IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

QUADRIPARTITIONED NEUTROSOPHIC CUBIC SET

J. Sharmila
Research Scholar, Nirmala college for women, Coimbatore.

Dr. F. Nirmala Irudayam
Associate Professor, Department of Mathematics, Nirmala college for women, Coimbatore.

Abstract :

The aim of this paper is introduce the concept of quadripartitioned neutrosophic cubic set. The notions of internal and external of truth, contradiction, ignorance, falsity quadripartitioned neutrosophic cubic sets are introduced and related properties are investigated.

Keywords: Quadripartitioned neutrosophic cubic set, truth internal (contradiction internal, ignorance internal, falsity internal) quadripartitioned to the cubic neutrosophic set, truth external (contradiction external, ignorance external, falsity external) quadripartitioned to the cubic neutrosophic set.

1. Introduction

Fuzzy sets, which were introduced by Zadeh, deal with possibilistic uncertainty, connected with imprecision of states, perceptions and preferences [1]. K.Atanassor [1,2,3]
in 1983 divided the idea of institutionstic fuzzy set on a universe X as a generation of fuzzy set. Here besides the degree of membership, a degree of non-membership for each element is also defined. The topological frame work of institutionstic fuzzy set was initiated by D.Coker[3] Based on the interval valued fuzzy sets, jun ct.al [2]. Introduced the notion of Internal and external cubic sets and investigated several properties.

As a generalization of institutionstic fuzzy sets, neutrosohic set was formulated by Smarandache [4,5] is more general platform which extends the concepts of the classic set and fuzzy set, institutionstic fuzzy set and interval valued institutionstic fuzzy set. Neutrosophic set theory is applied to various parts; refer the site (http://fs.gallups.unra .edu/neutrosophy.htm). Y.B. Jun et al [6] implemented a cubic set which is a combination of a fuzzy set with an interval valued fuzzy set [7]. Internal and external cubic sets also described and some properties were also described and some properties were studied.

Y.B. Jun, Smarandache and KIM [6] introduced neutrosophic sets and the concept of internal and external for truth, falsity and so many properties of P,R union, intersection for internal and external neutrosophic cubic sets.
The representation of combined concepts based on normal forms where linguistic connectives as well as variables are assumed to be fuzzy. Quadripartitioned neutrosophic set [8] is a mathematical tool, which is the extension of neutrosophic set and n-valued neutrosophic refined logic for dealing with real life problem. Interval quadripartitiond set [9] deal with concept of set theoretic operations.

In this paper, we introduce the concept of quadripartitioned to the neutrosophic cubic sets. We introduce the notions of truth internal, contradiction internal, ignorance internal, falsity internal on quadripartitioned nutrosophic cubic sets.

2. Preliminaries

Definition - 1.1[3]

A fuzzy set in a set X is defined by a function $\lambda : X \to [0,1]$. Denote by $[0,1]^X$ the collection of all fuzzy sets in X. Define a relation $\leq$ on $[0,1]^X$ as follows,
$(\forall \lambda, \mu \in [0,1]^X)(\lambda \leq \mu \leftrightarrow (\forall x \in X)(\lambda(x) \leq \mu(x))$.
The join ($\vee$) *and meet* ($\wedge$) *of* λ *and* μ *are defined by,*

$$(\lambda \vee \mu)(x) = max\{\lambda(x), \mu(x)\}$$

$$(\lambda \wedge \mu)(x) = min\{\lambda(x), \mu(x)\} \quad \forall x \in X$$

The complement of λ, denoted by λ^c, is defined by

$$(\forall x \in X)(\lambda^c(x)) = 1 - \lambda(x)$$

For a family $\{\lambda_i / i \in A\}$ of fuzzy sets in X,

$$(\bigvee_{i \in A} \lambda_i)(x) = \sup\{\lambda_i(x) / i \in A\}$$

$$(\bigwedge_{i \in A} \lambda_i)(x) = \inf\{\lambda_i(x) / i \in A\}$$

Definition - 1.2[3]

Let X be a non- empty set. An institutionstic fuzzy set A in X is an object having the form $A = \{ < \mu_A(x), \nu_A(x) >: x \in X\}$. where the functions $\mu_A(x), \nu_A(x) \to [0,1]$. Let the set A be the membership and non- membership of the element $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition - 1.3[3]

Let X be a non- empty fixed set. A neutrosophic set (NS) A is an object having the form $A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$ Where $T_A(x), I_A(x), and\ F_A(x)$ which represent the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy function (namely $\sigma_A(x)$), and the degree of non- membership function (namely $\gamma_A(x)$) respectively of each element x$\in X$ to the set A and $0 \leq \sup T_A(x) + supI_A(x) + supF_A(x) \leq 3$.

Definition - 1.4[9]

Let X be a space of points with a generic element in X denoted by x. An interval neutrosophic set (INS) A in X is characterized by truth membership function T_A, indeterminacy membership function I_A, and false membership function F_A. For each point x in X, T_A(x), I_A(x), F_A(x) $\subseteq$ [0,1].
When X is continuous, an INS A can be written as

$$A = \int_X < T(x), I(x), F(x) > x, x \in X$$

When X is discrete, an INS A can be written as

$$A = \sum_{i=1}^{n} < T(x_i), I(x_i), F(x_i) > /x_i, \ x \in X$$

Definition – 1.5[3]

Let X is a non- empty set. A cubic set in X is defined by

$$C = \{ (x, A(x), \lambda(x)) / x \in X \}$$

Where A is an interval valued fuzzy set in X and λ is a fuzzy set in X.

Definition – 1.6[3]

Let X be a non- empty set. A neutrosophic cubic set (NCS) in X is a pair of $\mathfrak{B} = (B, \Lambda)$ where B= $\{< x : B_T(x), B_I(x), B_F(x) > x \in X\}$ is an interval valued neutrosophic set in X and $\Lambda = \{< x : \lambda_T(x), \lambda_I(x), \lambda_F(x) > x \in X\}$ is a neutrosophic set in X.

Definition – 1.7[7]

A quadripartitioned means a division or distribution by four, or into four parts; also, a taking the fourth part of any quantity or number.

Definition – 1.8[7]

Let X be a non- empty fixed set. B in X is defined by quadripartitioned neutrosophic set $B = \{(x: B_T(x),\ B_C(x),\ B_I(x), B_F(x)\ /x \in X\}$ where $B_T,\ B_C,\ B_I, B_F \in [0,\ 1]$ are degrees of membership functions of truth, contradiction, ignorance, falsity membership functions respectively and $0 \leq Sup\ B_T(x) + Sup\ B_C(x) + Sup\ B_I(x) + Sup\ B_F(x) \leq 4$.

Definition – 1.9[7]

Let X be a non- empty fixed set. An interval quadripartitioned neutrosophic set B in X is defined by B = $\{(x: B_T(x),\ B_C(x),\ B_I(x), B_F(x)\ /x \in X)\}$ where $B_T,\ B_C,\ B_I, B_F \subseteq [0,1]$ are degrees of membership functions of truth, contradiction, ignorance, falsity membership functions respectively and $B_T(x)$ = [inf $B_T(x)$, sup $B_T(x)$], $B_C(x)$ = [inf $B_C(x)$, sup $B_C(x)$],
$B_I(x)$ = [inf $B_I(x)$, sup $B_I(x)$], $B_F(x)$ = [inf $B_F(x)$, sup $B_F(x)$] and
$0 \leq Sup\ B_T(x) + Sup\ B_C(x) + Sup\ B_I(x) + Sup\ B_F(x) \leq 4$.

3. Quadripartitioned neutrosophic cubic sets

Definition - 3.1

Let X be a non- empty set. A **quadripartitioned neutrosophic cubic set** (QNCS) in X is a pair $\mathfrak{B} = (B, \Lambda)$ where B = $\{< x : B_T(x),\ B_C(x),\ B_I(x), B_F(x) >/x \in X\}$ is a quadripartitioned interval neutrosophic set in X, where $B_T,\ B_C,\ B_I, B_F$ are the degrees of truth, contradiction, ignorance, falsity membership functions respectively and
$\Lambda = \{< x : \Lambda_T(x),\ \Lambda_C(x), \Lambda_I(x), \Lambda_F(x) >/x \in X\}$ is a quadripartitioned neutrosophic set in X, $\Lambda_T(x),\ \Lambda_C(x), \Lambda_I(x),\ \Lambda_F(x)$ are the degrees of truth, contradiction, ignorance, falsity membership functions respectively.

Example - 3.1

For X = {a,b,c}, the pair $\mathfrak{B} = (B, \Lambda)$ is represented by the Table 1 is a quadripartitioned neutrosophic cubic set in X.

Table 1. Table representation of $\mathfrak{B} = (B, \Lambda)$

X	B(X)	Λ(X)
a	([0.3,0.4], [0.5,0.6], [0.7,0.8], [0.1,0.9])	(0.2,0.3,0.5, 0.6)
b	([0.2,0.3], [0.1,0.7], [0.3,0.5], [0.6,0.7])	(0.1,0.4,0.2, 0.5)
c	([0.1,0.2], [0.5,0.6], [0.7,0.9], [0.4,0.8])	(0.7,0.3,0.6,0.9)

Example – 3.2

For a non- empty set X, we know that $\mathfrak{C} = (C, \lambda)_1$ = (B, Λ_1) and
$\mathfrak{C} = (C, \lambda)_0$ = (B, Λ_0) are quadripartitioned neutrosophic cubic set in X where
$\Lambda_1 = \{< x; 1,1,1,1 > / x\in X\}$ and $\Lambda_0 = \{< x; 0,0,0,0 > / x\in X\}$.
Let $\lambda_T(x) = \frac{A_T^-(x)+A_T^+(x)}{2}$, $\lambda_C(x) = \frac{A_C^-(x)+A_C^+(x)}{2}$, $\lambda_I(x) = \frac{A_I^-(x)+A_I^+(x)}{2}$, and
$\lambda_F(x) = \frac{A_F^-(x)+A_F^+(x)}{2}$, then $\mathfrak{B} = (B, \Lambda)$ is a quadripartitioned neutrosophic cubic set in X.

Definition – 3.2

A quadripartitioned neutrosophic cubic set in X is a pair of $\mathfrak{B} = (B, \Lambda)$, is said to be a empty set(null set), it is denoted by $\hat{0}$ iff Inf B_T(x) = sup B_T(x) = 0,
Inf B_C(x) = sup B_C(x) = 0, Inf B_I(x) = sup B_I(x) = 1, Inf B_F(x) = sup B_F(x) = 1.

$\hat{0}$= {[0,0], [0,0], [1,1], [1,1]} and Λ = (0,0,1,1).

Definition – 3.3

A quadripartitioned neutrosophic cubic set in X is a pair of $\mathfrak{B} = (B, \Lambda)$, is said to be an unity, it is denoted by $\hat{1}$ iff Inf B_T(x) = sup B_T(x) = 1,
Inf B_C(x) = Sup B_C(x) = 0, Inf B_I(x) = Sup B_I(x) = 1, Inf B_F(x) = Sup B_F(x) = 1.
$\hat{1}$= {[1,1], [1,1], [0,0], [0,0]} and Λ = (1,1,0,0).

Definition – 3.4

Let X be a non- empty set. Let $A, B \in \mathfrak{B}$ where A, B be the two quadripartitioned neutrosophic cubic set in X. If A contained in B is denoted by A⊆ B iff,
for any x ∈ X,

Inf B_{T_A}(x) ≤ Inf B_{T_B}(x), Sup B_{T_A}(x) ≤ Sup B_{T_B}(x),

Inf B_{C_A}(x) ≤ Inf B_{C_B}(x), Sup B_{C_A}(x) ≤ Sup B_{C_B}(x),

Inf B_{I_A}(x) ≥ Inf B_{I_B}(x), Sup B_{I_A}(x) ≥ Sup B_{I_B}(x),

Inf B_{F_A}(x) ≥ Inf B_{F_A}(x), Sup B_{F_A}(x) ≥ Sup B_{F_B}(x).

If Ψ contained in $\mathit{\Lambda}$ is denoted by Ψ ⊆ $\mathit{\Lambda}$ iff, for any x ∈ X,

Ψ_{T_A}(x) ≤ $\mathit{\Lambda}_{T_B}$(x)

Ψ_{C_A}(x) ≤ $\mathit{\Lambda}_{C_B}$(x)

Ψ_{I_A}(x) ≥ $\mathit{\Lambda}_{I_B}$(x)

Ψ_{F_A}(x) ≥ $\mathit{\Lambda}_{F_A}$(x)

Definition – 3.5

Let X be a non- empty set, X is a pair of $\mathfrak{B} = (B, \Lambda)$. Let B = $\{< x : B_T(x), B_C(x), B_I(x), B_F(x) > x \in X\}$ and $\mathit{\Lambda} = \{< x : \Lambda_T(x), \Lambda_C(x), \Lambda_I(x), \Lambda_F(x) > x \in X\}$ be a quadripartitioned neutrosophic cubic set. The complement of B is denoted by B^C is defined as: $B_T(x)$=$B_F(x)$, $B_C(x) = B_I(x)$, $B_I(x) = B_C(x)$, $B_F(x)$ =$B_T(x)$,

$B^C = \{< x : B_F(x), B_I(x), B_C(x), B_T(x) > x \in X\}$. Likewise,

$\Lambda^C = \{< x : \Lambda_F(x), \Lambda_I(x), \Lambda_C(x), \Lambda_T(x) > x \in X\}$.

Example – 3.3

Let A be a quadripartitioned neutrosophic cubic set of the form,
A = {[0.3, 0.45], [0.6,0.7], [0.55,0.9], [0.22,0.44]} then,
A^C= {[0.22, 0.44], [0.55,0.9], [0.6,0.7], [0.3,0.45]}.
Ψ = {0.4, 0.8, 0.3, 0.1}
Ψ^C= {0.1, 0.3, 0.8, 0.4}

Definition – 3.6

Let X be a non-empty set, X is a pair of $\mathfrak{B} = (A, \Psi), (B, \Lambda)$. Let $A, B \in \mathfrak{B}$ and $\Psi, \Lambda \in \mathfrak{B}$ where A,B, Ψ, Λ be the quadripartitioned neutrosophic cubic set in X. If union of A, B is denoted by A∪B and the union of Ψ, Λ is denoted by Ψ∪Λ iff, for any x ∈ X.
Let C = A∪B,

Let A = $\{< x : A_T(x), A_C(x), A_I(x), A_F(x) > x \in X\}$
Let B = $\{< x : B_T(x), B_C(x), B_I(x), B_F(x) > x \in X\}$

C = {(x, ([max (inf ($A_T(x)$, inf $B_T(x)$), max (sup($A_T(x)$, sup$B_T(x)$)],
[max (inf ($A_C(x)$, inf $B_C(x)$), max (sup($A_C(x)$, sup$B_C(x)$)],
[min (inf ($A_I(x)$, inf $B_I(x)$), min (sup($A_I(x)$, sup$B_I(x)$)],
[min (inf ($A_F(x)$, inf $B_F(x)$), min (sup($A_F(x)$, sup$B_F(x)$)]) : $x \in X$}

Let D = Ψ∪Λ

Let $\Psi = \{< x : \Psi_T(x), \Psi_C(x), \Psi_I(x), \Psi_F(x) > x \in X\}$

Let $\Lambda = \{< x : \Lambda_T(x), \Lambda_C(x), \Lambda_I(x), \Lambda_F(x) > x \in X\}$

$D = \{(x, (\max(\Psi_T(x), \Lambda_T(x))$
$\max(\Psi_C(x), \Lambda_C(x))$
$\min(\Psi_I(x), \Lambda_I(x))$
$\min(\Psi_F(x), \Lambda_F(x)) : x \in X\}$

Example – 3.4

Let X be a non- empty set, X is a pair of $\mathfrak{B} = (A, \Psi), (B, \Lambda)$. Let A, B ∈ $\mathfrak{B}$ where A, B be the two quadripartitioned neutrosophic cubic set in X.

A = {[0.6,0.8], [0.5,0.7], [0.25,0.3], [0.55,0.65]}
B = {[0.4,0.5],[0.4,0.5], [0.11,0.25], [0.65,0.85]}
C = A∪B = {[0.6,0.8], [0.5,0.7], [0.11,0.25], [0.55,0.65]}

Ψ = {0.4, 0.1, 0.2, 0.4}
Λ = {0.9, 0.2, 0.5, 0.1}
D = Ψ∪Λ = {0.9, 0.2, 0.2, 0.1}

Definition – 3.7

Let X be a non- empty set. Let A, B ∈ $\mathfrak{B}$ where A, B be the two quadripartitioned neutrosophic cubic set in X. If intersection of A, B is denoted by A∩B iff, for any x ∈ X,

Let C = A∩B,

Let $A = \{< x : A_T(x), A_C(x), A_I(x), A_F(x) > x \in X\}$

Let $B = \{< x : B_T(x), B_C(x), B_I(x), B_F(x) > x \in X\}$

$C = \{(x, ([\min(\inf(A_T(x), \inf B_T(x)), \min(\sup(A_T(x), \sup B_T(x))],$
$[\min(\inf(A_C(x), \inf B_C(x)), \min(\sup(A_C(x), \sup B_C(x))],$
$[\max(\inf(A_I(x), \inf B_I(x)), \max(\sup(A_I(x), \sup B_I(x))],$
$[\max(\inf(A_F(x), \inf B_F(x)), \max(\sup(A_F(x), \sup B_F(x))]) : x \in X\}$

Let D = Ψ∩Λ

Let $\Psi = \{< x : \Psi_T(x), \Psi_C(x), \Psi_I(x), \Psi_F(x) > x \in X\}$

Let $\Lambda = \{< x : \Lambda_T(x), \Lambda_C(x), \Lambda_I(x), \Lambda_F(x) > x \in X\}$

$D = \{(x, (\min(\Psi_T(x), \Lambda_T(x))$
$\min(\Psi_C(x), \Lambda_C(x))$
$\max(\Psi_I(x), \Lambda_I(x))$
$\max(\Psi_F(x), \Lambda_F(x)): x \in X\}$

Example – 3.5

Let X be a non- empty set. Let A, B ∈ $\mathfrak{B}$ where A, B be the two quadripartitioned neutrosophic cubic set in X.

A = {[0.5,0.8], [0.5,0.6], [0.25,0.35], [0.65,0.85]}
B = {[0.4,0.6],[0.2,0.5], [0.11,0.2], [0.4,0.75]}
C = A∩B = {[0.4,0.6], [0.2,0.5], [0.25,0.35], [0.65,0.85]}
Ψ = {0.2, 0.1, 0.3, 0.4}
Λ = {0.7, 0.8, 0.5, 0.1}
D = Ψ∩Λ = {0.2, 0.1, 0.5, 0.4}

Definition – 3.8

Let X be a non- empty set. Quadripartitioned neutrosophic cubic set $\mathfrak{B} = (B, \Lambda)$ in X is said to be,

- Truth- internal (T- internal), It is defined as,
  $((\forall x \in X)(B_T^-(x) \leq \lambda_T(x) \leq B_T^+(x)))$ (3.1)
- Contradiction- internal (C- internal), It is defined as ,
  $((\forall x \in X)(B_C^-(x) \leq \lambda_C(x) \leq B_C^+(x)))$ (3.2)
- Ignorance- internal (I- internal), It is defined as ,
  $((\forall x \in X)(B_I^-(x) \leq \lambda_I(x) \leq B_I^+(x)))$ (3.3)
- False- internal (F- internal), It is defined as ,

$$((\forall x \in X)(B_F^- (x) \leq \lambda_F(x) \leq B_F^+ (x)) \quad (3.4)$$

The above internals are satisfies the quadripartitioned neutrosophic cubic set in X .

Example - 3.6

For X = { a,b,c}, the pair $\mathfrak{B} = (B, \Lambda)$ with the tabular representation in Table 2 is an internal quadripartitioned neutrosophic cubic set in X .

Table 2. Tabular representation of $\mathfrak{B} = (B, \Lambda)$

X	B(X)	Λ(X)
a	([0.2,0.4], [0.5,0.7], [0.1,0.8], [0.1,0.3])	(0.30, 0.60, 0.45, 0.2)
b	([0.2,0.6], [0.3,0.7], [0.4,0.5], [0.6,0.9])	(0.40, 0.50, 0.45, 0.75)
c	([0.1,0.6], [0.5,0.7], [0.4,0.9], [0.4,0.8])	(0.35, 0.6, 0.65, 0.60)

Definition – 3.9

Let X be a non- empty set. Quadripartitioned neutrosophic cubic set $\mathfrak{B} = (B, \Lambda)$ in X is said to be,

- Truth- external (T- external), It is defined as ,
  $$((\forall x \in X) \ (\lambda_T(x) \notin (B_T^- (x), \ B_T^+(x)) \quad (3.5)$$
- Contradiction- external (C- external)
  It is defined as,
  $$((\forall x \in X) \ (\lambda_C (x) \notin (B_C^- (x), \ B_C^+(x)) \quad (3.6)$$
- Ignorance- external (I- external)
  It is defined as,
  $$((\forall x \in X) \ (\lambda_I (x) \notin (B_I^- (x), \ B_I^+(x)) \quad (3.7)$$
- False- external (F- external)
  It is defined as,
  $$((\forall x \in X) \ (\lambda_F(x) \notin (B_F^- (x), \ B_F^+(x)) \quad (3.8)$$

The above internals are satisfies the quadripartitioned neutrosophic cubic set in X .

Proposition- 3.1

Let $\mathfrak{B} = (B, \Lambda)$ be a quadripartitioned neutrosophic cubic set in a non- empty set X. which is not external. Then there exist $x \in X$ such that $\lambda_T(x) \in \{B_T^-(x) , B_T^+(x)\}$ or $\lambda_C(x) \in \{B_C^-(x) , B_C^+(x)\}$ or $\lambda_I(x) \in \{B_I^-(x) , B_I^+(x)\}$ or $\lambda_F(x) \in \{B_F^-(x) , B_F^+(x)\}$.

Proof :

The conditions (3.5), (3.6), (3.7), (3.8) are false then directly true for the conditions (3.1), (3.2), (3.3), (3.4) are in quadripartitioned neutrosophic cubic set in X.

Proposition- 3.2

Let $\mathfrak{B} = (B, \Lambda)$ be a quadripartitioned neutrosophic cubic set in a non-empty set X. If $\mathfrak{B} = (B, \Lambda)$ is both T-internal and T-external, then

$$(\forall x \in X) \ (\lambda_T(x) \in \{B_T^-(x)/x \in X\} \cup \{B_T^+(x)/x \in X\} \quad (3.9)$$

Proof :

Two conditions (3.1) and (3.5) imply that $B_T^- (x) \leq \lambda_T(x) \leq B_T^+ (x)$ and $\lambda_T(x) \notin (B_T^- (x), \ B_T^+(x))$ for all x$\in X$. It follows that $\lambda_T(x) = B_T^-(x)$ or $\lambda_T(x) = B_T^+(x)$, and so that $(\lambda_T(x) \in \{B_T^-(x) / x\in X\} \cup \{B_T^+(x) / x\in X\}$.

Proposition- 3.3

Let $\mathfrak{B} = (B, \Lambda)$ be a quadripartitioned neutrosophic cubic set in a non- empty set X. If $\mathfrak{B} = (B, \Lambda)$ is both C-internal and C-external, then

$$(\forall x \in X) (\lambda_C(x) \in \{B_C^-(x) / x\in X\} \cup \{B_C^+(x) / x\in X\} \quad (4.0)$$

Proof :

Two conditions (3.2) and (3.6) imply that $B_C^-(x) \leq \lambda_C(x) \leq B_C^+(x)$ and $\lambda_C(x) \notin (B_C^-(x), B_C^+(x)$ for all $x\in X$. It follows that $\lambda_C(x) = B_C^-(x)$ or $\lambda_C(x) = B_C^+(x)$, and so that $(\lambda_C(x) \in \{B_C^-(x) / x\in X\} \cup \{B_C^+(x) / x\in X\}$.

Proposition- 3.4

Let $\mathfrak{B} = (B, \Lambda)$ be a quadripartitioned neutrosophic cubic set in a non- empty set X. If $\mathfrak{B} = (B, \Lambda)$ is both I-internal and I-external, then

$$(\forall x \in X) (\lambda_I(x) \in \{B_I^-(x) / x\in X\} \cup \{B_I^+(x) / x\in X\} \quad (4.1)$$

Proof :

Two conditions (3.3) and (3.7) imply that $B_I^-(x) \leq \lambda_I(x) \leq B_I^+(x)$ and $\lambda_I(x) \notin (B_I^-(x), B_I^+(x)$ for all $x\in X$. It follows that $\lambda_I(x) = B_I^-(x)$ or $\lambda_I(x) = B_I^+(x)$, and so that $(\lambda_I(x) \in \{B_I^-(x) / x\in X\} \cup \{B_I^+(x) / x\in X\}$.

Proposition- 3.5

Let $\mathfrak{B} = (B, \Lambda)$ be a quadripartitioned neutrosophic cubic set in a non- empty set X. If $\mathfrak{B} = (B, \Lambda)$ is both F-internal and F-external, then

$$(\forall x \in X) (\lambda_F(x) \in \{B_F^-(x) / x\in X\} \cup \{B_F^+(x) / x\in X\} \quad (4.2)$$

Proof :

Two conditions (3.1) and (3.5) imply that $B_F^-(x) \leq \lambda_F(x) \leq B_F^+(x)$ and $\lambda_F(x) \notin (B_F^-(x), B_F^+(x)$ for all $x\in X$. It follows that $\lambda_F(x) = B_F^-(x)$ or $\lambda_F(x) = B_F^+(x)$, and so that $(\lambda_F(x) \in \{B_F^-(x) / x\in X\} \cup \{B_F^+(x) / x\in X\}$.

Definition – 3.10

Let X be a non- empty set. $\mathcal{A}, \mathfrak{B}$ Let $\mathcal{A} = (A, \Psi)$, and $\mathfrak{B} = (B, \Lambda)$ be quadripartitioned neutrosophic cubic set in X,

$A = \{< x : A_T(x), A_C(x), A_I(x), A_F(x) > x \in X\}$
$\Psi = \{< x : \Psi_T(x), \Psi_C(x), \Psi_I(x), \Psi_F(x) > x \in X\}$
$B = \{< x : B_T(x), B_C(x), B_I(x), B_F(x) > x \in X\}$
$\Lambda = \{< x : \lambda_T(x), \lambda_C(x), \lambda_I(x), \lambda_F(x) > x \in X\}$

Then we define equality P- order and R- order as follows

(a) (Equality) $\mathcal{A} = \mathfrak{B} \Leftrightarrow A = B$ and $\Psi = \Lambda$
(b) (P-order) $\mathcal{A} \subseteq_P \mathfrak{B} \Leftrightarrow A \subseteq B$ and $\Psi \leq \Lambda$
(c) (R-order) $\mathcal{A} \subseteq_R \mathfrak{B} \Leftrightarrow A \subseteq B$ and $\Psi \geq \Lambda$

Definition – 3.11

Let X is a non- empty set. Quadripartitioned neutrosophic cubic non- empty sets in X $\mathcal{A}_i = (A_i, \Psi_i)$ is defined as,

$A_i = \{< x : A_{iT}(x), A_{iC}(x), A_{iI}(x), A_{iF}(x) > x \in X\}$
$\Lambda_i = \{< x : \lambda_{iT}(x), \lambda_{iC}(x), \lambda_{iI}(x), \lambda_{iF}(x) > x \in X\}$

for i ϵ J and J is any index set, we define

(a) $\bigcup_{P, i\in J} \mathcal{A}_i = (\bigcup_{i\in J} A_i, \bigvee_{i\in J} \Psi_i)$ (P- union)

(b) $\bigcap_{P, i\in J} \mathcal{A}_i = (\bigcap_{i\in J} A_i, \bigwedge_{i\in J} \Psi_i)$ (P- intersection)

(c) $\bigcup_{R, i\in J} \mathcal{A}_i = (\bigcup_{i\in J} A_i, \bigwedge_{i\in J} \Psi_i)$ (R- union)

(d) $\bigcap_{P, i\in J} \mathcal{A}_i = (\bigcap_{i\in J} A_i, \bigvee_{i\in J} \Psi_i)$ (R- intersection)

Where

$$\bigcup_{i\in J} A_i = \{< x: (\bigcup_{i\in J} A_{iT})(x), (\bigcup_{i\in J} A_{iC})(x), (\bigcup_{i\in J} A_{iI})(x), (\bigcup_{i\in J} A_{iF})(x) > x \in X\},$$

$$\bigvee_{i\in J} \Lambda_i = \{< x: (\bigvee_{i\in J} \lambda_{iT})(x), (\bigvee_{i\in J} \lambda_{iC})(x), (\bigvee_{i\in J} \lambda_{iI})(x), (\bigvee_{i\in J} \lambda_{iF})(x) > x \in X\},$$

$$\bigcap_{i\in J} A_i = < x: (\bigcap_{i\in J} A_{iT})(x), (\bigcap_{i\in J} A_{iC})(x), (\bigcap_{i\in J} A_{iI})(x), (\bigcap_{i\in J} A_{iF})(x) > x \in X\},$$

$$\bigwedge_{i\in J} \Lambda_i = \{< x: (\bigwedge_{i\in J} \lambda_{iT})(x), (\bigwedge_{i\in J} \lambda_{iC})(x), (\bigwedge_{i\in J} \lambda_{iI})(x), (\bigwedge_{i\in J} \lambda_{iF})(x) > x \in X\}.$$

Theorem - 3.1

Let $\mathfrak{B}$ = (B, Λ) is a quadripartitioned neutrosophic cubic set in a non- empty set X. If $\mathfrak{B}$ = (B, Λ) is T- internal (resp. T- external), then the complement $\mathfrak{B}^C = (B^C, \Lambda^C)$ of $\mathfrak{B}$ = (B, Λ) is an T- internal (resp. T- external) quadripartitioned neutrosophic cubic set in X.

Proof :

Let X be a non- empty set. If $\mathfrak{B}$ = (B, Λ) is an T- internal (resp. T- external) quadripartitioed neutrosophic cubic set in X, then $B_T^- \le \lambda_T(x) \le B_T^+$ (resp, $\lambda_T(x) \notin (B_T^-(x) \le B_T^+(x))\ \forall\ x \in X$. It follows that $1 - B_T^-(x) \le 1 - \lambda_T(x) \le 1 - B_T^+(x)$ (resp, $1 - \lambda_T(x) \notin 1 - B_T^-(x),\ 1 - B_T^+(x)$). Therefore $\mathfrak{B}^C = (B^C, \Lambda^C)$ is a T- internal (resp.T- internal) quadripartitioned neutrosophic cubic set in X.

Similarly we have the following theorems.

Theorem - 3.2

Let $\mathfrak{B}$ = (B, Λ) is a quadripartitioned neutrosophic cubic set in a non- empty set X. If $\mathfrak{B}$ = (B, Λ) is C- internal (resp. C- external), then the complement $\mathfrak{B}^C = (B^C, \Lambda^C)$ of $\mathfrak{B}$ = (B, Λ) is an C- internal (resp.C- external) quadripartitioned neutrosophic cubic set in X.

Proof :

Let X be a non- empty set. If $\mathfrak{B}$ = (B, Λ) is an T- internal (resp. T- external) quadripartitioed neutrosophic cubic set in X, then $B_C^- \le \lambda_C(x) \le B_C^+$ (resp, $\lambda_C(x) \notin (B_C^- \le B_C^+(x))\ \forall\ x \in X$. It follows that $1 - B_C^-(x) \le 1 - \lambda_C(x) \le 1 - B_C^+(x)$ (resp, $1 - \lambda_C(x) \notin 1 - B_C^-(x),\ 1 - B_C^+(x)$). Therefore $\mathfrak{B}^C = (B^C, \Lambda^C)$ is a C- internal (resp. C - external) quadripartitioned neutrosophic cubic set in X.

Theorem - 3.3

Let $\mathfrak{B}$ = (B, Λ) is a quadripartitioned neutrosophic cubic set in a non- empty set X. If $\mathfrak{B}$ = (B, Λ) is I- internal (resp. I- external), then the complement $\mathfrak{B}^C = (B^C, \Lambda^C)$ of $\mathfrak{B}$ = (B, Λ) is an I- internal (resp. I- external) quadripartitioned neutrosophic cubic set in X.

Proof :

Let X be a non- empty set. If $\mathfrak{B}$ = (B, Λ) is an I- internal (resp. I- external) quadripartitioed neutrosophic cubic set in X, then $B_T^- \le \lambda_T(x) \le B_T^+$ (resp, $\lambda_I(x) \notin (B_I^- \le B_I^+(x))\ \forall\ x \in X$. It follows that $1 - B_I^-(x) \le 1 - \lambda_I(x) \le 1 - B_I^+(x)$ (resp, $1 - \lambda_I(x) \notin 1 - B_I^-(x),\ 1 - B_I^+(x)$). Therefore $\mathfrak{B}^C = (B^C, \Lambda^C)$ is an I- internal (resp. I- external) quadripartitioned neutrosophic cubic set in X.

Theorem - 3.4

Let $\mathfrak{B}$ = (B, Λ) is a quadripartitioned neutrosophic cubic set in a non- empty set X. If $\mathfrak{B}$ = (B, Λ) is F- internal (resp. F- external), then the complement $\mathfrak{B}^C = (B^C, \Lambda^C)$ of $\mathfrak{B}$ = (B, Λ) is an F- internal (resp.F- external) quadripartitioned neutrosophic cubic set in X.

Proof :

Let X be a non- empty set. If $\mathfrak{B}$ = (B, Λ) is an F - internal (resp. F - external) quadripartitioed neutrosophic cubic set in X, then $B_T^- \leq \lambda_T(x) \leq B_T^+$
(resp, $\lambda_F(x) \notin (B_F^- \leq B_F^+ (x)) \forall x \in X$. It follows that
$1 - B_F^- (x) \leq 1 - \lambda_F(x) \leq 1 - B_F^+(x)$ (resp, $1 - \lambda_F(x) \notin 1 - B_F^- (x), 1 - B_F^+(x)$). Therefore $\mathfrak{B}^C = (B^C, \Lambda^C)$ is an F - internal (resp. F - internal) quadripartitioned neutrosophic cubic set in X.

Corollary - 3.1

Let $\mathfrak{B}$ = (B, Λ) is a quadripartitioned neutrosophic cubic set in a non- empty set X.
If $\mathfrak{B}$ = (B, Λ) is an internal (resp. external), then the complement $\mathfrak{B}^C = (B^C, \Lambda^C)$ of
$\mathfrak{B}$ = (B, Λ) is an internal (resp. external) quadripartitioned neutrosophic cubic set in X.

Theorem - 3.5

Let $\{\mathfrak{B}_i = (B_i, \Lambda_i)$ / i∈J) is a family of T- internal quadripartitioned neutrosophic cubic set in a non- empty set in X, then the P- union and the P- intersection of $\{\mathfrak{B}_i = (B_i, \Lambda_i)$ / i∈J) are T- internal quadripartitioned neutrosophic cubic set in X.

Proof :

Let $\mathfrak{B}_i = (B_i, \Lambda_i)$ is an T- internal quadripartitioned neutrosophic cubic set in a non-empty set X, we have $B_{iT}^-(x) \leq \lambda_{iT} \leq B_{iT}^+(x)$ for i∈J. It follows that

$$(\bigcup_{i\in J} B_{iT})^- (x) \leq (\bigvee_{i\in J} \Lambda_{iT})(x) \leq (\bigcup_{i\in J} B_{iT})^+ (x) \quad \text{and}$$

$$(\bigcap_{i\in J} B_{iT})^- (x) \leq (\bigwedge_{i\in J} \Lambda_{iT})(x) \leq (\bigcap_{i\in J} B_{iT})^+ (x)$$

Therefore $\bigcup_{P\, i\in J} \mathfrak{B}_i = (\bigcup_{i\in J} A_i, \bigvee_{i\in J} \wedge_i)$ and $\bigcap_{P\, i\in J} \mathfrak{B}_i = (\bigcap_{i\in J} A_i, \bigwedge_{i\in J} \wedge_i)$ are T- internal

quadripartitioned neutrosophic cubic set in X.

Similarly we have the following theorems.

Theorem - 3.6

Let if $\{\mathfrak{B}_i = (B_i, \Lambda_i)$ / i∈J) is a C- internal quadripartitioned neutrosophic cubic set in a non- empty set in X, then the P- union and the P- intersection of $\{\mathfrak{B}_i = (B_i, \Lambda_i)$ / i∈J) are
C- internal quadripartitioned neutrosophic cubic set in X.

Proof :

Let $\mathfrak{B}_i = (B_i, \Lambda_i)$ is an C- internal quadripartitioned neutrosophic cubic set in a non-empty set X, we have $B_{iC}^-(x) \leq \lambda_{iC} \leq B_{iC}^+(x)$ for i∈J. It follows that

$$(\bigcup_{i\in J} B_{iC})^- (x) \leq (\bigvee_{i\in J} \Lambda_{iC})(x) \leq (\bigcup_{i\in J} B_{iC})^+ (x) \quad \text{and}$$

$$(\bigcap_{i\in J} B_{iC})^- (x) \leq (\bigwedge_{i\in J} \Lambda_{iC})(x) \leq (\bigcap_{i\in J} B_{iC})^+ (x)$$

Therefore $\bigcup_{P\, i\in J} \mathfrak{B}_i = (\bigcup_{i\in J} A_i, \bigvee_{i\in J} \wedge_i)$ and $\bigcap_{P} \mathfrak{B}_i = (\bigcap_{i\in J} A_i, \bigwedge_{i\in J} \wedge_i)$ are C- internal

quadripartitioned neutrosophic cubic set in X.

Theorem - 3.7

Let if $\{\mathfrak{B}_i = (B_i, \Lambda_i)$ / i∈J) is a I- internal quadripartitioned neutrosophic cubic set in a non- empty set in X, then the P- union and the P- intersection of $\{\mathfrak{B}_i = (B_i, \Lambda_i)$ / i∈J) are
I- internal quadripartitioned neutrosophic cubic set in X.

Proof :

Let $\mathfrak{B}_i = (B_i, \Lambda_i)$ is an I- internal quadripartitioned neutrosophic cubic set in a non-empty set X, we have $B_{iI}^-(x) \leq \lambda_{iI} \leq B_{iI}^+(x)$ for i∈J. It follows that

$$(\bigcup_{i\in J} B_{iI})^-(x) \leq \bigvee_{i\in J} \Lambda_{iI}(x) \leq (\bigcup_{i\in J} B_{iI})^+(x) \quad \text{and}$$

$$(\bigcap_{i\in J} B_{iI})^-(x) \leq \bigwedge_{i\in J} \Lambda_{iI}(x) \leq (\bigcap_{i\in J} B_{iI})^+(x)$$

Therefore $\bigcup_{P, i\in J} \mathfrak{B}_i = (\bigcup_{i\in J} A_i, \bigvee_{i\in J} \Lambda_i)$ and $\bigcap_{P, i\in J} \mathfrak{B}_i = (\bigcap_{i\in J} A_i, \bigwedge_{i\in J} \Lambda_i)$ are I- internal quadripartitioned neutrosophic cubic set in X.

Theorem - 3.8

Let if $\{\mathfrak{B}_i = (B_i, \Lambda_i)$ / i∈J) is a F- internal quadripartitioned neutrosophic cubic set in a non- empty set in X, then the P- union and the P- intersection of $\{\mathfrak{B}_i = (B_i, \Lambda_i)$ / i∈J) are F- internal quadripartitioned neutrosophic cubic set in X.

Proof :

Let $\mathfrak{B}_i = (B_i, \Lambda_i)$ is an F- internal quadripartitioned neutrosophic cubic set in a non-empty set X, we have $B_{iF}^-(x) \leq \lambda_{iF} \leq B_{iF}^+(x)$ for i∈J. It follows that

$$(\bigcup_{i\in J} B_{iF})^-(x) \leq (\bigvee_{i\in J} \Lambda_{iF})(x) \leq (\bigcup_{i\in J} B_{iF})^+(x) \quad \text{and}$$

$$(\bigcap_{i\in J} B_{iF})^-(x) \leq (\bigwedge_{i\in J} \Lambda_{iF})(x) \leq (\bigcap_{i\in J} B_{iF})^+(x)$$

Therefore $\bigcup_{P, i\in J} \mathfrak{B}_i = (\bigcup_{i\in J} A_i, \bigvee_{i\in J} \Lambda_i)$ and $\bigcap_{P, i\in J} \mathfrak{B}_i = (\bigcap_{i\in J} A_i, \bigwedge_{i\in J} \Lambda_i)$ are F- internal quadripartitioned neutrosophic cubic set in X.

Corollary - 3.2

Let if $\{\mathfrak{B}_i = (B_i, \Lambda_i)$ / i∈J) is a family of internal quadripartitioned neutrosophic cubic sets in a non-empty set in X, then the P- union and the P- intersection of $\{\mathfrak{B}_i = (B_i, \Lambda_i)$ / i∈J) are T- internal quadripartitioned neutrosophic cubic set in X.

Example - 3.7

Let $\mathcal{A} = (A, \Psi)$ and $\mathfrak{B} = (B, \Lambda)$ be a quadripartitioned neutrosophic cubic sets in [0,1] where A = {< x; [0.3,0.5], [0.4,0.6], [0.2,0.7], [0.6,0.9] > / x∈ [0,1]}

ψ = {< x; 0.2,0.3, 0.5,0.8 > / x∈ [0,1]}

B = {< x; [0.4,0.5], [0.2,0.3], [0.6,0.7], [0.7,0.8] > / x∈ [0,1]}

Λ = {< x; 0.1,0.5, 0.6,0.7 > / x∈ [0,1]}

Then $\mathcal{A} = (A, \Psi), \mathfrak{B} = (B, \Lambda)$ are F-external quadripartitioned neutrosophic cubic sets in [0,1], and $\mathcal{A}\ U_P\ \mathfrak{B}$ = (A ∩ B, ψ ∨ Λ) with

A∪B = {< x; [0.4,0.5], [0.4,0.6], [0.2,0.7], [0.6,0.9] > / x∈ [0,1]}

Ψ∨Λ = {< x; 0.2, 0.5, 0.5,0.7 > / x∈ [0,1]}

is not an F-external quadripartitioned neutrosophic cubic sets in [0,1], since

$(\psi_F \vee \lambda_F)$ (X) = 0.7 ∈ (0.6,0.9) = $((A_F \cup B_F)^-(x), (A_F \cup B_F)^+(x))$.

Also $\mathcal{A}\ \cap_P \mathfrak{B}$ = (A∪B, Ψ∨Λ) with

A∩B = {< x; [0.3,0.5], [0.2,0.3], [0.6,0.7], [0.6,0.9]> / x∈ [0,1]}

Ψ∧Λ = {< x; 0.1, 0.3, 0.6,0.8 > / x∈ [0,1]}

is not an F-external quadripartitioned neutrosophic cubic sets in [0,1], since

$(\psi_F \wedge \lambda_F)$ (X) = 0.8 ∈ (0.6,0.9) = $((A_F \cup B_F)^-(x), (A_F \cup B_F)^+(x))$.

Example - 3.8

For X = {a,b,c}, let $\mathcal{A} = (A, \Psi) and\ \mathfrak{B} = (B, \Lambda)$ be a quadripartitioned neutrosophic cubic sets in X. The table 2 and table 3 represents the $\mathcal{A} = (A, \Psi)\ and\ \mathfrak{B} = (B, \Lambda)$ respectively.

Table 2. Table representation of $\mathcal{A} = (A, \Psi)$

X	A(x)	Ψ(x)
a	([0.3,0.4], [0.2,0.6], [0.3,0.5], [0.1,0.3])	(0.25,0.35,0.15, 0.50)
b	([0.6,0.7], [0.1,0.3], [0.2,0.3], [0.6,0.7])	(0.45,0.40,0.20,0.25)
c	([0.0,0.2], [0.3,0.6], [0.0,0.1], [0.2,0.4])	(0.10,0.30,0.65,0.85)

Table 3. Table representation of $\mathfrak{B} = (B, \Lambda)$

X	B(x)	Λ (x)
a	([0.3,0.5], [0.2,0.5], [0.3,0.4], [0.1,0.4])	(0.15, 0.25, 0.35, 0.40)
b	([0.5,0.7], [0.2,0.4], [0.1,0.3], [0.2,0.5])	(0.50, 0.30, 0.40, 0.15)
c	([0.0,0.6], [0.2,0.6], [0.4,0.5], [0.2,0.3])	(0.20, 0.55, 0.35, 0.25)

Table 4. Table representation of $\mathcal{A} \cup_P \mathfrak{B} = (A \cup B, \psi \vee \Lambda)$

X	$(A \cup B)(x)$	$(\psi \vee \Lambda)$(x)
a	([0.3,0.5], [0.2,0.6], [0.3,0.4], [0.1,0.3])	(0.25, 0.35, 0.15, 0.40)
b	([0.6,0.7], [0.2,0.4], [0.1,0.3], [0.2,0.5])	(0.50, 0.40, 0.20, 0.15)
c	([0.0,0.6], [0.3,0.6], [0.0,0.1], [0.2,0.3])	(0.20, 0.55, 0.35, 0.25)

Table 5. Table representation of $\mathcal{A} \cap_P \mathfrak{B} = (A \cap B, \psi \wedge \Lambda)$

X	$(A \cap B)(x)$	$(\psi \wedge \Lambda)$(x)
a	([0.3,0.4], [0.2,0.5], [0.3,0.5], [0.1,0.3])	(0.15, 0.25, 0.35, 0.50)
b	([0.5,0.7], [0.1,0.3], [0.2,0.3], [0.6,0.7])	(0.45, 0.30, 0.40, 0.25)
c	([0.0,0.2], [0.2,0.6], [0.4,0.5], [0.2,0.4])	(0.10, 0.30, 0.60, 0.85)

Then $\mathcal{A} = (A, \Psi)$ and $\mathfrak{B} = (B, \Lambda)$ are both T- internal quadripartitioned neutrosophic cubic sets in X.

Table 4 and 5 are represented by $\mathcal{A} \cup_P \mathfrak{B} = (A \cup B, \psi \vee \Lambda)$ and $\mathcal{A} \cap_P \mathfrak{B} = (A \cap B, \psi \wedge \Lambda)$

Then $\mathcal{A} \cup_P \mathfrak{B} = (A \cap B, \psi \wedge \Lambda)$ is neither C-external nor T-external quadripartitioned neutrosophic cubic set in X.

$(\lambda_C \vee \psi_C)$ (a) = 0.35 ∈ (0.3,0.5) = $((A_C \cup B_C)^-(a), (A_C \cup B_C)^+(a))$ and
$(\lambda_T \wedge \psi_T)$ (b) = 0.45 ∈ (0.5,0.7) = $((A_C \cup B_C)^-(b), (A_C \cup B_C)^+(b))$

Example - 3.9

Let $\mathcal{A} = (A, \Psi) and\ \mathfrak{B} = (B, \Lambda)$ be a quadripartitioned neutrosophic cubic sets in [0,1] where A = { < x: [0.2,0.4], [0.3,0.4], [0.3,0.5], [0.4, 0.6] > / x ∈ [0,1]}

Ψ = { < x: 0.1, 0.2, 0.6, 0.5 > / x ∈ [0,1]}

B = { < x: [0.2,0.6], [0.4,0.9], [0.4,0.6] [0.4,0.7] > / x ∈ [0,1]}

Λ = { < x: 0.4, 0.7, 0.5, 0.6 > / x ∈ [0,1]}

Then $\mathcal{A} = (A, \Psi)$ and $\mathfrak{B} = (B, \Lambda)$ are T- internal quadripartitioned neutrosophic cubic sets in [0,1]. The R-union $\mathcal{A} \cup_R \mathfrak{B} = (A \cup B, \psi \wedge \Lambda)$ of $\mathcal{A} = (A, \Psi)$ and $\mathfrak{B} = (B, \Lambda)$ is given below,

A∪ B = { < x: [0.2,0.6], [0.4,0.9], [0.3,0.5] [0.4,0.6] > / x ∈ [0,1]}

$\psi \wedge \Lambda$ = { < x: 0.1, 0.2, 0.6, 0.6 > / x ∈ [0,1]}

$(\lambda_T \wedge \psi_T)$ (x) = 0.1 < 0.2 = $(A_T \cup B_T)^-\ (x) and\ (\lambda_C \wedge \psi_C)$ (x) = 0.2 < 0.3 = $(A_C \cup B_C)^-\ (x)$.
Hence, $\mathcal{A} \cup_R \mathfrak{B} = (A \cup B, \psi \wedge \Lambda)$ is neither a C-internal nor I-internal quadripartitioned neutrosophic cubic sets in [0,1].

$\mathcal{A} \cup_R \mathfrak{B} = (A \cup B, \psi \wedge \Lambda)$ is a T-internal quadripartitioned neutrosophic cubic sets in [0,1].
The R-intersection $\mathcal{A} \cap_R \mathfrak{B} = (A \cap B, \psi \vee \Lambda)$ of $\mathcal{A} = (A, \Psi)$ and $\mathfrak{B} = (B, \Lambda)$ is given below,

A∩B = {< x: [0.2,0.4], [0.3,0.4], [0.4,0.6] [0.4,0.7] > / x ∈ [0,1]}

$\psi \vee \Lambda$ = {< x: 0.4, 0.7, 0.5, 0.5 > / x ∈ [0,1]}

Since, $(A_F \cap B_F)^-$(x) $\leq (\lambda_F \vee \psi_F) \leq (A_F \cap B_F)^+$(x) for all x ∈ [0,1].
$\mathcal{A} \cap_R \mathfrak{B} = (A \cap B, \psi \vee \Lambda)$ is a F-internal quadripartitioned neutrosophic cubic sets in [0,1].
But it is not for T-internal, C-internal, F-internal quadripartitioned neutrosophic cubic sets in [0,1].

References

1) Dogan Coker, An introduction to institutionstic fuzzy topological spaces, fuzzy sets and systems, 88, 81-89 (1997).
2) Y.B. Jun, C.S. Kim and K.O. Yang, cubic sets, Ann. Fuzzy Math. Inform 4(1) (2012) 83-98.
3) Y.B. Jun, F. Smarandache and C.S. Kim, Neutrosophic cubic set, New Mathematics and Natural Computation, 13(1). PP. 41-54,2017, and Natural Computation, 13(1)
41-54, 2017.
4) F. Smarandache, A Unifying Field of logics: Neutrosophic logic. Neutrosophy, Neutrosophic set, Neutrosophic probality. (American Research Press. Rehoboth, NM, 1999).
5) F. Smarandache, Neutrosophic set – a generalization of the institutionstic fuzzy set. Int. J. Pure Appl.Math. 24(3) (2005) 287-297.
6) Suman Das, Rakhal Das and Carlos Granados, Topology on Quadripartioned Neutrosophic sets.
7) Surapati Parmanik, Interval quadripartioned Neutrosophic sets.
8) Turksen, Interval valued fuzzy sets and systems, 20, PP. 191-210(1968).
9) Wang, H., Madiraju, P., Zhang, Y. Q., & Sunderraman, R. (2005). Interval neutrosophic sets. International Journal of Applied Mathematics & Statistics, 3, (5),
1-18.
10) L.A. Zadeh, Fuzzy sets, information and control 8(1965) 338-353.