IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

MULTI-LEVEL LINEAR AND NON LINEAR PROGRAMMING PROBLEM BASED ON FUZZY GOAL PROGRAMMING

[1] Anil Kumar Yadav,[2]Dr. Savita Mishra, [3]Dr. Prabhat Kumar Singh

[1] Research Scholar, University Department of Mathematics, Kolhan University, Chaibasa, Jharkhand, India [2]Assistant professor, Department of Mathematics, The Graduate School College For Women, Jamshedpur, KolhanUniversity, Jharkhand, [3]Assistant Professor, Department of Statistics Jamshedpur co-operative college Jamshedpur, Jharkhand ,India.

ABSTRACT:

In this paper we will apply fuzzy goal programming (FGP) approach to solve a Multi-level Linear and Non Linear programming problem. The main advantage of the FGP approach presented here is that the computational load with re-evaluation of the problem again and again by re-defining the elicited membership values of the DMs for searching higher degree of satisfaction does not arise in the solution search process. The proposed approach can be extended to solve fuzzy multi-objective MLPPs. An extension of the approach for fuzzy multi-level decentralized is one of the current research problems. However, it is hoped that proposed approach can contribute to future study in the field of practical hierarchical decision making problems.

Key words: Artificial Intelligence Multi-level linear and non linear programming problems, Fuzzy and Fuzzy goal programming, Fuzzy membership functions,satisfactory solution.

I . INTRODUCTION

The real situations of making decision in an organization involve a diversity of evaluation such as evaluating alternatives and attaining several goals at the same time. In many practical decision making activities, decision making structure has been changing from a single decision maker with single criterion to multiple decision makers with multi-criteria and even to multi-level situations. A resource planning problem in an organization usually consists of several objectives and requires a compromise among several committing individuals or units. Typically, these groups of decision making are arranged in an administrative hierarchical structure to supervise the independent and perhaps conflicting objectives. In this type of multi-level organization, decision planning should concern issues of central administration and coordination of decision making among lower-level activities to achieve the overall organization target. Each decision maker is responsible for one decision making unit of the hierarchical decision-making levels and controls a decision to optimize the objectives at each level. Although the execution of decision moves sequentially from an upper level to a lower level, the reaction, behavior and decision of a lower-level decision maker should affect the optimization of the decision at an upper level decision maker (Baky, 2010; Ahlatcioglu and Tiryaki, 2007; Anandalingam, 1988; Mesarovic et al., 1970). Because of conflicting objectives over different levels, the dissatisfaction with the decision results is often observed among the decision makers. In such cases, a proper distribution of decision authority must be established among the decision levels for most multi-level decision situations. A mathematical multi-level multi-objective programming has often served as a basis for structuring the underlying goals and hierarchical decision making situation of such organizations (Sinha and Sinha, 2004, Shih et al., 1996). Subsequently, a multi objective linear programming problem aims to optimize various conflicting linear objective functions simultaneously under given linear constraints to find compromise solutions (Yu, 1985 and Zeleny, 1982). Linear Programming (LP) problems has been used to model many serious real -life decision making problem s such as management, economic, transportation , data envelopment analysis, railways , agricultural , and many industrial applications . LP problem s involve the optimization of a linear objective function, subject to some linear equality and inequality constraint s involving some continuous /discrete non -negative decision variables. A typical model of an LP problem is:

$$\text{Min} \quad Z(x) = \sum_{j=1}^{n} c_i x_j$$

$$\text{s.t.} \qquad \sum_{j=1}^{n} a_{ij} x_j \geq b_i \ , \quad i = 1,2,\ldots,m$$

$$x_j \geq 0 \ , \qquad j = 1,2,\ldots,n.$$

To ensure all decisions are made in cooperatively and decision authorities are distributed properly in the organization, the satisfaction of decision makers at the lower level must be considered. From this standpoint, it is desirable to develop a fuzzy programming method that facilitates multiple objectives.

II. A MULTI-LEVEL PROGRAMMING PROBLEM [MLLP] IS MATHEMATICALLY FORMULATED AS:

Multi- level programming problem (MLPP is identified as a mathematical programming that solves decentralized planning problems with multiple executors in a multi-level or hierarchical organization and each decision maker (DM) has linear or non-linear objective functions.

A MLPP can be represented as :

$$\underset{x_1}{Min/Max} \quad z_1(\bar{x})$$

$$\underset{x_2}{Min/Max} \quad z_2(\bar{x})$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\underset{x_p}{Min/Max} \quad z_p(\bar{x})$$

$$subject \quad to \quad g_i(x) \leq b_i, \quad i = 1,2,....,m; \quad x_1 \geq 0, x_2 \geq 0,........, x_p \geq 0.$$

Where, $z_1(x)$, $z_2(x)$, ... $z_p(x)$ and $g_i(x)$, ($i = 1,2,\ldots,m,$) are linear or non-linear objective functions and linear or non-linear constraints respectively.

$\{x_1, x_2, ...x_p\}$ are decision vectors under the control of the upper and the lower level DMs.

III. FUZZY GOAL PROGRAMMING APPROACH TO MLPP

Since fuzzy logic has proven to be a very useful tool for representing human knowledge by means of mathematical expressions, the optimization of the involved parameters has been one of the most investigated problems in the theory of fuzzy expert systems. Genetic algorithms are optimization methods which are based on the mechanisms of natural evolution, such as selection, mutation, or sexual reproduction. Genetic algorithms were introduced approximately 25 years ago and turned out to be a very promising approach to the solution of many problems in artificial intelligence. During the last years the combination of fuzzy logic and GAs has come into fashion.

IV. CONSTRUCTION OF MEMBERSHIP FUNCTION OF MLPP

4.1 Membership function using product operator:

$$\mu_{f_p}[f_p(\bar{x})] = \prod_{p=1}^{P} \mu_p(\bar{x})$$

Where $\mu_p(\bar{x})$ is the membership function of the p^{th} objective function of a VMP and is given as:

$$\mu_p(\bar{x}) = \begin{cases} 0 & , \quad f_p(\bar{x}) < f_p^L \\ \dfrac{f_p(\bar{x}) - f_p^L}{f_p^U - f_p^L} & , \quad f_p^L \le f_p(\bar{x}) \le f_p^U \\ 1 & , \quad f_p(\bar{x}) > f_p^U \end{cases}$$

Where f_p^U and f_p^L are the upper and lower bounds on $f_p(\bar{x})$.

4.2 *Piece-wise Linear membership function:*

$$\mu_{f_p}[f_p(\bar{x})] = \sum_{j=1}^{N} \alpha_j \left| f_p(\bar{x}) - f_j \right| + \beta f_p(\bar{x}) + \gamma$$

Where, $\alpha_j = (t_{j+1} - t_j)/2$, $\beta = (t_{N+1} + t_1)/2$, $\gamma = (s_{N+1} + s_1)/2$ and $\mu_{f_p}[f_p(\bar{x})] = t_i f_p(\bar{x}) + s_i$ for each segment i, $f_{i-1} \le f_p(\bar{x}) \le f_i \forall_i$. Here, t_i is the slope and s_i is the y-intercept for the section of the curve initiated at f_{i-1} and terminated at f_i.

4.3 Linear membership function:

$$\mu_{f_p}[f_p(\bar{x})] = \begin{cases} 0 & , \quad f_p(\bar{x}) < f_p^L \\ [f_p(\bar{x}) - f_p^L]/[f_p^U - f_p^L] & , \quad f_p^L \le f_p(\bar{x}) \le f_p^U \\ 1 & , \quad f_p(\bar{x}) > f_p^U \end{cases} \qquad (1)$$

4.3 Quadratic membership function:

$$\mu_{f_p}[f_p(\bar{x})] = a[f_p(\bar{x})]^2 + bf_p(\bar{x}) + c$$

Assuming that

$$\mu_{f_p}[f_p(\bar{x})] = \begin{cases} 0, & f_p(\bar{x}) < f_p^L \\ 1/2, & f_p(\bar{x}) = (f_p^U + f_p^L)/2 \\ 1, & f_p(\bar{x}) > f_p^U \end{cases} \qquad (2)$$

The values of a, b and c can be determined by solving the equations:

$$a[f_p^L]^2 + bf_p^L + c = 0$$
$$a[(f_p^U + f_p^L)/2]^2 + b(f_p^U + f_p^L)/2 + c = 1/2$$
$$a[f_p^U]^2 + bf_p^U + c = 1.$$

V. FUZZY GOAL PROGRAMMING APPROACH FOR SOLVING MLPP

In fuzzy goal programming approaches, the highest degree of membership function is one. So, for the defined membership function in (1) and (2), the flexible membership goals with aspiration levels 1 can be expressed as :

$\mu_{fij}(f_{ij}(x)) + d^-_{ij} - d^+_{ij} = 1,$ i=1,2,…p, j=1,2,…,m_i , (3)

$\mu_{xik}(x_{ik})) + d^-_{iK} - d^+_{ik} = 1,$ i=1,2,…p-1,kj=1,2,…,n_i , (4)

or can be written as :

$$\frac{u_{ij} - (c_1^{ij} x_1 + c_2^{ij} x_2 + ...+ cp^{ij} x_p}{u_{ij} - g_{ij}} + d_{ij}^- - d_{ij}^+ = 1, \quad i=1,2,…p,\ j=1,2,…,m_i, \qquad (5)$$

$$\frac{x_{ik} - (x_{ik}^{iL} - t_k^{iL}}{t_k^{iL}} + d^{L-}{}_{ik} - d_{ik}^{L+} = 1, \quad i=1,2,…p-1,\ k=1,2,…,n_i, \qquad (6)$$

$$\frac{(x_{ik}^{iR} + t_k^{iR}) \_ x_{ik}}{t_k^{iR}} + d_{ik}^{R-} - d_{ik}^{R+} = 1, \quad i=1,2,…,p-1,\ k=1,2,…,n_i, \qquad (7)$$

Where $d_{ik}^- = (d_{ik}^{L-}, d_{ik}^{R-}), d_{ik}^+ = (d_{ik}^{L+}, d_{ik}^{R+}), and d_{ij}^-, d_{ik}^{L-}, d_{ik}^{R-}, d_{ij}^+, d_{ik}^{L+}, d_{ik}^{R+} \geq 0$ with $\frac{F_{i(x)} - t_i}{g_i - t_i} + d_i^- - d_i^+ = 1 \frac{t_i(x) - F_{i(x)}}{t_i - g_i} + d_i^- + d_i^+ = 1$, $d_{ik}^{L-} \times d_{ik}^{L+} = 0$, and $d_{ik}^{R-} \times d_{ik}^{R+} = 0$, i=1,2,…,p-1 , k=1,2,…,$n_i$,represent the under and over –deviations .resp. , from the aspired levels.

In conventional goal programming (GP), the under- and/or over-deviational variables are included in the achievement function for minimizing them and that depends upon the type of the objective functions to be optimized. In the proposed procedures, the over-deviational variables for the fuzzy goals of objective functions, d_{ij}^+, i=1,2,…p , j=1,2,…,m_i, and the over-deviational and the under-deviational variables for the fuzzy goals of the decision variables, d_{ik}^{L-}, d_{ik}^{L+}, d_{ik}^{R-}, d_{ik}^{R+} , i=1,2,…p-1 , k=1,2,…,n_i , are required to be minimized to achieve the aspired levels of the fuzzy goals. It may be noted that any under-deviation from a fuzzy goal indicates the full achievement of the membership value . In addition, d_{ij}^+ =0 (when a membership goal is fully achieved), and d_{ij}^+ =1 (are when its achievement is zero) are found in the solution .

VI. CONCLUSION

The main advantage of the FGP approach presented here is that the computational load with re-evaluation of the problem again and again by re-defining the elicited membership values of the DMs for searching higher degree of satisfaction does not arise in the solution search process. A fuzzy goal programming model is developed to minimize the group regret of degree of satisfactions of all the decision makers, and to achieve the highest degree (unity) of each of the defined membership function goals to the extent possible by minimizing their deviational variables and thereby obtain the most satisfactory solution for all the decision makers

VII. Acknowledgment

The authors take this opportunity to thank the Kolhan University for providing the facilities and for encouragement to carry out this work thoroughly.

REFERENCES

[1] Compose, L. and Verdegay, J.L, J.L. (1989), Linear programming problems and ranking of fuzzy number, Fuzzy Sets and Systems, 32, 1 -11.

[2] I.A. BAKY, Solving multi-level multi-objective linear programming problems through fuzzy goal programming approach. Appl. Math. Model. 34 (2010) 2377-2387

[3] Ignizio, J.P. (1984), Linear programming in single -and multiple –objective systems, Journal of Policy Analysis and Management 3(3), 477 -489.

[4] Kim JS, Whang K-S. 1998. A tolerance approach to the fuzzy goal programming problems with unbalanced triangular membership function. European Journal of Operational Research 107: 614–624

[5] Lai, Y. J. 1996. Hierarchical optimization: a satisfactory solution. Fuzzy Sets and Systems 77, 321-335.

[6] Baky, I. A. 2010. Solving multi-level multi-objective linear programming problems through fuzzy goal programming approach. Applied Mathematical Modelling 34 (9), 2377-2387.

[7]Lachhwani, K. 2014. On solving multi-level multi objective linear programming problems through fuzzy goal programming approach. OPSEARCH 51(4), 624–637.

[8] Moitra, B. N. & Pal, B. B., (2002) A fuzzy goal programming approach for solving bilevel programming problems, in: Pal, N. R. & Sugeno, M. (Eds.), AFSS 2002, INAI 2275, Springer- Verlag, Berlin, Heidelberg, 2002, pp.91-98.

[9] Pramanik, S. & Roy, T. K., (2007) "Fuzzy goal programming approach to multi-level programming problem" *European Journal of Operational Research*, Vol. 176, No.2, pp. 1151-1166.

[10]Shih, H. S. , Lai, Y. J. , and Lee, E. S. 1996. Fuzzy approach for multi-level programming problems. Computers & Operations Research 23 (1), 73-91.

[11] S. Sinha, Fuzzy programming approach to multi-level programming problems, Fuzzy Sets Syst. 136 (2003) 189–202.

[12] Wang. X. and E. Kerre, E. (2001), Reasonable properties for the ordering of fuzzy quantities (2 parts), Fuzzy Sets and Systems, 118, 375 -405.

[13]Zimmermann, H.J. (1978), Fuzzy programming and linear programming with several objective function and, Fuzzy Sets and Systems, 1, 45 -55.