



# **A STUDY ON COMPARISION OF TRANSPORTATION PROBLEMS IN OPERATIONS RESEARCH**

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## **ABSTRACT**

The transportation problem in Operations Research has wide applications in inventory control, production planning, scheduling, personal allocation and so forth. The objective is to minimize the cost of distribution a product from a number of sources or origins to a number of destinations. The characteristic of a transportation problem are such that it usually solved by a specialized method rather than by simplex method.

Key words: Operations Research, Transportation problem, Inventory control, production plan.

## **TRANSPORTATION PROBLEM**

The transportation problem in operational research is concerned with finding the minimum cost of transporting a single commodity from a given number of sources to a given number of destinations. These types of problems can be solved by general network methods, but here we use a specific transportation algorithm.

The data of the model include

1. The level of supply at each source and the amount of demand at each destination.
2. The unit transportation cost of the commodity from each source to each destination.

Since there is only one commodity, a destination can receive its demand from more than one source. The objective is to determine how much should be shipped from each source to each destination so as to minimise the total transportation cost.

Mathematically the problem may be stated as follows:

$$\text{Minimize } z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} x_{ij}$$

Subject to the constraints:

$$x_{i1} + x_{i2} + x_{i3} + \dots + x_{in} = a_i ; \text{ where } i=1, 2, 3, \dots, m$$

$$x_{1j} + x_{2j} + x_{3j} + \dots + x_{mj} = b_j ; \text{ where } j=1, 2, 3, \dots, n$$

And  $x_{ij} \geq 0$ , for all  $i$  and  $j$ .

For a feasible solution to exist, it is necessary that total supply equals total requirement.

$$(\text{ie}); \sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

Types of transportation problem in Operation research:

**a. Balanced Transportation Problem:** In transportation problem when the total supply from all the sources is equal to the total demand in all destinations is said to be an balanced transportation problem.

$$\sum_{i=1}^n a_i = \sum_{j=1}^n b_j$$

**b. Unbalanced Transportation Problem:** In transportation problem when the sum of supply available from all sources is not equal to the sum of demands of all destinations is said to be an unbalanced transportation problem.

$$\sum_{i=1}^n a_i \neq \sum_{j=1}^n b_j$$

## SOLUTION OF THE TRANSPORTATION PROBLEM

1. Finding an initial basic feasible solution.
2. Checking for optimality.

Finding an initial basic feasible solutions there are three methods:

### a. Northwest Corner Method

The method starts at the northwest corner cell of the tableaux 11.

1. Select the upper left corner cell of the transportation matrix and allocate  $\min(S_1, d_1)$
2. A. Subtract this value from supply and demand of respective row and column.  
B. If the supply is 0, then cross that row and move down the next cell.  
C. If the demand is 0, then cross that column and move right to the next cell.

D. If supply and demand both are 0 then cross both row and column and move diagonally to the next cell.

3. Repeat these steps until all supply and demand values are 0.

### b. Least Cost Method

The least cost method by assigning possible to the cell with smallest unit cost.

1. Find the cell with the least (minimum) cost in the transportation table.
2. Allocate the maximum feasible quantity to the cell.
3. Eliminate the row or column where an allocation is made.
4. Repeat the above steps for the reduced transportation table until all the allocations are made.

### c. Vogel's Approximation Method

Vogel's Approximation Method is an improved version of the least cost method that generally produces better starting solutions.

1. Calculate the penalties for each row and each column. Here penalty means the difference between the two successive least cost in row and in column.
2. Select the row or column with the largest penalty.
3. In the selected row or column, allocate the maximum feasible quantity to the cell with the minimum cost.
4. Eliminate the row or column where all the allocations are made.
5. Write the reduced transportation table and repeat the step 1 to 4.
6. Repeat the procedure until all the allocations are made.

### ILLUSTRATIVE EXAMPLE

Solve the following transportation Problem:

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	Supply
S <sub>1</sub>	19	30	50	10	7
S <sub>2</sub>	70	30	40	60	9
S <sub>3</sub>	40	8	70	20	18
Demand	5	8	7	14	34

Solution:

Since  $\sum a_i = \sum b_j = 34$ .

There exists a feasible solution to the transportation problem. We obtain an initial basic feasible solution.

### 1. North west corner method:

Given transportation problem is

19	30	50	10	7
70	30	40	60	9
40	8	70	20	18
5	8	7	14	

Table 1: First allocation

19(5)	30	50	10	7/2
70	30	40	60	9
40	8	70	20	18
5/0	8	7	14	

Table 2: Second allocation

30(2)	50	10	2/0
30	40	60	9
8	70	20	18
8/6	7	14	

Table 3: Third allocation

30(6)	40	60	9/3
8	70	20	18
6/0	7	1	

Table 4: Fourth allocation

40(3)	60	3/0
70	20	18
7/4	14	

Table 5: Fifth allocation

70(4)	20	18/14
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4/0      14

Table 6: Sixth allocation

20(14)	14
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14

Final table is

19(5)	30(2)	50	10	7
70	30(6)	40(3)	60	9
40	8	70(4)	20(14)	18
5	8	7	14	

The associated objective function is

$$(19 \times 5) + (30 \times 2) + (30 \times 6) + (40 \times 3) + (70 \times 4) + (20 \times 14) = 1015$$

Solution is non-degenerate.

## 2. Least cost method

Table 1: First allocation

19	30	50	10	7
70	30	40	60	9
40	8	70	20	18/10
5	8/0	7	14	

Table 2: Second allocation:

19	50	10(7)	7/0
70	40	60	9
40	70	20	10
5	7	14/7	

Table 3: third allocation:

70	40	60	9
40	70	20(7)	10/3

5          7          7/0

Table 4: Fourth allocation

70	40(7)	9/2
40	70	3

5          7/0

Table 5: Fifth allocation

70	2
40(3)	3/0

5/2

Table 6: Sixth allocation

70(2)	2
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2

Final table is

19	30	50	10(7)	7
70	30	40(7)	60	9
40(3)	8(8)	70(2)	20(7)	18

5          8          7          14

Hence the associated objective function is

$$(10 \times 7) + (40 \times 7) + (40 \times 3) + (8 \times 8) + (70 \times 2) + (20 \times 7) = 814$$

Solution is non-degenerate.

## 3.Vogels Approximation (or) Penalty method

Given transportation problem is

19	30	50	10	7
70	30	40	60	9
40	8	70	20	18
5	8	7	14	

Table 1: First allocation

P<sub>1</sub>

19	30	50	10	7	(9)
70	30	40	60	9	(10)
40	8	70	20	18/10	(12)
5	8/0	7	14		
(21)	(22)	(10)	(10)		

Table 2: Second allocation

P<sub>2</sub>

19(5)	50	10	7/2	(9)
70	40	60	9	(20)
40	70	20	10	(20)
5/0	7	14		
(21)	(10)	(10)		

Table 3: Third allocation

P<sub>3</sub>

50	10	2	(40)
40	60	9	(20)
70	20(10)	10/0	(50)
7	14/4		
(10)	(10)		

Table 4: Fourth allocation  $P_4$ 

50	10(2)	2/0	(40)
40	60	9	(20)
7	4/2		
(10)	(50)		

Table 5: Fifth allocation  $P_5$ 

40	60 (2)	9/7	(20)
7	2/0		
(40)	(60)		

Table 6: Sixth allocation

40(7)	7
7	

Final Table is

19(5)	30	50	10(2)	7
70	30	40(7)	60(2)	9
40	8	70	20(10)	18
5	8	7	14	

Hence the associated objective function is

$$(19 \times 5) + (10 \times 2) + (40 \times 7) + (60 \times 2) + (8 \times 8) + (20 \times 10) = 779$$

Solution is non-degenerate.

### Comparison between the three methods:

North West Corner method is used when the purpose of completing demand and the next and is used when the purpose of completing the supply then the next. Advantage of NWC is quick solution because computation takes short time but yields a bad solution because it is very far from Optimum solution.

Least Cost method and Vogel's approximation method are used to obtain the shortest route. Advantage of LCM and VAM is the best starting basic solution because gives initial solution near to optimum solution but VAM takes a long time.



**CONCLUSION:**

The transportation problem is one of the most frequently encountered applications in real life situations. The transportation problem indicates the amount of consignment to be transported from various origins to different destinations so that the total transportation cost is minimized without violating the availability constraints and the requirement constraints.

**REFERENCES:**

- [1] S. I. Gass, On solving the transportation problem, Journal of Operational Research Society 35(12), 1113-1114, 1990.
- [2] R. R.K. Sharma and K. D. Sharma, A new dual based procedure for the transportation problem, European Journal of Operations Research 144, 560-564, 2003.
- [3] H. Wagner, Principles of Operations Research, New Jersey: Prentice- Hall, Englewood cliffs, 1969.
- [4] W. L. Winston, Operations Research Applications and Algorithms, California: Wadsworth Publishing, 1991.
- [5] P.K Gupta and D.S. Hira, Operation Research, S. Chand, New Delhi, India, 1992.

