



## A Study Of Bio-Porous Convection Of Gyrotactic Micro-Organisms In A Thermally Stratified Sparsely Packed Porous Layer Of Finite Depth By Using A Modified Galerkin Technique

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**Abstract:** Bioconvection is a new area of fluid mechanics and it describes the phenomenon of spontaneous pattern formation in suspensions of micro-organisms like *Bacillus subtilis* and algae. The term "bioconvection" referred to macroscopic convection induced in water by the collective motion of a large number of self propelled motile micro-organisms. The direction of swimming micro-organisms is dictated by the different tactic nature of the micro-organisms. They may be gyrotactic, phototactic, chemotactic, etc.. The present study deals with the linear stability analysis of a suspension of gyrotactic micro-organisms in a horizontal sparsely packed porous fluid layer of finite depth subject to adverse temperature gradient. This problem is relevant to certain species of thermophilic micro-organisms that live in hot environment. Finally it is concluded that the permeability as well as the modulation parameters have strong opposing influences on the onset of bio-convection. Thus, by controlling these parameters it is possible to enhance or suppress bio-convection.

**Keywords:** Bio-porous convection, Galerkin technique, Trial functions microorganism, critical Rayleigh number and critical wave number.

### I. INTRODUCTION

Bioconvection is a new area of fluid mechanics and it describes the phenomenon of spontaneous pattern formation in suspensions of micro-organisms like *Bacillus subtilis* and algae (Pedleys and Kessler 1987,1992). The term "bioconvection" referred to macroscopic convection induced in water by the collective motion of a large number of self propelled motile micro-organisms. To the best of authors' knowledge the effect of uniform rotation about the vertical axis on the interaction of bioconvection and natural convection has not been studied earlier. The effect of gravity inclination on BPC is studied by Srimani and Roopa (2011a, 2011b). The direction of swimming micro-organisms is dictated by the different tactic nature of the micro-organisms. They may be gyrotactic, phototactic, chemotactic, etc.. The present study deals with the linear stability analysis of a suspension of gyrotactic micro-organisms in a horizontal fluid layer of finite depth subject to adverse temperature gradient. The present study has several geophysical applications. For example, this problem is relevant to certain species of thermophilic micro-organisms that live in hot environment.

### II MATHEMATICAL FORMULATION

Under the following major assumptions viz. (i) heating from below is sufficiently weak so that the micro-organisms will not be killed and they would retain their gyrotactic nature (ii) due to the small flow velocity associated with the phenomenon, the inertia terms in the Navier-Stoke's equations are neglected and (iii) the system is subject to a uniform rotation about the vertical axis. The governing equations are:

$$\rho_w \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{\mu}{K} u \quad (1)$$

$$\rho_w \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) - \frac{\mu}{K} v \quad (2)$$

$$\rho_w \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - n\theta\Delta\rho g + \rho_w g \beta (T - T_o) \quad (3)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (4)$$

$$c_p \rho_w \left( \left( \frac{\partial T}{\partial t} \right) + u \left( \frac{\partial T}{\partial x} \right) + v \left( \frac{\partial T}{\partial y} \right) + w \left( \frac{\partial T}{\partial z} \right) \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad (5)$$

$$\frac{\partial n}{\partial t} = -\text{div}(\vec{j}) \quad (6)$$

where,

$$\vec{j} = n\vec{v} + nW_c\vec{P} - D\nabla n \quad (7)$$

Here,  $\rho_w$ : The density of water,  $\vec{v} = (u, v, w)$ : The fluid convection velocity vector,  $t$ : The time,  $p$ : The excess pressure above the hydrostatic,  $\hat{p}$ : The unit vector in the direction of swimming of micro-organism,  $(x, y, z)$ : Cartesian co-ordinates (space variables),  $\mu$ : The dynamic viscosity of the suspension,  $W_c\vec{P}$ : The vector of average swimming velocity relative to the fluid ( $W_c = \text{constant}$ ),  $\beta$ : The volume expansion coefficient of water at constant pressure,  $\Delta\rho = \rho_{cell} - \rho_w$  (the density difference),  $\vec{j}$ : The flux of micro-organisms due to microscopic convection of the fluid, self propelled swimming of micro-organisms and the diffusion of micro-organisms (further, it is assumed that all random motions of micro-organisms can be approximated by a diffusive process),  $c_p$ : The specific heat of water,  $D$ : The diffusivity of microorganisms;  $g$  is acceleration due to gravity,  $k$ : The conductivity of water,  $n$ : The number density of motile organisms,  $Q$ : The bioconvection Peclet number,  $\vec{g} = g_o G^*$  where  $G^*$ : modulation parameter  $G^* = 1 + (g(t)/g_o)$  where  $g = g(t) + g_o$  is the acceleration due to gravity,  $P_L = \frac{H^2}{K}$

The permeability parameter

### A Boundary Conditions

At the bottom of the layer (assumed to be rigid), the following conditions are satisfied.

$$@ z=0: u=v=w=0, T = T_o + \Delta T, j \cdot \hat{k} = 0 \quad (8)$$

where  $\hat{k}$  is the vertically upward unit vector.

The upper surface of the layer is assumed to be rigid as well because, according to Hill et.al. [1989], even if it is opened to the air, micro-organisms tend to collect at the surface forming a packed layer. Under this assumption, the boundary conditions at the upper surface of the layer are,

$$@ z=H: u=v=w=0, T = T_o, j \cdot \hat{k} = 0 \quad (9)$$

## III SOLUTION PROCEDURE

### A Basic state

In the basic state the equation of continuity admit a steady state solution where the fluid is motionless,  $n_b$  is the number density of the micro-organisms,  $p_b$  is the pressure and  $T_b$  is the temperature which are functions of  $z$  only.

In this case equations (6) and (7) reduced to

$$n_b W_c = D \frac{\partial n_b}{\partial z} \quad (10)$$

The solution of this equation is

$$n_b(z) = V \exp\left(\frac{W_c z}{D}\right) \quad (11)$$

The integration constant  $V$ , which represents the value of the basic number density at the bottom of the layer, is related to the average concentration

$$\bar{n} = \frac{1}{H} \int_0^H n_b(z) dz = \frac{V}{H} \int_0^H \exp\left(\frac{W_c z}{D}\right) dz \quad (12)$$

and is given by,

$$V = \frac{\bar{n} Q}{\exp(Q) - 1} \quad (13)$$

where the bioconvection Peclet number  $Q$  is defined by

$$Q = \frac{W_c H}{D} \quad (14)$$

From equations (5), (8), (9), the temperature distribution in the basic state is

$$T_b = -\frac{\Delta T}{H} Z + T_0 + \Delta T \quad (15)$$

From (3) the pressure distribution in the basic state is found to be

$$\frac{\partial p}{\partial z} = -\nu \theta \Delta \rho g \exp\left(\frac{W_c z}{D}\right) + \rho_w g \beta \Delta T \left(1 - \frac{z}{H}\right) \quad (16)$$

$$p_b - p_0 = \nu \theta \Delta \rho g \frac{D}{w_c} \left[ \exp - \exp w_c \frac{z}{D} \right] - \rho_w g \beta \Delta T \left[ H - z - \frac{1}{2H} (H^2 - z^2) \right] \quad (17)$$

### B Linear stability analysis

The perturbations are introduced as follows:

$$(n, v, p, T)(t, x, y, z) = (n_b(z), 0, p_b(z), T(z)_b, \hat{k}) + \varepsilon(n^*, v^*, p^*, T^*, \hat{p}^*)(t, x, y, z) \quad (18)$$

where \* denotes a perturbation quantity and  $\varepsilon$  is the small perturbation amplitude. Substitution of equations (18) into equations (1) - (7) and linearizing, result in the following equations for perturbations:

$$\rho_w \frac{\partial u^*}{\partial t} = -\frac{\partial p^*}{\partial x} + \mu \left( \frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial y^2} + \frac{\partial^2 u^*}{\partial z^2} \right) - \frac{\mu}{K} u^* \quad (19)$$

$$\rho_w \frac{\partial v^*}{\partial t} = -\frac{\partial p^*}{\partial y} + \mu \left( \frac{\partial^2 v^*}{\partial x^2} + \frac{\partial^2 v^*}{\partial y^2} + \frac{\partial^2 v^*}{\partial z^2} \right) - \frac{\mu}{K} v^* \quad (20)$$

$$\rho_w \varepsilon \frac{\partial u^*}{\partial t} = -\frac{\partial(p_b + \varepsilon p^*)}{\partial x} + \mu \varepsilon \left( \frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial y^2} + \frac{\partial^2 u^*}{\partial z^2} \right) - \frac{\mu}{K} \varepsilon u^* \quad (21)$$

$$\rho_w \frac{\partial u^*}{\partial t} = -\frac{\partial p^*}{\partial x} + \mu \left( \frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial y^2} + \frac{\partial^2 u^*}{\partial z^2} \right) - \frac{\mu}{K} u^* \quad (22)$$

Similarly,

$$\rho_w \frac{\partial v^*}{\partial t} = -\frac{\partial p^*}{\partial x} + \mu \left( \frac{\partial^2 v^*}{\partial x^2} + \frac{\partial^2 v^*}{\partial y^2} + \frac{\partial^2 v^*}{\partial z^2} \right) - \frac{\mu}{K} v^* \quad (23)$$

$$\frac{\partial p^*}{\partial y} + \mu \left( \frac{\partial^2 w^*}{\partial x^2} + \frac{\partial^2 w^*}{\partial y^2} + \frac{\partial^2 w^*}{\partial z^2} \right) - n^* \theta \Delta \rho g + \rho_w g \beta T^* - \frac{\mu}{K} w^* \quad (24)$$

$$\left( \frac{\partial^2 v^*}{\partial x^2} + \frac{\partial^2 v^*}{\partial y^2} + \frac{\partial^2 v^*}{\partial z^2} \right) = 0 \quad (25)$$

$$c_p \rho_w \left( \frac{\partial T^*}{\partial t} - w^* \frac{\Delta T}{H} \right) = k \left( \frac{\partial^2 T^*}{\partial x^2} + \frac{\partial^2 T^*}{\partial y^2} + \frac{\partial^2 T^*}{\partial z^2} \right) \quad (26)$$

$$\frac{\partial n^*}{\partial t} = -\text{div}[n_b(v^* + W_c \hat{p}^*) + n^* W_c \hat{k} - D \nabla n^*] \quad (27)$$

The elimination of  $u^*$ ,  $v^*$ , and  $p^*$  from eqs. 23a,b,c,d -26 result in,

$$\left( \rho_w \frac{\partial}{\partial t} - \mu \nabla^2 \right)^2 \nabla^2 w^* + 4\Omega^2 \frac{\partial^2 w^*}{\partial z^2} = -\theta \Delta \rho g \left( \rho_w \frac{\partial}{\partial t} - \mu \nabla^2 \right) \nabla_1^2 n^* + \rho_w g \beta \left( \rho_w \frac{\partial}{\partial t} - \mu \nabla^2 \right) \nabla_1^2 T^* \quad (28)$$

Since it is assumed that the temperature variation within the fluid layer do not influence the gyrotactic behavior of micro-organisms, (according to Pedley et al.[1988]), it is considered,

$$\hat{p}^* = B(\eta, -\xi, 0) \quad (29)$$

where

$$\xi = (1 - \alpha_0) \frac{\partial w^*}{\partial y} - (1 + \alpha_0) \frac{\partial v^*}{\partial z} \quad (30)$$

$$\eta = -(1 - \alpha_0) \frac{\partial w^*}{\partial y} + (1 + \alpha_0) \frac{\partial u^*}{\partial z} \quad (31)$$

$$\alpha_0 = \frac{a^2 - b^2}{a^2 + b^2} \quad (32)$$

$$B = \frac{\alpha_{\perp} \mu}{2h\rho_0 g} \quad (33)$$

where a and b are the semi-major and minor axes of the spheroidal cell,  $\alpha_0$  is a measure of the cell eccentricity, B is the gyrotactic orientation parameter which is introduced by Pedley and Kessler [1987] which has the dimension of time;  $\alpha_{\perp}$  is a dimensionless constant relating to viscous torque to the relative angular velocity of the cell; and h is the displacement of center of mass of the cell from the center of buoyancy.

Thus equation (27) can be written as :

$$\frac{\partial n^*}{\partial t} = -w^* \frac{\partial n^*}{\partial z} - w_c \frac{\partial n^*}{\partial z} - w_c B n_b \left[ \frac{\partial \eta}{\partial x} - \frac{\partial \xi}{\partial y} \right] + D \Delta^2 n^* \quad (34)$$

With the consideration of eqns.(30) and (31) eqn..34) can be recasted as

$$\frac{\partial n^*}{\partial t} = -w^* \frac{\partial n_b}{\partial z} - w_c \frac{\partial n^*}{\partial z} + w_c B n_b \left[ (1 - \alpha_0) \left( \frac{\partial^2 w^*}{\partial x^2} + \frac{\partial^2 w^*}{\partial y^2} \right) + (1 + \alpha_0) \frac{\partial^2 w^*}{\partial z^2} \right] + D \Delta^2 n^* \quad (35)$$

A normal mode expansion is introduced in the following form:

$$[w^*, n^*, T^*] = [W(z), N(z), \Theta(z)] f(x, y) \exp(\sigma t) \quad (36)$$

The function f(x,y) satisfies the following equation:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = -m^2 f \quad (37)$$

where m is the horizontal wave number (used as separation constant). Substituting eqn. (36) into eqns. (26), (28), (35) and by using eqn. (37), the following equations for the amplitudes W,  $\Theta$ , and N, are obtained.

$$-\theta \Delta \rho g N - (\mu n^4 + \rho_w m^2 \sigma + \frac{\mu}{K} m^2 W) + \rho_w g \beta m^2 \theta + (2m^2 \mu \rho_w \sigma + \frac{\mu}{K}) W^{11} - \mu W^{1v} \quad (38)$$

$$-c_p \Delta T \rho_w W + H[(km^2 + c_p \rho_w \sigma) \Theta - k \Theta'] = 0 \quad (39)$$

$$D(Dm^2 + \sigma)N + D(W_c N' - DN'') - \exp\left(\frac{W_c z}{D}\right) W_{c,v} \times [-(1 + BDm^2(1 - \alpha_0))W + BD(1 + \alpha_0)W''] = 0 \quad (40)$$

Introducing the following dimensionless variables,

$$\bar{z} = \frac{z}{H}, a = mH, \bar{W} = \frac{v \theta W_c H^2}{D^2} W, \bar{N} = N \theta, \bar{\Theta} = \beta \Theta$$

eqs. (38) - (40) can be re-casted as

$$\begin{aligned} & -a^2 R_b Q \left[ (a^2 + \frac{\rho_w H^2 \sigma}{\mu}) \bar{N} - \bar{N}^{11} \right] + \frac{a^2 R_b Q}{\omega} \left[ (a^2 + \frac{\rho_w H^2 \sigma}{\mu}) \bar{\Theta} - \bar{\Theta}^{11} \right] - a^2 \\ & \left( a^2 + \frac{\rho_w H^2 \sigma}{\mu} \right)^2 \bar{W} - a^2 R_b Q \bar{N} - \left( a^4 + a^2 \frac{\rho_w H^2}{\mu} \sigma + \frac{a^2 H^2}{K} \right) \bar{W} + a^2 \frac{R_b Q}{\omega} \bar{\Theta} \\ & + \left( 2a^2 + \frac{\rho_w H^2}{\mu} \sigma + \frac{H^2}{K} \right) \bar{W}'' - \bar{W}^{IV} = 0 \end{aligned} \quad (41)$$

$$-\frac{Ra}{Q} \frac{\omega}{Rb} \bar{W} + \left[ \left( a^2 + \frac{H^2 c_p \rho_w}{k} \sigma \right) \bar{\Theta} - \bar{\Theta}'' \right] = 0 \quad (42)$$

$$\left( a^2 + \frac{H^2}{D} \sigma \right) \bar{N} + (Q \bar{N}' - \bar{N}'') - \exp(Q \bar{z}) \left[ -(1 + G(1 - \alpha_0) a^2) \bar{W} + G(1 + \alpha_0) \bar{W}'' \right] = 0 \quad (43)$$

Where  $Rb = \Delta \rho g v \theta H^3 / (\mu D)$  is the bioconvection Rayleigh number,  $\omega = \Delta \rho / \rho_w$  is the measure of density of micro-organisms,  $Ra = g \beta \Delta T H^3 \rho_w^2 c_p / (\mu k)$  is the traditional Rayleigh number associated with natural convection, and  $G = BD / H^2$  is the gyrotaxis number.

Setting  $\sigma = 0$  the following equations are obtained,

$$-a^2 Rb Q \bar{N} - (a^4 + \frac{a^2 H^2}{K}) \bar{W} + a^2 \frac{Rb Q}{\omega} \bar{\Theta} + \left( 2a^2 + \frac{H^2}{K} \right) \bar{W}'' - \bar{W}^{N'} = 0 \quad (44)$$

$$-\frac{Ra}{Q} \frac{\omega}{Rb} \bar{W} + a^2 \bar{\Theta} - \bar{\Theta}'' = 0 \quad (45)$$

$$a^2 \bar{N} + (Q \bar{N}' - \bar{N}''') - \exp(Q \bar{z}) \left[ -(1 + G(1 - \alpha_0) a^2) \bar{W} + G(1 + \alpha_0) \bar{W}'' \right] = 0. \quad (46)$$

Eqs.(44)–(46) should be solved subject to the following boundary conditions:

$$z = 0 : \bar{W} = 0, \frac{d\bar{W}}{d\bar{z}} = 0, \bar{\Theta} = 0, Q \bar{N} = \frac{d\bar{N}}{d\bar{z}} \quad (47)$$

$$z = H : \bar{W} = 0, \frac{d\bar{W}}{d\bar{z}} = 0, \bar{\Theta} = 0, Q \bar{N} = \frac{d\bar{N}}{d\bar{z}} \quad (48)$$

$$z = 0 : \bar{W} = 0, \frac{d^3 \bar{W}}{d\bar{z}^3} = 0 \quad (49)$$

$$z = H : \bar{W} = 0, \frac{d^3 \bar{W}}{d\bar{z}^3} = 0 \quad (50)$$

For the solution of this system, a simple Galerkin method (Finlayson 1972) is employed. Suitable trial functions (satisfying the boundary conditions) are obtained as follows:

### C Galerkin procedure

The boundary conditions and required set of functions which were symmetric about  $z=0$  were considered as a power series in  $\bar{z}$  as follows.

$$\left. \begin{aligned} \bar{W}_1(\bar{z}, t) &= \sum_{i=1}^N A_i(t) W_i(\bar{z}) \\ \bar{\Theta}_1(\bar{z}, t) &= \sum_{i=1}^N B_i(t) \Theta_i(\bar{z}) \\ \bar{N}_1(\bar{z}, t) &= \sum_{i=1}^N N_i^*(t) N_i(\bar{z}) \end{aligned} \right\} \quad (51)$$

Trial functions were,

$$\bar{W}_1 = \bar{z}^4 - \bar{z}^2 \quad (52)$$

$$\bar{\Theta}_1 = \bar{z} - \bar{z}^2 \quad (53)$$

$$\bar{N}_1 = 2 - Q(1 - 2\bar{z}) - Q^2(\bar{z} - \bar{z}^2) \quad (54)$$

For the application of Galerkin method it is necessary to compute various integrals involving the trial functions. Thus the following functions are defined :

$$\partial(m, n) = \int_0^1 \bar{z}^{m+2} (\bar{z}^2 - 1)^n d\bar{z} \quad (55)$$

$$\partial(m^*, n) = \frac{-2n}{m + 2n + 3} \partial(m^*, n - 1) \quad (56)$$

where  $m^* = m + 2$

Using the recurrence relation the values for  $\partial_{(m^*, n)}$  is computed . Finally

$$\begin{aligned} Rb_{crit} &= 15.01785623 ((.0015 a^4 + .038096 a^2 + .79) (-.03333 a^2 - .3333) (-a^2 \\ &((2 - Q)^2 + 1.0 Q (2 - Q)^2 + .3333 Q^2 (2 - Q) (4 - Q) + .50 Q^3 (2 - Q) + .2 Q^4) \\ &- Q (Q (2 - Q)^2 + 1.0 Q^2 (2 - Q) + .5 Q^2 (2 - Q)^2 + .9999 Q^3 (2 - Q) + .50 Q^4) \\ &+ 2 Q^2 (2 - Q) + 1.0 Q^3 (2 - Q) + .6666 Q^4) + .000051408 a^2 Ra (-a^2 \end{aligned}$$

$$\begin{aligned} &((2 - Q)^2 + 1.0 Q (2 - Q)^2 + .3333 Q^2 (2 - Q) (4 - Q) + .50 Q^3 (2 - Q) + .2 Q^4) \\ &- Q (Q (2 - Q)^2 + 1.0 Q^2 (2 - Q) + .5 Q^2 (2 - Q)^2 + .9999 Q^3 (2 - Q) + .50 Q^4) \\ &+ 2 Q^2 (2 - Q) + 1.0 Q^3 (2 - Q) + .6666 Q^4) / (Q a^2 (-.03333 a^2 - .3333) (G \\ &(1 + \alpha_0) ( \end{aligned}$$

$$\begin{aligned}
 &.0008 - .0004 Q + .0004 Q (2 - Q) - .30008 Q^2 + .0666 Q^2 (2 - Q) - .214152 Q^3) \\
 &- (1 + G (1 - \alpha_0)) a^2 (.0666 - .0333 Q + .03332 Q (2 - Q) + .009524 Q^2 (2 - Q) \\
 &+ .009524 Q^2 + .005953 Q^3))
 \end{aligned} \tag{57}$$

We computed all the terms. Extensive mathematical calculations and computations are involved. Finally the Critical Bio-Rayleigh number is computed for suitable values of the parameters and the results are presented through graphs (1) to (6).

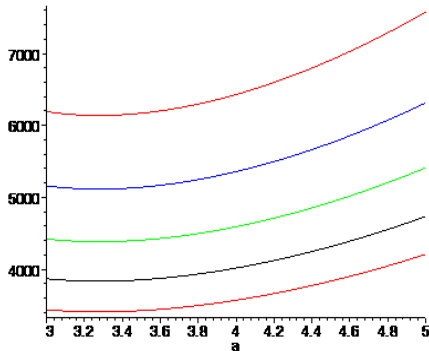


Fig1:  $Ra$  vs  $a$ ,  $Rb = 0$ ,  $Q = 0.1$ ,  $G = 10$ ,  $P_l = 100$ ,  
 $\alpha_0 = 0.2$ ,  $G^* = 1, 1.2, 1.4, 1.6$ , and  $1.8$

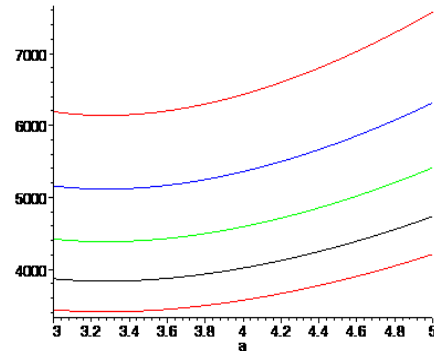


Fig2:  $Ra$  vs  $a$ ,  $Rb = 0$ ,  $Q = 0.05$ ,  $G = 10$ ,  $P_l = 100$ ,  
 $\alpha_0 = 0.2$ ,  $G^* = 1, 1.2, 1.4, 1.6$ , and  $1.8$

**IV RESULTS AND DISCUSSION:** The computed results are presented in figure 1 & 2 for the values of bioconvection Peclet number  $Q=0.1$  &  $Q=0.05$ , the gyrotaxis number  $G=10$ , the permeability parameter  $P_l=100$  the cell eccentricity  $\alpha_0 = 0.2$  and the modulation parameter  $G^* = 1.2$  to  $1.8$ . The important observations are made: In the figure the graphs of  $Ra$  vs  $a$  (the traditional Rayleigh number vs wavenumber) is presented for  $Q=0.1$ ;  $P_l = 100$ ,  $\alpha_0 = 0.2$ ,  $Rb=0$  and  $G^* = 1.2$  to  $1.6$ . and  $1.8$ . The effect of  $Q$  is negligibly small.

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