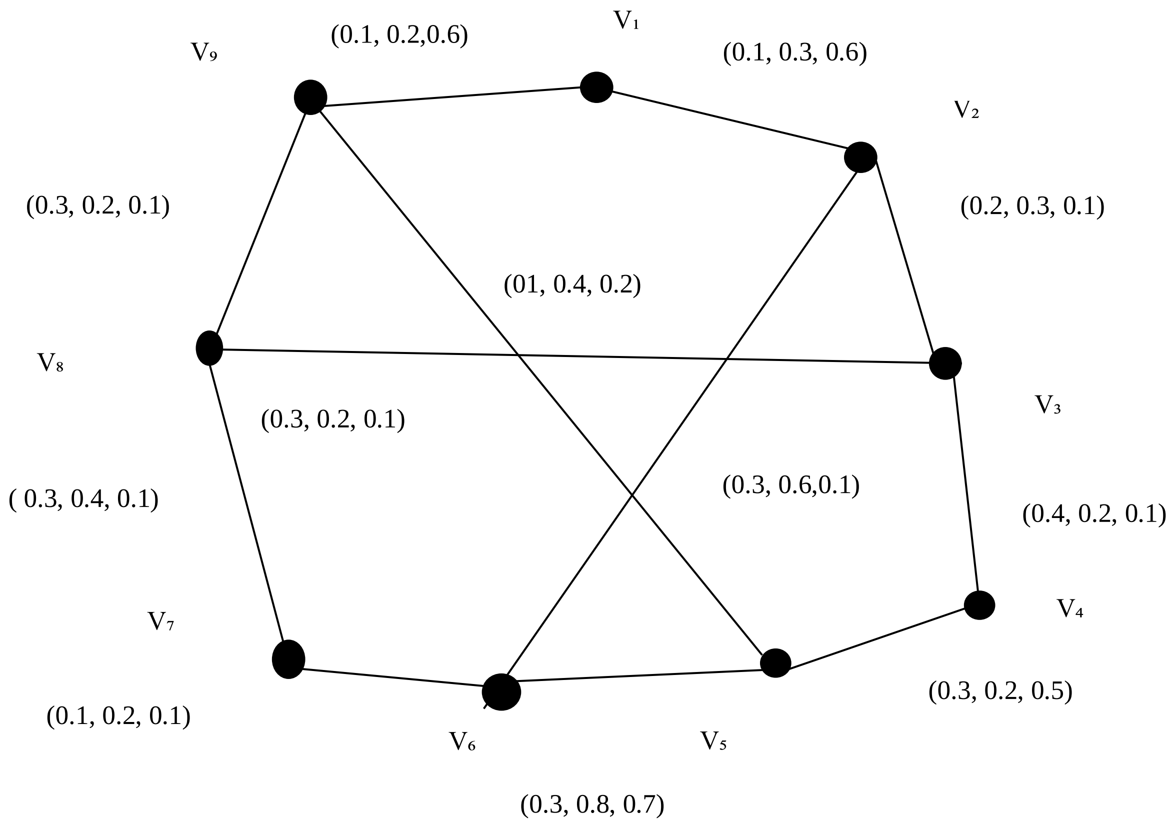
It need not be highly irregular r- polar fuzzy graph that is any adjacent vertices of the graph G have the same degree. Other than any adjacent vertices of the graph G have the distinct degree.

Definition: 11

Let G be a connected r- polar fuzzy graph. Then G is called highly irregular r- polar fuzzy graph if every vertex of G is adjacent to vertices with distinct degrees.

Example:

Highly Irregular r- polar fuzzy graph.



d (V_1) = (0.2,0.5,1.2), d (V_2) = (0.6 1.2,0.8), d (V_3) = (0.9,0.7,0.3)

d (V_4) = (0.7,0.4,0.6), d (V_5) = (0.6,1.0,1.2), d (V_6) = (0.7,1.6,0.9),

d (V_7) = (0.4,0.6,0.2), d(V_8) = (0.9, 0.7 0.3), d (V_9) = (0.5,0.8,0.9)

$d(V_3) = d(V_8)$ $d(V_1) \neq d(V_2) \neq d(V_4) \neq d(V_5) \neq d(V_6) \neq d(V_7) \neq d(V_9)$

This example for 3^3 polar fuzzy graph.

Theorem: 1

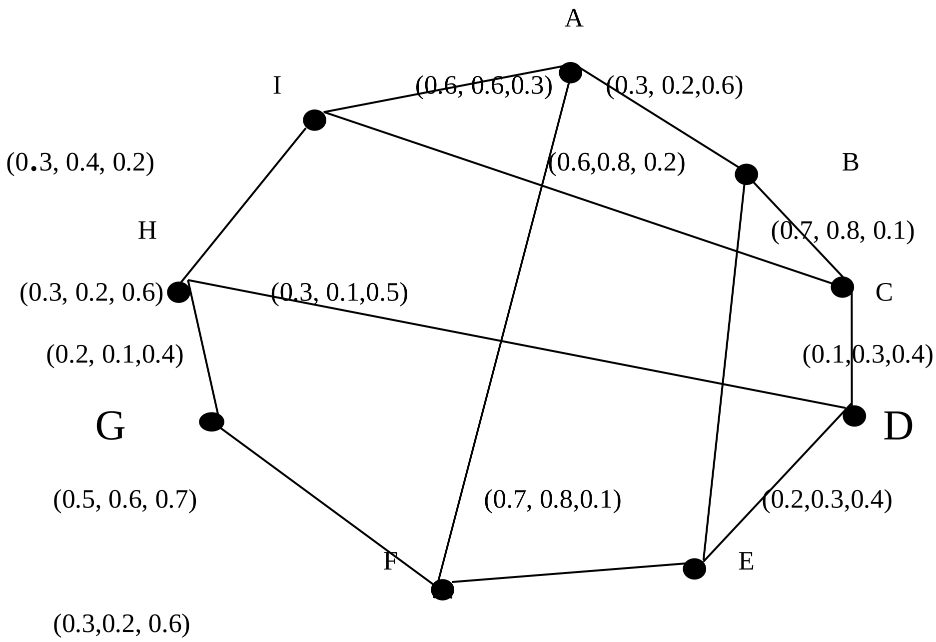
Let G = (A, B, C) be r- polar fuzzy graph. Then G is highly irregular r- polar fuzzy graph and neighbourly irregular r- polar fuzzy graph iff the degrees of all vertices of G are distinct.

Proof:

Let G =(A, B, C) be r- polar fuzzy graph where $A:V\to[0,1]^r$ is a r- polar fuzzy set in V and $B:\overline{V^2}\to[0,1]^r$ is a r- polar fuzzy set in $\overline{V^2}$, $C:\overline{V^3}\to[0,1]^r$ is a r- polar fuzzy set in $\overline{V}^3$ be three r- polar fuzzy sets on a non–empty finite set V and $E\subseteq V\times V\times V$ respectively.

Let $V=\{v_1, v_2, v_3, \ldots\ldots v_n\}$. We assume that G is highly irregular and neighbourly irregular r- polar fuzzy graph.

To prove if degree of all vertices of G is distinct. Let the adjacent vertices V_1 be $V_2,V_3,V_4,\ldots\ldots V_n$ with degrees $d(V_1),d(V_2),d(V_3),\ldots\ldots d(V_n)$ respectively.



We know that G is highly irregular r- polar fuzzy graph and neighbourly irregular r- polar fuzzy graph so,

$d(V_1)\neq d(V_2)\neq d(V_3)\neq \ldots\ldots\ldots\ldots\ldots\ldots\ldots \neq d(V_n)$.

So it is obvious that all vertices are of distinct degrees. Conversely, we assume that the degrees of all vertices of G distinct.

This means that every three adjacent vertices have distinct degrees and to every vertex the adjacent vertices have distinct degrees.

Hence G is highly irregular and neighbourly irregular r- polar fuzzy graph.

Theorem: 2

Let G be a r- polar fuzzy graph if G is neighbourly irregular then G is a neighbourly irregular then G is a neighbourly totally irregular r- polar fuzzy graph.

Proof:

Let G = (A,B,C) be a r- polar fuzzy graph where A: $V \to [0,1]^r$ is a r- polar fuzzy set in V and B:$\overline{V^2} \to [0,1]^r$ is a r- polar fuzzy set in $\overline{V}^2$, C:$\overline{V^3} \to [0,1]^r$ is a r- polar fuzzy set in $\overline{V^3}$ be three r- polar fuzzy sets on a non–empty finite set V and E$\subseteq V\times V\times V$ respectively.

Assume that G is a neighbourly irregular r- polar fuzzy graph.

To Prove

G is a neighbourly totally irregular r- polar fuzzy graph.

If graph G is neighbourly irregular r- polar fuzzy graph.

That is degree of every three adjacent vertices distinct.

The adjacent vertices are u_1 and u_2, u_3 with distinct degrees d (u_1) and $d(u_2)$, $d(u_3)$ respectively.

And also let,

$r(u_1) = a_1, a_2$

$r(u_2) = a_3, a_4$

$r(u_3) = a_5, a_6$. r - Membership function. $a_1, a_2, a_3, a_4, a_5, a_6$ are constants

$\therefore Td(u_1) = d(u_1) + a_1 + a_2$

$Td(u_2) = d(u_2) + a_3 + a_4$

$Td(u_3) = d(u_3) + a_5 + a_6$

Clearly,

$Td(u_1) \neq Td(u_2) \neq Td(u_3)$

For any three adjacent vertices v_1, v_2, v_3 with distinct degrees, It is total degrees are also distinct.

Hence G is a neighbourly totally irregular r- polar fuzzy graph.

Theorem: 3

Let G be r-polar fuzzy If G is neighbourly totally irregular then G is neighbourly irregular r- polar fuzzy graph.

Proof:

Let G = (A, B, C) be a r- polar fuzzy graph where A: $V \rightarrow [0,1]^r$

Is a r- polar fuzzy set in V and B: $\overline{V^2} \rightarrow [0,1]^r$ is a r- polar fuzzy set in $\overline{V}^2$, C: $\overline{V}^3 \rightarrow [0,1]^r$ is a r – polar fuzzy set in $\overline{V^3}$ be three r – polar fuzzy sets on a non- empty finite set V and E ⊆ V×V× V respectively.

Assume that G is a neighbourly totally irregular fuzzy graph.

i.e) Total degrees of every three adjacent vertices are distinct.

To Prove

G is neighbourly irregular r- polar fuzzy graph.

Consider three adjacent vertices v_1 and v_2, v_3 with degrees (x_1, x_1, x_1) (x_2, x_2, x_2) (x_3, x_3, x_3) respectively.

And also assume that $V_1 = a_1, a_2, a_3$ where a_1, a_2, a_3 are constant.

Then,

$Td (V_1) \neq Td (V_2) \neq Td(V_3)$

$d (V_1) \neq d (V_2) \neq d (V_3)$

$x_1 + a_1 \neq x_2 + a_2 \neq x_3 + a_3$

$x_1 \neq x_2 \neq x_3$ and $a_1 \neq a_2 \neq a_3$

Hence the degree of adjacent vertices of G is distinct.

For, the every pair of the adjacent vertices in G.

Remark:-

Also we have to prove G is neighbourly irregular r- polar fuzzy graph, then. G is neighbourly totally irregular fuzzy graph.

Theorem: 4

The size of a (k_1, k_2, k_3) regular r- polar fuzzy graph is $\frac{qk_1}{3}, \frac{qk_2}{3}, \frac{qk_3}{3}$ where $q = |v|$. That is q- vertex.

Proof:

Let G = (A, B, C)be a r- polar fuzzy graph where $A:V \rightarrow [0,1]^r$ is a r- polar fuzzy sets in V and $B:\overline{V}^2 \rightarrow [0,1]^r$ is a r- polar fuzzy sets in $\overline{V}^2$

and $C:\overline{V}^3 \rightarrow [0,1]^r$ is a r- polar fuzzy sets in $\overline{V}^3$.be three r- polar fuzzy sets on a non – empty finite set V and $E \subseteq V\times V\times V$ respectively.

The size of G is

$$S(G) = \sum \frac{1+\sum_{i=1}^{r} q_i \circ B, C(xyz)}{3}$$

we've

$\sum d(V) = 3\,S(G)$

$3S(G) = (\sum k_1, \sum k_2, \sum k_3)$

$3S(G) = q_{k_1}, q_{k_2}, q_{k_3}$

$S(G) = \frac{qk_1}{3}, \frac{qk_2}{3}, \frac{qk_3}{3}$

Hence prove the theorem.

Theorem: 5

Let G = (A, B, C) be a r- polar fuzzy graph. Then G is highly regular r- polar fuzzy graph and neighbourly regular r – polar fuzzy graph iff the degrees of all vertices of G are same.

Proof:

Let G = (A,B,C) be a r- polar fuzzy graph, where $A: V \rightarrow [0,1]^r$ is a r- polar fuzzy sets in V and $B:\overline{V}^2 \rightarrow [0,1]^r$ is a r- polar fuzzy sets in V^2 , $C: \overline{V}^3 \rightarrow [0,1]^r$ is a r- polar fuzzy sets in $\overline{V}^3$, be three r- polar fuzzy sets on a non- empty finite set V and $E \subseteq V\times V\times V$ respectively.

Let V= {v_1, v_2, v_3, …….v_n}.

We assume that G is highly regular and neighbourly regular r- polar fuzzy graph.

To prove

If the degrees all vertices of G are same. Let the adjacent vertices v_1 and v_2, v_3……v_n with degrees d (v_1), d v_2), ………d (v_n) respectively.

We know that G is highly regular r- polar fuzzy graph and neighbourly regular r- polar fuzzy graph so,

$d(v_1) = d(v_2) = d(v_3) = \ldots\ldots\ldots\ldots = d(v_n)$,

So,

it obvious that all vertices are of same degrees.

Conversely,

We assume that the degrees of all vertices of G are same. That means that every three adjacent vertices have same degrees and to every vertex the adjacent vertices have same degrees.

Hence G is highly regular and neighbourly regular r- polar fuzzy graph.

Conclusion:

Graph theory is an extremely useful tool for solving problems in different areas Because, research on modelling of real world problems often involve multi- agent, multi- attribute, multi- object, multi- index, multi – polar information, uncertainly and process limits r- polar fuzzy graph are very helpful.

In this paper we have described order size of r- polar fuzzy graphs. The necessary and sufficient conditions for r- polar fuzzy graph to be the regular r- polar and irregular r- polar fuzzy graphs have been presented. Size of r- polar fuzzy graphs, relation between size and order of r-polar fuzzy graphs has been calculated.

We have define an irregular, neighbourly irregular totally and highly irregular r- polar fuzzy graphs, regular and totally regular r- polar fuzzy graphs. Some relations about the defined graphs have been proved.

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