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THE DEGREE OF AN EDGE IN UNION AND JOIN OF THREE FUZZY GRAPHS

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Abstract.

A fuzzy graph can be obtained from three given fuzzy graphs using union and join. In this paper, we find the degree of an edge in fuzzy graphs formed by these operations in terms of the degree of edges in the given fuzzy graphs in some particular cases.

Keywords:

Degree of a vertex, degree of an edge, union and join.

Introduction

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975[4].Through it is very young, it has been growing fast and has numerous applications in various fields. During the same time Y eh and Bang have also introduced the concepts in connectedness in fuzzy graphs[9]. Mordeson and Peng introduced the concept of operations on fuzzy graphs. Sunitha and Vijayakumar discussed about complementary of the operations of union, join, Cartesian product and composition on two fuzzy graphs[8]. The degree of a vertex in fuzzy graphs which are obtained from two given fuzzy graphs using these operations were discussed by Nagoor Gani and Radha[3]. Radha and Kumaravel introduced the concept of degree of an edge and total degree of an edge in fuzzy graphs[5]. We study about the degree of an edge in fuzzy graphs which are obtained from three given fuzzy graphs using the operations of union and join. In general, the degree of an edge in union and join of three fuzzy graphs G_1, G_2 and G_3 cannot be expressed in terms of these in G_1, G_2 and G_3. In this paper, we find the degree of an edge in union and join of three fuzzy graphs G_1, G_2 and G_3 in terms of the degree of edges of G_1, G_2 and G_3 in some particular cases. First we go through some basic concepts.

Definition 1.1

A fuzzy subset of a set V is a mapping σ from V to [0, 1]. A fuzzy graph G is a pair of functions G:(σ,μ) where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ, (i.e.) $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all x, y∈V. The underlying crisp graph of G: (σ, μ) is denoted by G*: (V,E) where E V×V.

Throughout this paper, $G_1: (\sigma_1,\mu_1)$,$G_2: (\sigma_2,\mu_2)$ and $G_3:(\sigma_3,\mu_3)$ denote three fuzzy graphs with underlying crisp graphs $G_1^*: (V_1,E_1)$, $G_2^*: (V_2,E_2)$ and $G_3^*: (V_3,E_3)$ with $|V_i|=P_i$, i=1,2,3. Also $d_{G_i^*}(u_i)$ denotes the degree of u_i in G_i^*.

Definition 1.2

Let G: (σ, μ) be a fuzzy graphs on G*: (V, E). The degree of a vertex u is $d_G(u) = \sum_{u \neq v} \mu(uv) = \sum_{uv \in E} \mu(uv)$.

The minimum degree of G is $\delta(G) = \wedge \{d_G(v): v \in V\}$.

The maximum degree of G is $\Delta(G) = \vee \{d_G(v): v \in V\}$.

Definition 1.3

The total degree of a vertex $u \in V$ is defined by

$$td_G(u) = \sum_{u \neq v} \mu(uv) + \sigma(u) = d_G(u) + \sigma(u).$$

Definition 1.4

Let G*: (V, E) be a graph and let e = u v be an edge in G*. Then the degree of an edge $e = uv \in E$ is defined by

$$d_{G^*}(uv) = d_{G^*}(u) + d_{G^*}(v) - 2.$$

Definition 1.5

Let $G_1:(\sigma_1,\mu_1)$, $G_2:(\sigma_2,\mu_2)$ and $G_3:(\sigma_3,\mu_3)$ be three fuzzy graphs with underlying graphs $G_1^*:(V_1,E_1)$, $G_2^*:(V_2,E_2)$ and $G_3^*:(V_3,E_3)$ and let $G^* = G_1^* \cup G_2^* \cup G_3^* = (V_1 \cup V_2 \cup V_3, E_1 \cup E_2 \cup E_3)$ be the union of G_1^*, G_2^* and G_3^*. Then the union of three fuzzy graphs G_1, G_2 and G_3 is a fuzzy graphs $G = G_1 \cup G_2 \cup G_3 : (\sigma_1 \cup \sigma_2 \cup \sigma_3, \mu_1 \cup \mu_2 \cup \mu_3)$ defined by

$$(\sigma_1 \cup \sigma_2 \cup \sigma_3)(u) = \begin{cases} \sigma_1(u), & \text{if } u \in V_1 - V_2 \\ \sigma_2(u), & \text{if } u \in V_2 - V_3 \\ \sigma_3(u), & \text{if } u \in V_3 - V_1 \\ \sigma_1(u) \vee \sigma_2(u) \vee \sigma_3(u), & \text{if } u \in V_1 \cap V_2 \cap V_3 \end{cases}$$

$$(\mu_1 \cup \mu_2 \mu_3)(u\,v) = \begin{cases} \mu_1(u\,v), & \text{if } u\,v \in E_1 - E_2 \\ \mu_2(u\,v), & \text{if } u\,v \in E_2 - E_3 \\ \mu_3(u\,v), & \text{if } u\,v \in E_3 - E_1 \\ \mu_1(u\,v) \vee \mu_2(u\,v) \vee \mu_3(u\,v), & \text{if } u\,v \in E_1 \cap E_2 \cap E_3. \end{cases}$$

Definition 1.6

Let $G_1:(V_1,E_1)$, $G_2:(V_2,E_2)$ and $G_3:(V_3,E_3)$ be three fuzzy graphs with underlying graphs $G_1^*:(V_1,E_1)$, $G_2^*:(V_2,E_2)$ and $G_3^*:(V_3,E_3)$ with $V_1 \cap V_2 \cap V_3 = \phi$ and let $G^* = G_1^*+G_2^*+G_3^* = (V_1 \cup V_2 \cup V_3, E_1 \cup E_2 \cup E_3 \cup E')$ be the join of all G_1^*, G_2^* and G_3^*,where E is the set of all edges joining the vertices of V_1, V_2 and V_3. Then the join (sum) of three fuzzy graphs $G = G_1+G_2+G_3$: $(\sigma_1+\sigma_2+\sigma_3, \mu_1+\mu_2+\mu_3)$ defined by

$(\sigma_1+\sigma_2+\sigma_3)(u) = (\sigma_1 \cup \sigma_2 \cup \sigma_3)(u)$, $\forall\, u \in V_1 \cup V_2 \cup V_3$ and

$(\mu_1+\mu_2+\mu_3)(u\,v\,w) = (\mu_1 \cup \mu_2 \cup \mu_3)(u\,v\,w)$, if $u\,v\,w \in E_1 \cup E_2 \cup E_3$

$\sigma_1(u) \wedge \sigma_2(v) \wedge \sigma_3(w)$, if $u\,v\,w \in E'$.

Definition 1.7

The order of a fuzzy graphs G is defined by $O(G) = \sum_{u \in v} \sigma(u)$.

Definition 1.8

The size of a fuzzy graph G is defined by $S(G) = \sum_{uv \in E} \mu(uv)$.

Definition 1.9

Let G: (σ, μ) be a fuzzy graph on G^*: (V, E). The degree of an edge u v is

$$d_G(u\,v) = d_G(u) + d_G(v) - 2\mu(u\,v)$$

$$= \sum_{\substack{uw \in E \\ w \neq v}} \mu(uw) + \sum_{\substack{wv \in E \\ w \neq u}} \mu(wv).$$

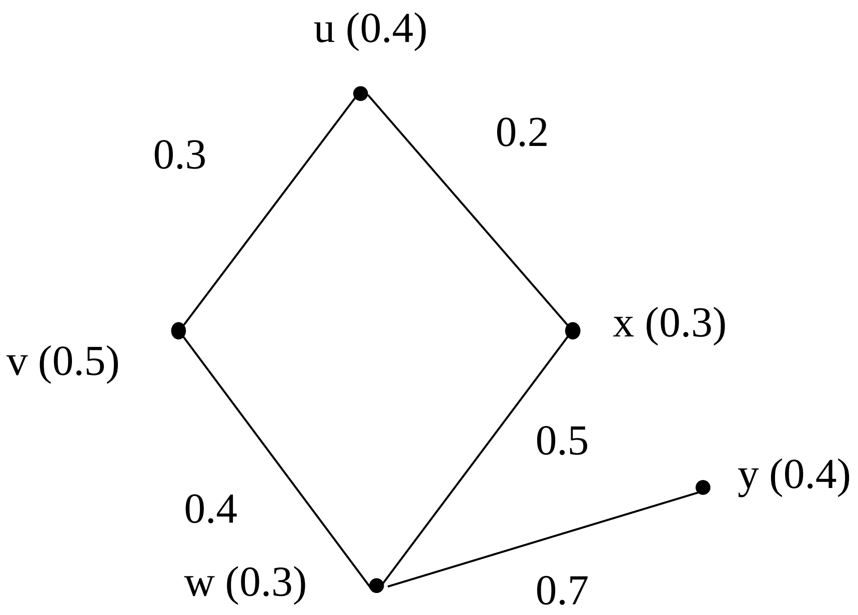
The minimum degree of G is $\delta_E(G) = \wedge\{d_G(u\ v): u\ v \in E\}$. The maximum degree of G is $\Delta_E(G) = \vee\{d_G(u\ v): u\ v \in E\}$.

Definition 1.10

Let G: (σ, μ) be a fuzzy graphs on G*: (V, E). The total degree of an edge $u\ v \in E$ is defined by

$$td_G(u\ v) = d_G(u\ v) + \mu(u\ v) = d_G(u) + d_G(v) - \mu(u\ v).$$

Example 1.11



$d_G(u) = \mu(u\ v) + \mu(u\ x) = 0.3 + 0.2 = 0.5,$

$td_G(u) = d_G(u) + \sigma(u) = 0.5 + 0.4 = 0.9$

$\delta(G) = \wedge\{d_G(v), \forall\ v \in V\} = \wedge\{0.5, 0.7, 1.6, 0.7, 0.7\} = 0.5 = d_G(u).$

$\Delta(G) = \vee\{d_G(v), \forall\ v \in V\} = \vee\{0.5, 0.7, 1.6, 0.7, 0.7\} = 1.6 = d_G(w).$

$d_G(u\ v) = \sum_{\substack{uw \in E \\ w \neq v}} \mu(ux) + \sum_{\substack{vw \in E \\ w \neq u}} \mu(vw) .= 0.2 + 0.4 = 0.6.$

$t\ d_G(u\ v) = d_G(u\ v) + \mu(u\ v) = 0.6 + 0.3 = 0.9.$

$\delta_E(G) = \wedge\{d_G(u\ v), \forall\ u\ v \in E = \wedge\{0.6, 1.5, 1.3, 0.8, 0.9\}$

$= 0.6 = d_G(u\ v).$

$\Delta_E(G) = \vee\{d_G(u\ v), \forall\ u\ v \in E = v\{0.6, 1.5, 1.3, 0.8, 0.9\}$

$= 1.5 = d_G(u\ w).$

2. Degree of an edge in union

For any u v w$\in E_1 \cup E_2 \cup E_3$, fix u$\in V_1 \cup V_2 \cup V_3$ we have three cases to consider.

Case 1: $V_1 \cap V_2 \cap V_3 = \varphi$

Let u v w$\in E_1 \cup E_2 \cup E_3$ be any edge.

Hence $E_1 \cap E_2 \cap E_3 = \varphi$

Therefore u v$\in E_1$, v w$\in E_2$ or w u$\in E_3$, but not both.

$$\text{So } (\mu_1 \cup \mu_2 \cup \mu_3)(u\,v) = \begin{cases} \mu_1(u\,v), \text{ if } u\,v \in E_1 - E \\ \mu_2(u\,v), \text{ if } u\,v \in E_2 - E_3 \\ \mu_3(u\,v), \text{ if } u\,v \in E_3 - E_1 \end{cases}$$

By definition,

$$d_{G1 \cup G2 \cup G3}(u\,v) = \sum_{\substack{uw \in E1 \cup E2 \cup E3 \\ w \neq v}} (\mu_1 \cup \mu_2 \cup \mu_3)(u\,w) +$$

$$\sum_{\substack{wv \in E1 \cup E2 \cup E3 \\ w \neq u}} (\mu_1 \cup \mu_2 \cup \mu_3)(w\,v) +$$

$$\sum_{\substack{uv \in E1 \cup E2 \cup E3 \\ u \neq w}} (\mu_1 \cup \mu_2 \cup \mu_3)(u\,v)$$

If u v$\in E_1$,

$$d_{G1 \cup G2 \cup G3}(u\,v) = \sum_{\substack{uw \in E' \\ w \neq v}} \mu_1(u\,w) + \sum_{\substack{wv \in E' \\ w \neq u}} \mu_1(w\,v) +$$

$$\sum_{\substack{uv \in E' \\ u \neq w}} \mu_1(u\,v)$$

$$\therefore d_{G1 \cup G2 \cup G3}(u\,v) = d_{G1}(u\,v)$$

Similarly,

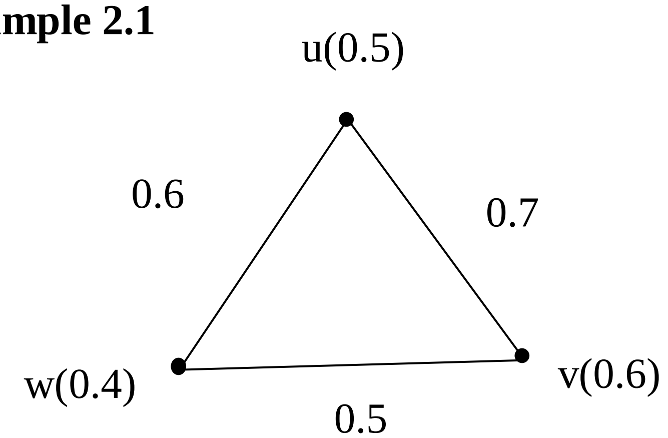
If w v$\in E_2$,

$$\therefore d_{G1 \cup G2 \cup G3}(u\,v) = d_{G2}(u\,v)$$

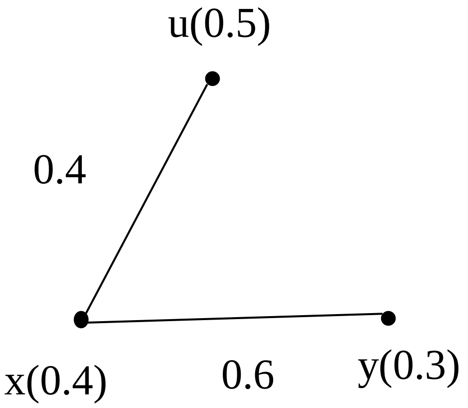
If u v$\in E_3$,

$$d_{G1 \cup G2 \cup G3}(u\,v) = d_{G2}(u\,v)$$

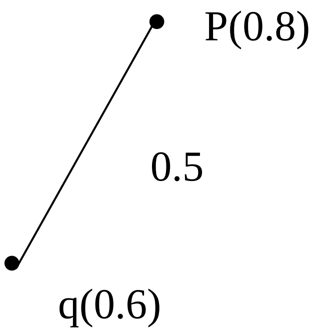
Example 2.1



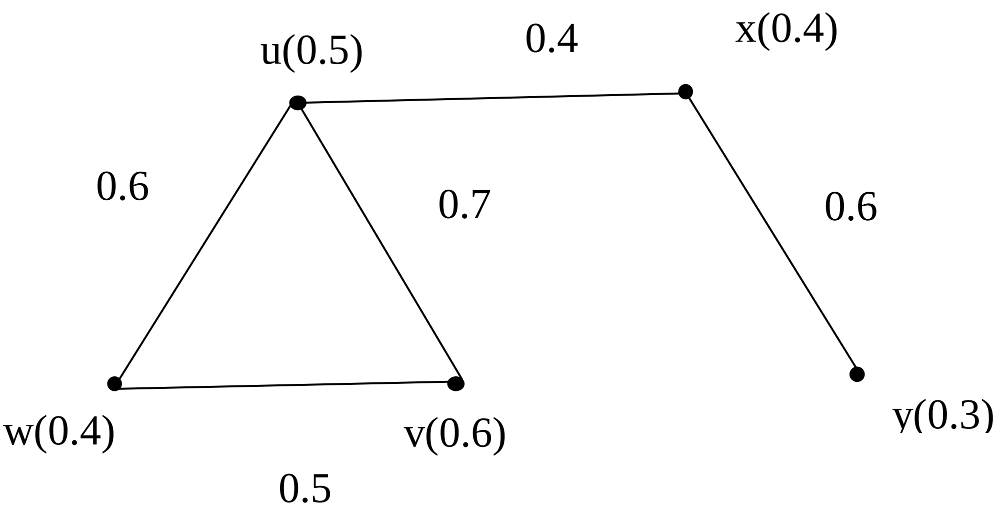
G_1



G_2



G_3



$G_1 \cup G2 \cup G3$

Here, u v$\in E_1$

Then $d_{G1\cup G2\cup G3}$ (u v) = d_{G1}(u v) = 0.6+0.5 = 1.1

Case 2: $V_1 \cap V_2 \cap V_3 \neq \varphi$, $E_1 \cap E_2 \cap E_3 = \varphi$.

Then u v$\in E_1$, w v$\in E_2$ or w u$\in E_3$.

Choose u v$\in E_1$, w v$\in E_2$. Then u v E_3.

Also, if both, u, v, w$\notin V_1 \cap V_2 \cap V_3$. Then it is of case 1.

So, consider either u$\in V_1 \cap V_2 \cap V_3$ or v$\in V_1 \cap V_2 \cap V_3$, w$\in V_1 \cap V_2 \cap V_3$ or both u, v, w$\in V_1 \cap V_2 \cap V_3$.

Subcase 1:

u $\in V_1 \cap V_2 \cap V_3$ or v$\in V_1 \cap V_2 \cap V_3$. When u$\in V_1 \cap V_2 \cap V_3$.

By definition,

$$d_{G1 \cup G2 \cup G3}(u\ v\ w) = d_{G1 \cup G2 \cup G3}(u) + d_{G1 \cup G2 \cup G3}(v) + d_{G1 \cup G2 \cup G3}(w) - 3(\mu_1 \cup \mu_2 \cup \mu_3)$$

$$= [d_{G1}(u) + d_{G2}(u) + d_{G3}(u)] + d_{G1}(v) + d_{G1}(w) - 3\ \mu_1(u\ v\ w)$$

$$= [d_{G1}(u) + d_{G1}(v) + d_{G1}(w) - 3\ \mu_1(u\ v\ w)] + d_{G2}(u) + d_{G3}(w)$$

$$= d_{G1}(u\ v) + d_{G2}(u) + d_{G3}(w), \text{ if } u \in V_1 \cap V_2 \cap V_3.$$

Similarly, $d_{G1 \cup G2 \cup G3}(u\ v\ w) = d_{G1}(u\ v) + d_{G2}(v) + d_{G3}(w)$,

if v$\in V_1 \cap V_2 \cap V_3$.

Thus,

$$d_{G1 \cup G2 \cup G3}(u\ v\ w) = \begin{cases} d_{G1}(u\ v) + d_{G2}(u), \text{ if } u \in V_1 \cap V_2 \cap V_3 \\ d_{G1}(u\ v) + d_{G2}(v), \text{ if } v \in V_1 \cap V_2 \cap V_3 \end{cases}$$

In a similar way,

If v w$\in E_2$,

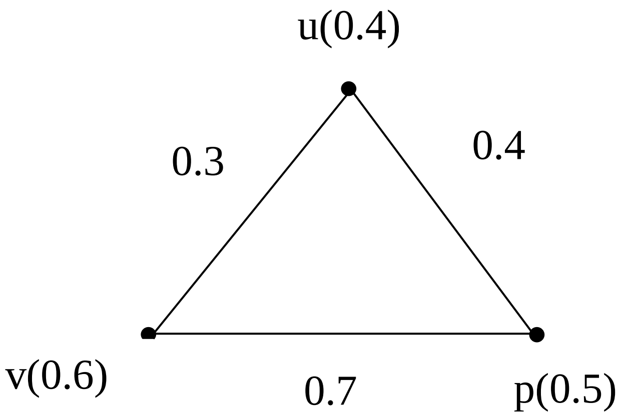
$$\text{Then } d_{G1 \cup G2 \cup G3}(u\ v\ w) = \begin{cases} d_{G2}(v\ w) + d_{G3}(v), \text{ if } v \in V_1 \cap V_2 \cap V_3 \\ d_{G2}(v\ w) + d_{G3}(w), \text{ if } w \in V_1 \cap V_2 \cap V_3 \end{cases}$$

In a similar way,

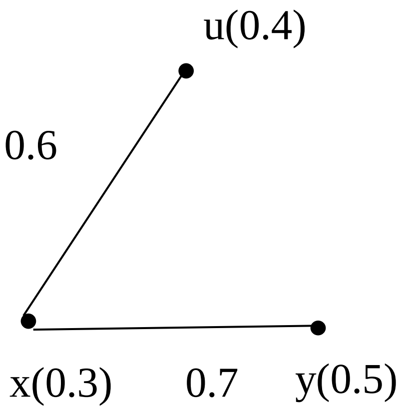
If w u$\in E_3$,

$$\text{Then } d_{G1 \cup G2 \cup G3}(u\ v\ w) = \begin{cases} d_{G3}(w\ u) + d_{G1}(w), \text{ if } w \in V_1 \cap V_2 \cap V_3 \\ d_{G3}(w\ u) + d_{G1}(u), \text{ if } u \in V_1 \cap V_2 \cap V \end{cases}$$

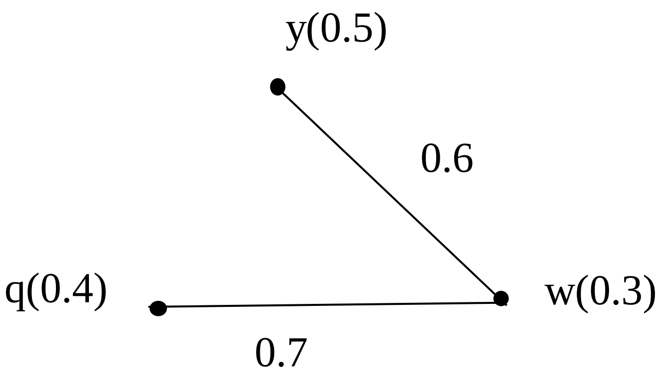
Example 2.2



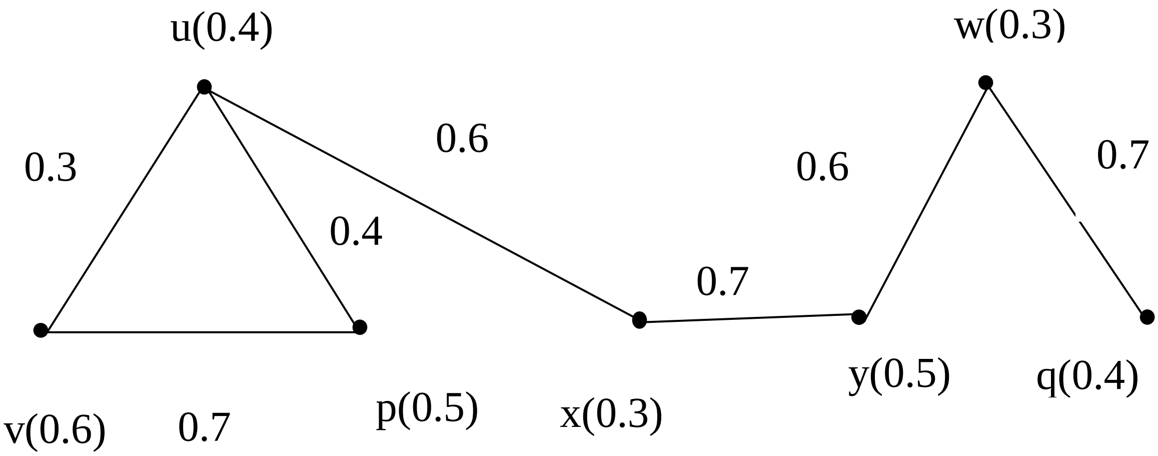
G_1



G_2



G_3



$G_1 \cup G_2 \cup G_3$

Here u v$\in E_1$ and u$\in V_1 \cap V_2 \cap V_3$.

Then $d_{G_1 \cup G_2 \cup G_3}$ (u v w) = d_{G_1}(u v)+ d_{G_2}(u)+ d_{G_2}(w)

=(0.4+0.7+0.6+0.7+0.6)=3.0

Subcases 2: u, v, w$\in V_1 \cap V_2 \cap V_3$, u v$\in E_1$.

Since $E_1 \cap E_2 \cap E_3 = \varphi$, no edge incident at u or v or w is in $E_1 \cap E_2 \cap E_3$.

Since the edge incident at u, v & w in G_1, G_2 and also in G_3 appear with the same membership values in $G_1 \cup G_2 \cup G_3$.

By definition,

$$d_{G1 \cup G2 \cup G3}(u\ v\ w) = d_{G1 \cup G2 \cup G3}(u) + d_{G1 \cup G2 \cup G3}(v) +$$

$$d_{G1 \cup G2 \cup G3}(w) - 3(\mu_1 \cup \mu_2 \cup \mu_3)(u\ v\ w)$$

$$= d_{G1}(u) + d_{G2}(u) + d_{G3}(u) + d_{G1}(v)\ ) + d_{G2}(v) + d_{G3}(v) +$$

$$d_{G1}(w)\ ) + d_{G2}(w) + d_{G3}(w) - 3\mu_1(u\ v\ w)$$

$$= [d_{G1}(u) + d_{G1}(v) + d_{G1}(w) - 3\ \mu_1(u\ v\ w)] + d_{G2}(u) +$$

$$d_{G2}(v) + d_{G3}(w).$$

$$\therefore d_{G1 \cup G2 \cup G3}(u\ v\ w) = d_{G1}(u\ v) + d_{G2}(u) + d_{G2}(v) + d_{G2}(w).$$

In a similar way,

If, v w$\in E_2$,

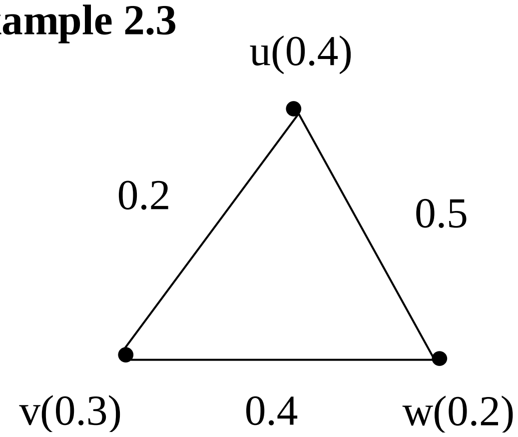
Then, $d_{G1 \cup G2 \cup G3}(v\ w) = d_{G2}(v\ w) + d_{G3}(v) + d_{G3}(w)$.

In a similar way,

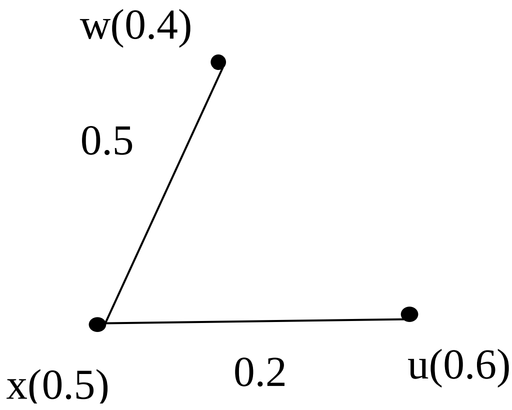
If, w u$\in E_3$,

Then, $d_{G1 \cup G2 \cup G3}(w\ u) = d_{G2}(w\ u) + d_{G1}(w) + d_{G1}(u)$.

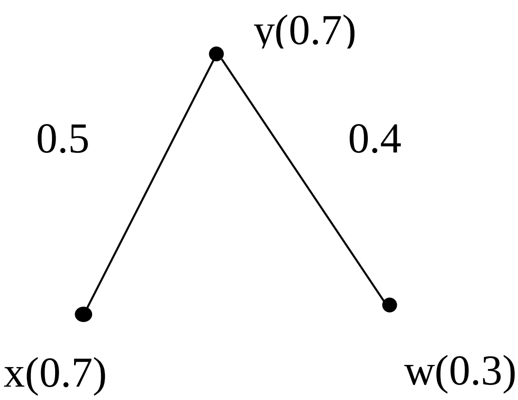
Example 2.3



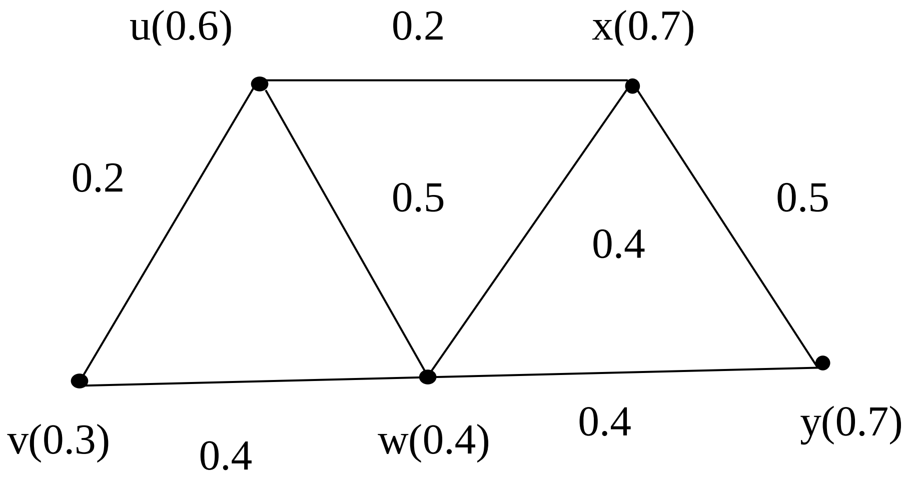
G_1



G_2



G_3



$G_1 \cup G_2 \cup G_3$

Here u v $\in E_1$ and u, v, w $\in V_1 \cap V_2 \cap V_3$. Then $d_{G_1 \cup G_2 \cup G_3}(u\ v\ w) = d_{G_1}(u\ v) + d_{G_2}(u) + d_{G_2}(w) + d_{G_3}(w)$

$= 0.5+0.4+0.2+0.3+0.4=1.8.$

Case 3: $V_1 \cap V_2 \cap V_3 \neq \varphi$, $E_1 \cap E_2 \cap E_3 \neq \varphi$.

Then u v w$\in E_1 \cap E_2 \cap E_3$. Therefore u v$\in E_1$, v w$\in E_2$ and w u$\in E_3$.

So, consider either no edge incident at u, v& w other than u v w is in $E_1 \cap E_2 \cap E_3$ or some of the edges incident at u, v& w other than u v w are in $E_1 \cap E_2 \cap E_3$.

Subcase 1:

No edge incident at u, v& w other than u v w is in $E_1 \cap E_2 \cap E_3$.

Then the edge incident at u or v or w is in either E_1 or E_2 or E_3, those edge appear with the same membership value in $G_1 \cap G_2 \cap G_3$.

By definition,

$$d_{G_1 \cup G_2 \cup G_3}(u\ v\ w) = \sum_{uw \in E1 \cup E2 \cup E3} (\mu_1 \cup \mu_2 \cup \mu_3)(u\ w) + \sum_{wv \in E1 \cup E2 \cup E3)} (\mu_1 \cup \mu_2 \cup \mu_3)(w\ v)$$

$$= \sum_{uw \in E1-E2} \mu_1(u\ w) + \sum_{uw \in E2-E3} \mu_2(u\ w) + \sum_{uw \in E3-E1} \mu_3(u\ w) + \sum_{wv \in E1-E2} \mu_1(w\ v) + \sum_{wv \in E2-E3} \mu_2(w\ v) + \sum_{wv \in E3-E1} \mu_3(w\ v).$$

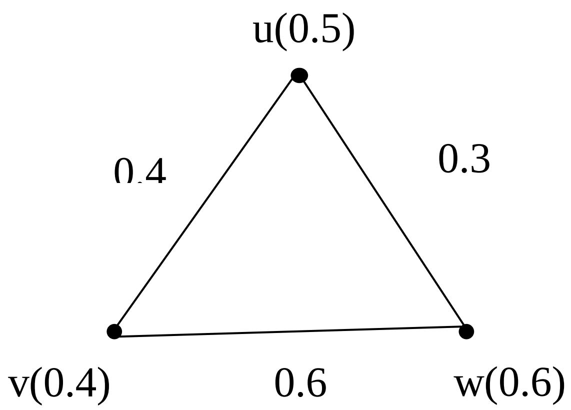
$$= \sum_{uw \in E1-E2} \mu_1(u\ w) + \sum_{wv \in E1-E2} \mu_1(w\ v) +$$

$$\sum_{uw \in E2-E3} \mu_2(u\,w) + \sum_{wv \in E2-E3} \mu_2(w\,v) +$$

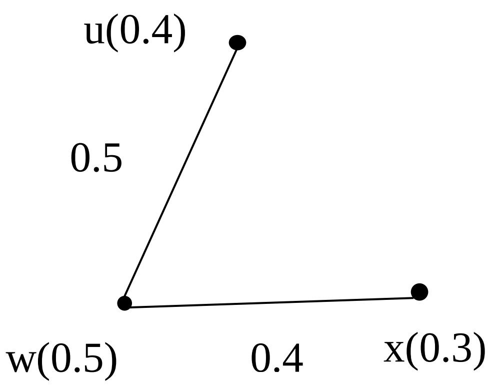
$$\sum_{uw \in E3-E1} \mu_3(u\,w) + \sum_{wv \in E3-E1} \mu_3(w\,v).$$

$d_{G1 \cup G2 \cup G3}(u\,v\,w) = d_{G1}(u\,v) + d_{G2}(v\,w) + d_{G3}(w\,u)$

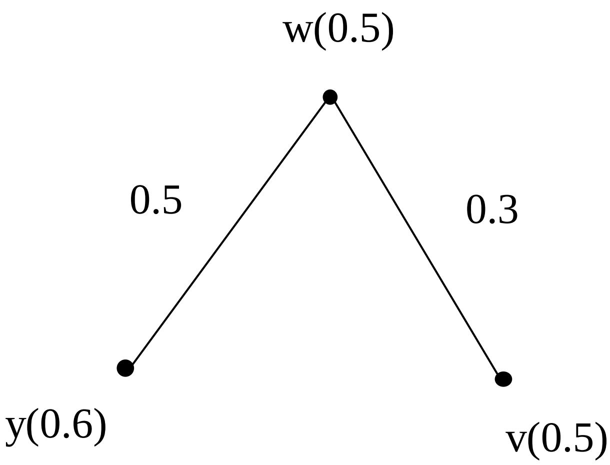
Example 2.4



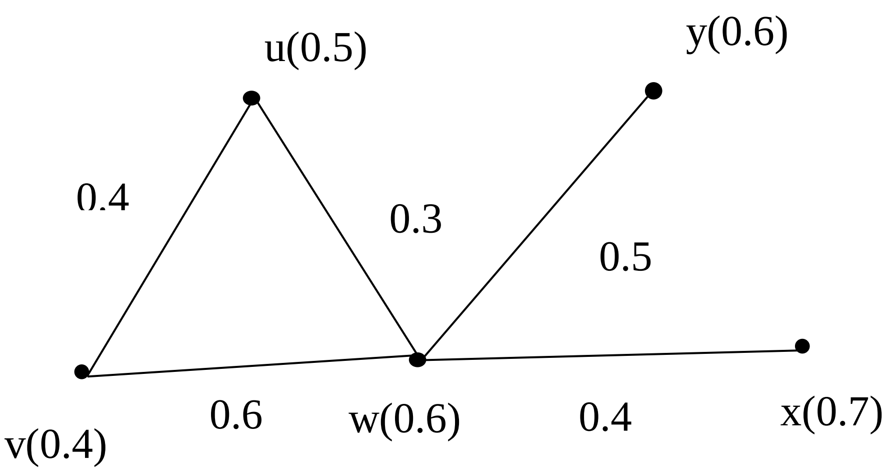
G_1



G_2



G_3



$G_1 \cup G_2 \cup G_3$

Here u v w$\in E_1 \cap E_2 \cap E_3$

Then $d_{G1} \cup_{G2} \cup_{G3}$ (u v w) = d_{G1}(u v)+ d_{G2}(v w)+ d_{G3}(w u).

= 0.6+0.3+0.4+0.5 = 1.8

Subcase 2:

Some of the edges incident u, v& w other than u v w are in $E_1 \cap E_2 \cap E_3$.

By definition,

$$d_{G1} \cup_{G2} \cup_{G3} (u\ v\ w) = \sum_{\substack{uw \in E1 \cup E2 \cup E) \\ w \neq v}} (\mu_1 \cup \mu_2 \cup \mu_3)(u\ w) +$$

$$\sum_{\substack{wv \in E1 \cup E2 \cup E3) \\ w \neq u}} (\mu_1 \cup \mu_2 \cup \mu_3)(w\ v)$$

$$= \sum_{\substack{uw \in E1-E2 \\ w \neq v}} \mu_1(u\ w) + \sum_{\substack{uw \in E2-E3 \\ w \neq v}} \mu_2(u\ w) + \sum_{\substack{uw \in E3-E1 \\ w \neq v}} \mu_3(u\ w) +$$

$$\sum_{\substack{wv \in E1-E2 \\ w \neq u}} \mu_1(w\ v) + \sum_{\substack{wv \in E2-E3 \\ w \neq u}} \mu_2(w\ v) + \sum_{\substack{wv \in E3-E1 \\ w \neq u}} \mu_3(w\ v) +$$

$$\sum_{\substack{uw \in E1 \cap E2 \cap E3) \\ w \neq v}} (\mu_1(u\ w) \vee \mu_2(u\ w) \vee \mu_3(u\ w) + \sum_{\substack{wv \in E1 \cap E2 \cap E3) \\ w \neq u}} (\mu_1(w\ v) \vee$$

$$\mu_2(w\ v) \vee \mu_3(w\ v).$$

$$= \sum_{\substack{uw \in E1-E2 \\ w \neq v}} \mu_1(u\ w) + \sum_{\substack{vw \in E1-E2 \\ w \neq u}} \mu_1(w\ v) +$$

$$\sum_{\substack{uw \in E2-E3 \\ w \neq v}} \mu_2(u\ w) + \sum_{\substack{wv \in E2-E3 \\ w \neq u}} \mu_2(w\ v) +$$

$$\sum_{\substack{wv \in E2-E3 \\ w \neq v}} \mu_3(u\ w) + \sum_{\substack{wv \in E3-E1 \\ w \neq u}} \mu_3(w\ v) +$$

$$\sum_{\substack{uw \in E1 \cap E2 \cap E3) \\ w \neq v}} (\mu_1(u\ w) \vee \mu_2(u\ w) \vee \mu_3(u\ w) +$$

$$\sum_{\substack{wv \in E1 \cap E2 \cap E3) \\ w \neq u}} (\mu_1(w\ v) \vee \mu_2(w\ v) \vee \mu_3(w\ v) +$$

$$\sum_{\substack{uw \in E1 \cap E2 \cap E3) \\ w \neq v}} (\mu_1(u\ w) \wedge \mu_2(u\ w) \wedge \mu_3(u\ w) +$$

$$\sum_{\substack{wv \in E1 \cap E2 \cap E3) \\ w \neq u}} (\mu_1(w\ v) \wedge \mu_2(w\ v) \wedge \mu_3(w\ v)-$$

$$\sum_{\substack{uw \in E1 \cap E2 \cap E3) \\ w \neq v}} (\mu_1(u\ w) \wedge \mu_2(u\ w) \wedge \mu_3(u\ w)-$$

$\sum_{\substack{wv\in E1\cap E2\cap E3)\\ w\neq u}}(\mu_1(w\ v)\wedge\mu_2(w\ v)\wedge\mu_3(w\ v).$

$= \sum_{\substack{uw\in E1\\ w\neq v}} \mu_1(u\ w) + \sum_{\substack{wv\in E1\\ w\neq u}} \mu_1(w\ v) + \sum_{\substack{uw\in E2\\ w\neq v}} \mu_2(u\ w) +$

$\sum_{\substack{wv\in E2\\ w\neq u}} \mu_2(w\ v) + \sum_{\substack{uw\in E3\\ w\neq v}} \mu_3(u\ w) + \sum_{\substack{wv\in E3\\ w\neq u}} \mu_3(w\ v) -$

$\sum_{\substack{uw\in E1\cap E2\cap E3)\\ w\neq v}}(\mu_1(u\ w)\wedge\mu_2(u\ w)\wedge\mu_3(u\ w)-$

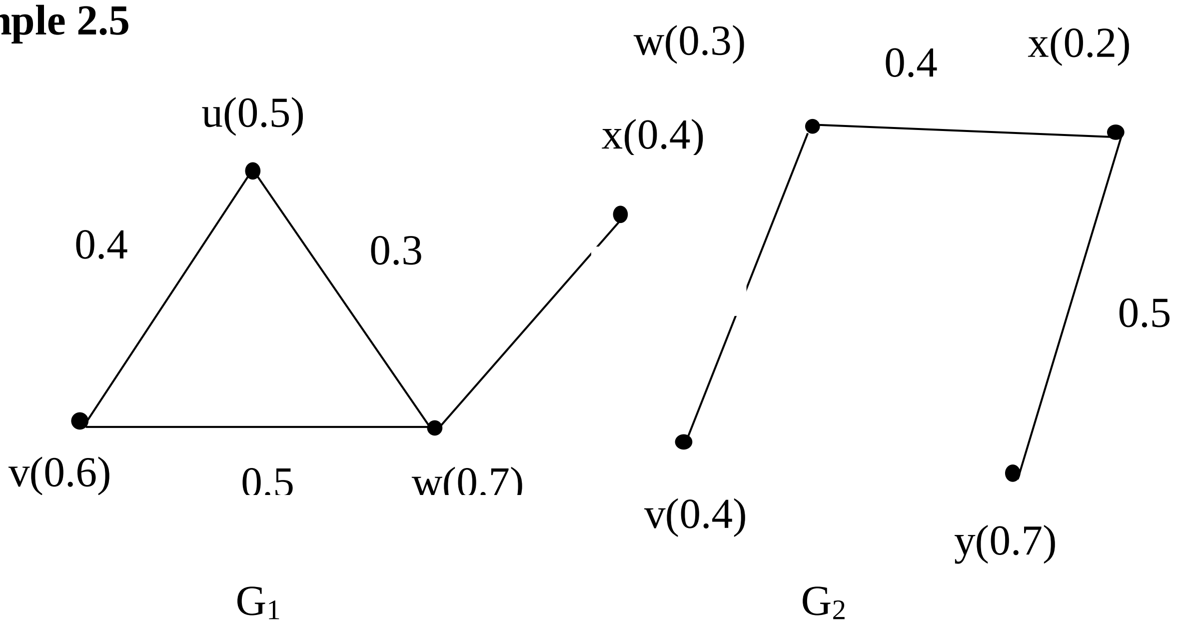
$\sum_{\substack{wv\in E1\cap E2\cap E3)\\ w\neq u}}(\mu_1(w\ v)\wedge\mu_2(w\ v)\wedge\mu_3(w\ v).$

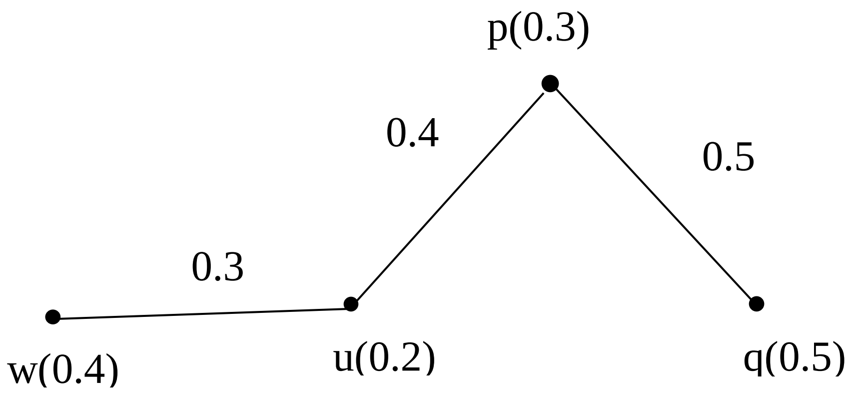
$d_{G1}\cup_{G2}\cup_{G3}\ (u\ v\ w) = d_{G1}(u\ v)+ d_{G2}(v\ w)+ d_{G3}(w\ u)-$

$\sum_{\substack{uw\in E1\cap E2\cap E3)\\ w\neq v}}(\mu_1(u\ w)\wedge\mu_2(u\ w)\wedge\mu_3(u\ w)-$

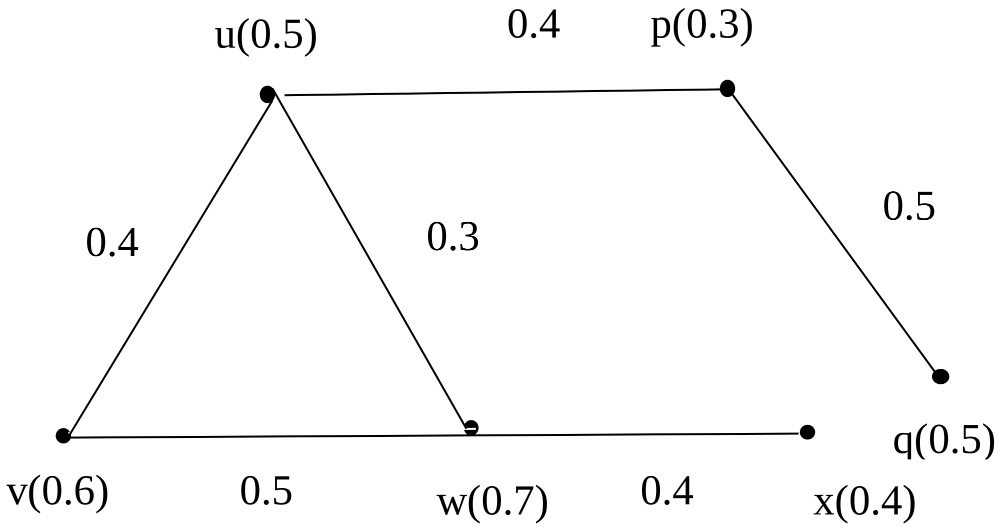
$\sum_{\substack{wv\in E1\cap E2\cap E3)\\ w\neq u}}(\mu_1(w\ v)+\mu_2(w\ v)+\mu_3(w\ v).$

Example 2.5





G_3



$G_1 \cup G_2 \cup G_3$

Here u v w$\in E_1 \cap E_2 \cap E_3$.

Then $d_{G1 \cup G2 \cup G3}$ (u v w) = d_{G1}(u v)+ d_{G2}(v w)+ d_{G3}(w u))-

$$\sum_{uw \in E1 \cap E2 \cap E3)} (\mu_1(u\ w) \wedge \mu_2(u\ w) \wedge \mu_3(u\ w)-$$

$$\sum_{wv \in E1 \cap E2 \cap E3)} (\mu_1(w\ v)+\mu_2(w\ v)+\mu_3(w\ v)\ d_{G1 \cup G2 \cup G3}\ (u\ v\ w)$$

$$= 0.3+0.5+0.4+0.5+0.4+0.5-0-0=2.7$$

Degree of an edge in join

Here $V_1 \cap V_2 \cap V_3 = \varphi$.

Hence $E_1 \cap E_2 \cap E_3 = \varphi$.

$$(\mu_1+\mu_2+\mu_3)(u\ v\ w) = \begin{cases} \mu_1(u\ v), \text{ if } u\ v \in E_1 \\ \mu_2(v\ w), \text{ if } v\ w \in E_2 \\ \mu_3(w\ u), \text{ if } w\ u \in E_3 \\ \sigma_1(u) \wedge \sigma_2(v) \wedge \sigma_3(w), \text{ if } u\ v\ w \in E' \end{cases}$$

By definition,

$$d_{G1+G2+G3}(u\ v\ w) = \sum_{\substack{uw \in E1 \cup E2 \cup E3 \cup E') \\ w \neq v}} (\mu(u\ w) + \sum_{\substack{wv \in E1 \cup E2 \cup E3 \cup E') \\ w \neq u}} \mu(w\ v) + \sum_{wu \in E1 \cup E2 \cup E3 \cup E')} \mu(w\ u)$$

$$\text{(i.e.) } d_{G1+G2+G3}(u\ v) = \sum_{\substack{uw \in E1 \cup E2 \cup E3) \\ w \neq v}} \mu(u\ w) + \sum_{\substack{wv \in E1 \cup E2 \cup E3) \\ w \neq u}} \mu(w\ v) + \sum_{\substack{wu \in E1 \cup E2 \cup E3) \\ w \neq v}} \mu(w\ u) + \sum_{\substack{uw \in E' \\ w \neq u}} \mu(w\ v)$$

For any $u\ v \in E_1$,

$$d_{G1+G2+G3}(u\ v) = \sum_{\substack{uw \in E1) \\ w \neq v}} \mu 2(u\ w) + \sum_{\substack{wv \in E1) \\ w \neq u}} (\mu_1(w\ v) + \sum_{uw \in E'} \sigma_1(u) \wedge \sigma_2(w) + \sum_{vw \in E'} \sigma_1(v) \wedge \sigma_2(w)$$

$$d_{G1+G2+G3}(u\ v) = d_{G1}(u\ v) + \sum_{vw \in E'} \sigma_1(u) \wedge \sigma_2(w) + \sum_{vw \in E'} \sigma_1(v) \wedge \sigma_2(w)$$

Similarly, for any $u\ v \in E_2$,

$$d_{G1+G2+G3}(u\ v) = d_{G2}(u\ v) + \sum_{vw \in E'} \sigma_1(v) \wedge \sigma_2(w) + \sum_{uw \in E'} \sigma_1(u) \wedge \sigma_2(w)$$

Similarly, for any $u\ v \in E_3$,

$$d_{G1+G2+G3}(u\ v\ w) = d_{G2}(u\ v) + \sum_{uw \in E1} \mu(u\ w) + \sum_{wv \in E2} \mu(w\ v) + \sum_{uv \in E3} \mu(u\ v) + \sum_{uw \in E'} \sigma_1(u) \wedge \sigma_2(w) + \sum_{wv \in E'} \sigma_1(w) \wedge \sigma_2(v) + \sum_{vv \in E'} \sigma_1(u) \wedge \sigma_2(v)$$

$= d_{G1}(u)+d_{G2}(v)+d_{G3}(w) + \sum_{uw\in E'} \sigma_1(u)\wedge\sigma_2(w) +$

$\sum_{wv\in E'} \sigma_1(w)\wedge\sigma_2(v) + \sum_{vv\in E'} \sigma_1(u)\wedge\sigma_2(v)$

Definition 3.1

The relation $\sigma_1\geq\sigma_2\geq\sigma_3$ means that $\sigma_1(u) \geq\sigma_2(v) \geq \sigma_3(w)$, for every $u\in V_1$ and for every $v\in V_2$, $w\in V_3$.

Where σ_1 is a fussy subset of V_i, i=1,2,3.

Theorem 3.2

Let $G_1: (\sigma_1,\mu_1)$, $G_2:(\sigma_2,\mu_2)$ and $G_3:(\sigma_3,\mu_3)$ be three fuzzy graphs.

1. If $\sigma_1\geq\sigma_2\geq\sigma_3$, then

$$d_{G1+G2+G3}(u\ v\ w) = \begin{cases} d_{G1}(u\ v)+ 3\ O(G_3), \text{ if } u\ v\ w \in E_1 \\ d_{G2}(v\ w)+ P_1(\sigma_2(v)+ \sigma_2(w)), \text{ if } u\ v\ w \in E_2 \\ d_{G3}(w\ u)+ P_2(\sigma_3(w)+ \sigma_3(u)), \text{ if } u\ v\ w \in E_3 \\ d_{G1}(u)+ d_{G2}(v)+ d_{G3}(w)+ O(G_3)+ (P_2-3)\sigma_3(v) \text{ if } u\ v\ w\in E'. \end{cases}$$

2.If $\sigma_3\geq\sigma_2\geq\sigma_1$, then

$$d_{G1+G2+G3}(u\ v\ w) = \begin{cases} d_{G1}(u\ v)+ P_3(\sigma_2(u)+ \sigma_2(v)), \text{ if } u\ v\ w \in E_1 \\ d_{G2}(u\ v)+ P_2(\sigma_1(u)+ \sigma_1(v)), \text{ if } u\ v\ w \in E_2 \\ d_{G3}(u\ v)+ 3\ O(G_2), \text{ if } u\ v\ w \in E_3 \end{cases}$$

$d_{G1}(u)+ d_{G2}(v)+ d_{G3}(w)+ O(G_2)+ (P_3-3)\sigma_1(u)$ if $u\ v\ w\in E'$.

Proof:

We have $\sigma_1\geq\sigma_2\geq\sigma_3$.

For any $u\ v\ w\in E_1$,

$d_{G1+G2+G3}(u\ v\ w) = d_{G1}(u\ v)+ \sum_{uw\in E'} \sigma_2(u)\wedge\sigma_3(w)+ \sum_{vw\in E'} \sigma_2(v)\wedge\sigma_3(w)$

$= d_{G1}(u\ v)+\sum_{w\in v3} \sigma_3(w)+ \sum_{w\in v3} \sigma_3(w)$

$= d_{G1}(u\ v)+3\ O(G_3)$,

For any $u\ v\ w\in E_2$,

$d_{G1+G2+G3}(u\ v\ w) = d_{G2}(v\ w) + \sum_{uw\in E'} \sigma_2(w)\wedge\sigma_3(v) +$

$\sum_{wv\in E'} \sigma_3(w)\wedge\sigma_2(v)$

$= d_{G2}(v\ w) + \sum_{w\in v1} \sigma_2(v) + \sum_{w\in v1} \sigma_2(w)$

$= d_{G2}(v\ w) + P_1\sigma_2(v) + P_1\sigma_2(w)$

$= d_{G2}(v\ w) + P_1(\sigma_2(v) + \sigma_2(w)).$

For any u v w$\in E_3$,

$d_{G1+G2+G3}(u\ v\ w) = d_{G3}(wu) + \sum_{wu\in v2} \sigma_3(w)\wedge\sigma_2(u) +$

$\sum_{wv\in v2} \sigma_3(u)\wedge\sigma_2(w)$

$= d_{G3}(w\ u) + \sum_{w\in v3} \sigma_3(w) + \sigma_3(u)$

$= d_{G3}(w\ u) + P_2\sigma_3(w) + P_2\sigma_3(u)$

$= d_{G3}(w\ u) + P_2(\sigma_3(w) + \sigma_3(u)).$

For any u v w$\in E'$,

$d_{G1+G2+G3}(u\ v\ w) = d_{G1}(u) + d_{G2}(v) + d_{G3}(w) + \sum_{uw\in E'} \sigma_2(u)\wedge\sigma_3(w) +$

$\sum_{uw\in E'} \sigma\ (w)\wedge\sigma_3(v) + \sum_{uv\in E'} \sigma_3(u)\wedge\sigma_1(v)$

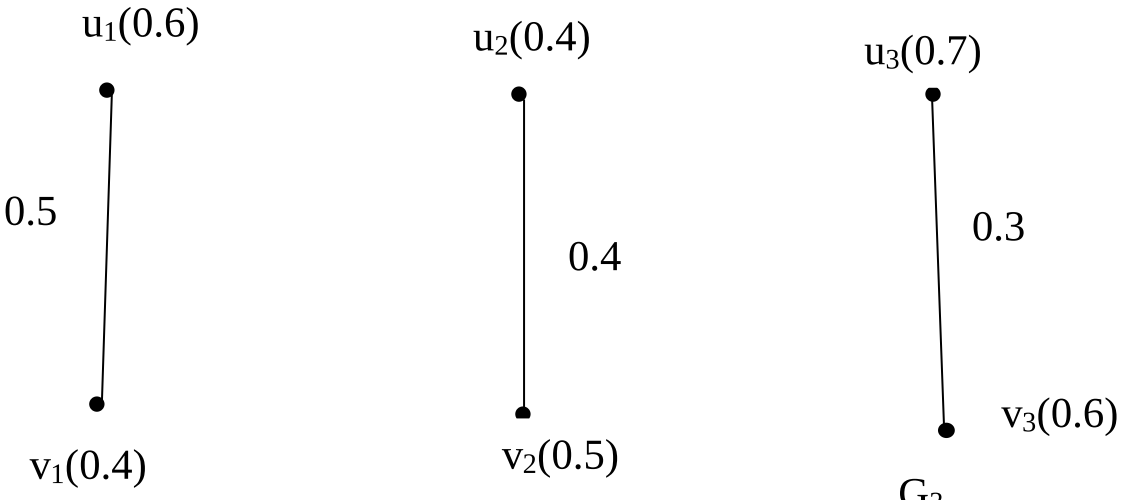
$= d_{G1}(u) + d_{G2}(v) + d_{G3}(w) + \sum_{w\in v2} \sigma_3(w) + \sum_{w\in v1}(\sigma_3(v)\text{-}3\sigma_3(v))$

$= d_{G1}(u) + d_{G2}(v) + d_{G3}(w) + O(G_3) + (P_2\text{-}3)\sigma_3(v).$

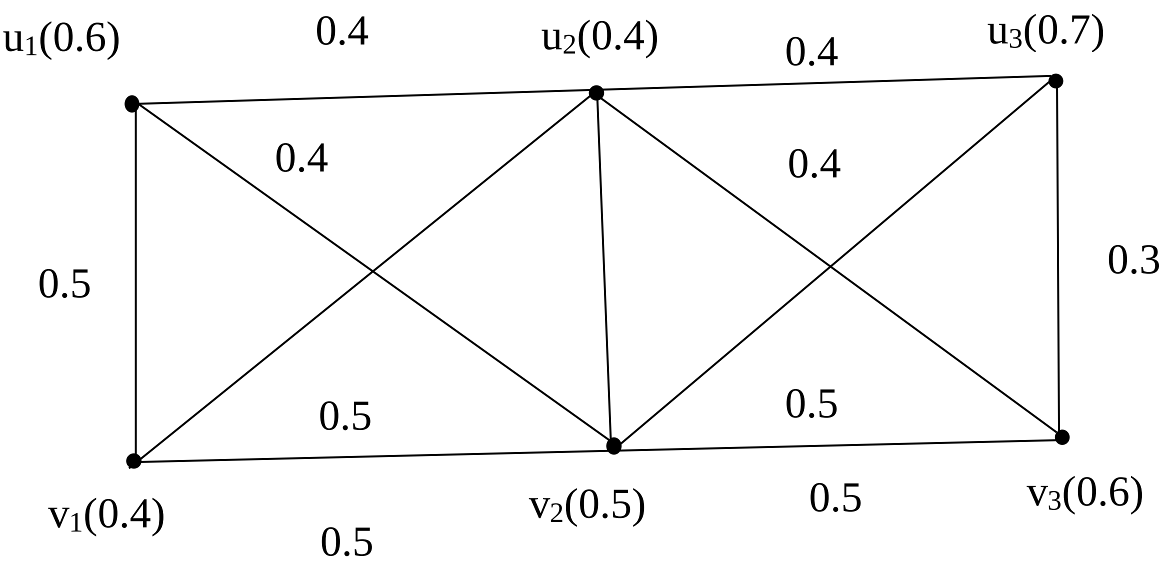
Proof of (2) is similar to the proof of (1).

Example 3.3

Consider G_1, G_2 and G_3 in figure 3.1



G_1 G_2 G_3



$G_1+G_2+G_3$

We have $\sigma_3 \geq \sigma_2 \geq \sigma_1$. So by (2) of theorem 3.2

$$d_{G1+G2+G3}(u_1\ v_1\ w_1) = d_{G1}(u_1 v_1) + P_3(\sigma_2(u) + \sigma_2(v))$$

$$= 0+3(0.6+0.4) = 3.0$$

$$d_{G1+G2+G3}(u_2\ v_2\ w_2) = d_{G2}(u_2 v_2) + P_2(\sigma_2(u_2) + \sigma_1(v_1))$$

$$= 0+2(0.6+0.4) = 2.0$$

$$d_{G1+G2+G3}(u_3\ v_3\ w_3) = d_{G3}(u_3 v_3)+3\ O(G_2)$$

$$= 0+3(0.9) = 2.7$$

$$d_{G1+G2+G3}(u_1\ v_2\ w_3) = d_{G1}(u_1)+ d_{G2}(v_2)+O(G_3)+(P_3-3)\ \sigma_1(u_1)$$

$$= 0.5+0.4+0.7+0.6+(3-3)(0.6) = 2.2$$

The above degrees can be verified in the figure of $G_1+G_2+G_3$ given in figure 3.1.

Theorem 3.4

Let $G_1:(\sigma_1,\mu_1)$, $G_2:(\sigma_2,\mu_2)$ and) $G_3:(\sigma_3,\mu_3)$ be three fuzzy graphs such that $\sigma_1 \wedge \sigma_2 \wedge \sigma_3$ is a constant function.

Then

$$d_{G1+G2+G3}(u\ v\ w) = \begin{cases} d_{G1}(u\ v)+3cp_3, \text{ if } u\ v \in E_1 \\ d_{G2}(u\ v)+3cp_2, \text{ if } u\ v \in E_2 \\ d_{G3}(u\ v)+3cp_1, \text{ if } u\ v \in E_3 \\ d_{G1}(u)+ d_{G2}(v)+ c(p_1+p_2+p_3-3), \text{ if } u\ v \in E' \end{cases}$$

Where $\sigma_1(u) \wedge \sigma_2(v) \wedge \sigma_3(w) = c$ is a constant, for all $u \in V_1, v \in V_2$ and $w \in V_3$.

Proof:

Let $\sigma_1(u) \wedge \sigma_2(v) \wedge \sigma_3(w) = c$, for all $u \in V_1, v \in V_2$ and $w \in V_3$, where c is a constant.

For any $u\ v \in E_1$

$$d_{G1+G2+G3}(u\ v\ w) = d_{G1}(u\ v) + \sum_{uw \in E'} \sigma_1(u) \wedge \sigma_2(w) +$$

$$\sum_{vw \in E'} \sigma_1(v) \wedge \sigma_2(w) + \sum_{wu \in E'} \sigma_1(w) \wedge \sigma_2(u)$$

$$= d_{G1}(u\ v)+\sum_{w \in V3} c+\sum_{w \in V3} c+\sum_{w \in V3} c$$

$$= d_{G1}(u\ v)+cp_3+cp_3+cp_3$$

$= d_{G1}(u\,v) + 3cp_3.$

For any $u\,v \in E_2$,

$d_{G1+G2+G3}(u\,v\,w) = d_{G2}(u\,v) + \sum_{wu\in E'} \sigma_1(w) \wedge \sigma_2(u) +$

$\sum_{wv\in E'} \sigma_1(w) \wedge \sigma_2(v) + \sum_{uw\in E'} \sigma_1(u) \wedge \sigma_2(w)$

$= d_{G2}(u\,v) + \sum_{w\in V2} c + \sum_{w\in V2} c + \sum_{w\in V2} c$

$= d_{G2}(u\,v) + cp_2 + cp_2 + cp_2$

$= d_{G2}(u\,v) + 3cp_2.$

For any $u\,v \in E_3$,

$d_{G1+G2+G3}(u\,v\,w) = d_{G3}(u\,v) + \sum_{uv\in E'} \sigma_1(u) \wedge \sigma_2(v) +$

$\sum_{vw\in E'} \sigma_1(v) \wedge \sigma_2(w) + \sum_{wu\in E'} \sigma_1(w) \wedge \sigma_2(u)$

$= d_{G3}(u\,v) + \sum_{w\in V1} c + \sum_{w\in V1} c + \sum_{w\in V1} c$

$= d_{G3}(u\,v) + cp_1 + cp_1 + cp_1$

$= d_{G3}(u\,v) + 3cp_1.$

For any $u\,v \in E'$, with $u\in V_1, v\in V_2$ and $w\in V_3$.

$d_{G1+G2+G3}(u\,v\,w) = d_{G1}(u) + d_{G2}(v) + \sum_{uw\in E'} \sigma_1(u) \wedge \sigma_2(w) +$

$\sum_{wv\in E'} \sigma_1(w) \wedge \sigma_2(v) + \sum_{uv\in E'} \sigma_1(u) \wedge \sigma_2(v)$

$= d_{G1}(u) + d_{G2}(v) + \sum_{w\in V3} c - \sigma_1(u) \wedge \sigma_2(v) +$

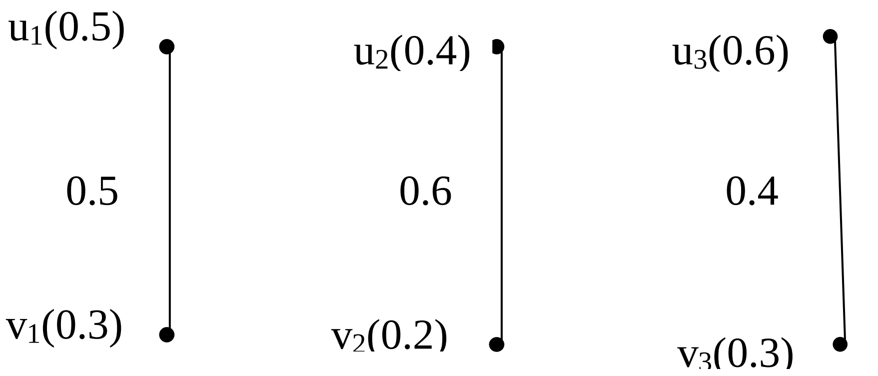
$\sum_{w\in V2} c - \sigma_1(u) \wedge \sigma_2(v) + \sum_{w\in V1} c - \sigma_1(u) \wedge \sigma_2(v)$

$= d_{G1}(u) + d_{G2}(v) + p_3c + p_2c + p_1c - 3c$

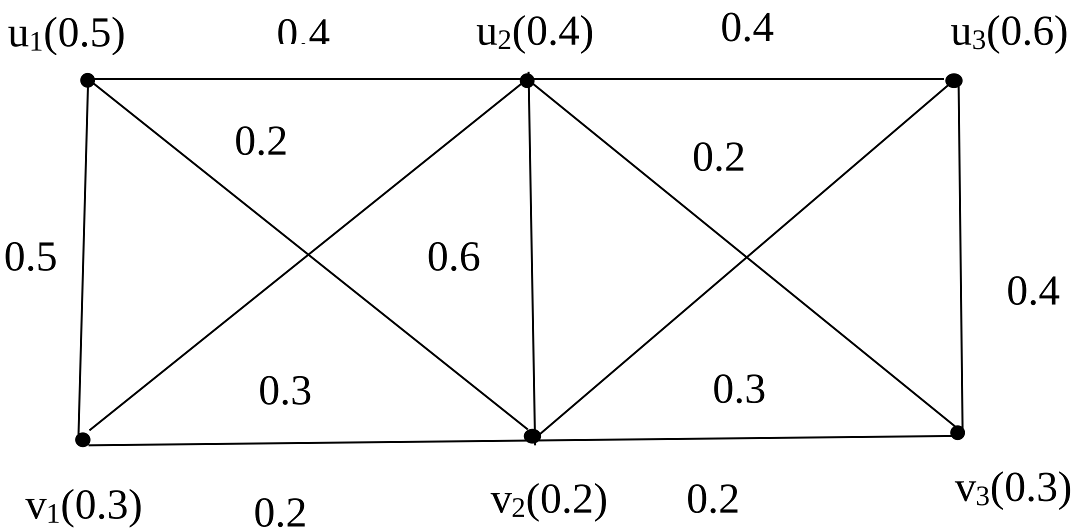
$= d_{G1}(u) + d_{G2}(v) + c(p_1 + p_2 + p_3 - 3).$

Example 3.5

The following figure illustrating the above theorem.



G_1 G_2 G_3



$G_1+G_2+G_3$

By theorem 3.4,

(1). $d_{G1+G2+G3}(u_1 v_1) = d_{G1}(u_1 v_1)+3cp_3 = 0+3(0.2)3=1.8$

(2). $d_{G1+G2+G3}(u_2 v_2) = d_{G2}(u_2 v_2)+3cp_2 = 0+3(0.2)3=1.8$

(3). $d_{G1+G2+G3}(u_3 v_3) = d_{G3}(u_3 v_3)+3cp_1 = 0+3(0.2)3=1.8$

(4). $d_{G1+G2+G3}(u_1 v_2 u_3) = d_{G1}(u_1)+d_{G2}(v_1)$

$+ d_{G3}(u_3)+c(p_1+p_2+p_3-3)$

$= 0.5+0.6+0.4+0.2(3+3+3-3) = 2.7$

Conclusion

In this paper, we have found the degree of edges in $G_1 \cup G_2 \cup G_3$ in terms of G_1,G_2 and G_3, the degree of edges in $G_1+G_2+G_3$ in terms of the degree of vertices and edges in G_1,G_2 and G_3 and also in terms of the degree of vertices in G_1^*,G_2^* and G_3^* under some conditions and illustrated them through examples. They will be more helpful especially when the graphs are very large. Also they will be useful in studying various condition, properties of union and join of three fuzzy graphs.

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