IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

A New Multi-Criteria Risk Assessment Model Combining GRA Techniques with Hesitant Bipolar Intuitionistic Fuzzy Entropy based TOPSIS method

R. Rajalakshmi[1*], Dr. K. Julia Rose Mary[2]

[1]*Research Scholar, Department of mathematics, Nirmala college for women, Tamil Nadu, India-641018.*
[2]*Associate Professor, Department of mathematics, Nirmala college for women, Tamil Nadu, India-641018.*

ABSTRACT

In this paper, we introduce a new multi criteria decision making method with Hesitant Bipolar Intuitionistic Fuzzy set namely, HBIF-TOPSIS. The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) has shown compromise solution for the various multi criteria problems. In the proposed model, HBIF-TOPSIS is analysed with the modified form of TOPSIS method with the aid of Grey Relational Analysis (GRA) techniques. The Efficiency of the new method is being tested on Environmental pollution problems. In this study, the assessment of the "environmental risks" such as the risks of air pollution due to CO_2 emissions, water pollution, land/soil pollution, noise pollution, light pollution is considered. Here, we weighed these five types of pollutions as alternatives. Further, every alternative is ranked based on the criteria weights. Finally, one of these alternatives is sorted out by using Entropy weights, for selecting the most devastating one. Moreover, Sensitivity Analysis is created to look at the distinction of different ranks.

Keywords: Hesitant bipolar, Hesitant Bipolar Intuitionistic Fuzzy, Entropy, TOPSIS, GRA techniques.

1. Introduction

Everyday life is intertwined with consistent results. The right decisions made at the right time are the most important reason for success. There are many aspects to consider in every decision of a person. Some features are considered directly and others, indirectly. Many of the decisions are with some hesitations. For example, the items for breakfast, selection to wear dress and the choice of vehicle for travel etc., many of the results in the above cases do not have a significant impact on life. Thus, the scientific result emerged as a field of great interest both in education as well as in industry. MCDM allows to take a decision in a complex situation. Bellman and Zadeh [1] worked on fuzzy sets with MCDM which played a key role in influential work.

Bipolar Fuzzy is a set introduced by Zhang [2] and he investigated both positive and negative membership functions. He stated that positive membership in the interval [0, 1] and negative membership in the interval [-1, 0]. Atanossov [3] generalized the concept of intuitionistic fuzzy set. He developed the idea of intuitionistic fuzzy set (IFs) which was the generalization of intuitionistic fuzzy set under the restriction that the sum of its membership and non-membership degrees is less than or equal to 1. Further in the new extensions of fuzzy set theory, Torra [4] studied the concept of hesitant fuzzy sets. Motivated by the above theories many authors contributed their works in our math world. D. Ezhilmaran & K. Sankar [5] developed the Morphism of bipolar intuitionistic graphs and discussed the fuzzy relations and its properties. Muhammad Sajjad et al., [6] promoted the Pythagorean hesitant fuzzy sets (PHFs) and he discussed the distance measure between the Pythagorean hesitant fuzzy numbers. Decui Lianga, Zeshui Xub [7] focused the distance measure of Hesitant Pythagorean fuzzy sets (HPFs) in advance and explored the application HPFs to Multi criteria decision making (MCDM) by employing the new extension of TOPSIS method. Based on that Rajalakshmi & K. Julia Rose Mary [8] proposed new score function, weighted arithmetic operator and weighted geometric operator of hesitant bipolar valued intuitionistic fuzzy information. In this research paper, we extended a new methodology for multi criteria decision making methods with HBIFs.

The MCDM problem is one of the problem-solving Technique. In general, MCDM process has two components of problem-solving Techniques. One is weight finding technique and another one is ranking part. For these two solving parts, there are several methods are used. For example, ENTROPY, Simple additive weighting (SAW), analytic hierarchy process (AHP), Decision Making Trial and Evaluation Laboratory model (DEMATEL), Data envelopment analysis (DEA) for weight finding methods. TOPSIS, Simple Multi Attribute Rating Technique (SMART), Elimination Et Choix Traduisant la REalite (ELECTRE) for rank finding methods (Velasquez and Patrick, [10]).

We now look at some of the applications for hesitant fuzzy set theory in relation to the theme of this research and MCDM. Sorin Borza et al., [9] analysed the level of pollution caused by vehicular traffic in the geographical area and he estimated criteria by AHP process and ranking by TOPSIS method. Ferhat Duran and İlke and Bereketli Zafeirakopoulos [10] discussed the many environmental risk factors in the E-waste field due to reverse logistics process by using COPRAS method. Yigit Kazancoglu et al., [11] calculated the risk to the economy posed by E-waste recycling using the integration method. Hameem Bin Hameed et al., [12] studied the environmental hazards posed by e-waste in developing countries using the modified SIRA method. Hiral J. Jariwala et al., [13] reviewed Noise pollution and their effects of human health. Vlachokostaset et. al. [14] mentioned the foremost polluted cities in Europe, particularly regarding airborne particle pollutants in those cities and natural gas saturation in buildings and subway constructions. Also, Lizhong Tong et al., [15] used fuzzy TOPSIS method to select the best equipment maintenance service supplier in the chemical industry.

After investigating these consequences, we considered to do the research in the above problem. Hence, the authors bring in HBIF-TOPSIS method. The HBIF set is characterized as the sum of its positive membership and non-membership degree is less than or equal to the interval [0,1]. Similarly, sum of its negative membership and non-membership degree is less than or equal to [-1,0].

From the above discussions and the literature, we notice that the environmental risk factors are examined with help of MCDM methods. TOPSIS has its drawbacks because decision makers need to subjectively judge the weighting of various criteria. The criteria weights are calculated by Entropy method. Thus, in this research paper, the calculation of hesitant bipolar intuitionistic fuzzy entropy is considered which is different from the general measurement of entropy weight. GRA is a measure of the nonlinear relationship between sequences and moreover compensate for the deficiencies of TOPSIS method. Based on the above analysis, in our work, Hesitant bipolar intuitionistic fuzzy multi-criteria decision making with entropy-based TOPSIS method along with GRA technique is proposed.

In this research paper in section 2, review of literature related to Entropy, TOPSIS and GRA techniques are studied. In section 3. hesitant bipolar intuitionistic fuzzy entropy based TOPSIS with GRA is proposed. Section .4 deals with a numerical example. In Section .5, we describe the analysis and discussion of the results associated with our approach. Finally, in Section .6 the conclusion of the study is explained.

2. Preliminaries

Definition 1 (Torra [4])

Let S be a set. The hesitant fuzzy set (HFS) S is in terms of a function h, when applied to S returns a subset of [0, 1], which may be expressed as

$$A = (< s, h(s) > s \in S) \tag{1}$$

where h(s) may be a set of some values in [0, 1], denoting the possible membership degrees of the element $s \in S$ to the set A we call h(s) is the hesitant fuzzy element (HFE).

Definition 2 (Zhang [2])

Let V be a reference set. Then a bipolar fuzzy set A on V is defined as,

$$A = \{(v, \theta_b^+(v), \theta_b^-(v))/v \in V\} \tag{2}$$

where $\theta_b^+(v): V \to [0,1]$ and $\theta_b^-(v): V \to [-1,0]$.

Definition 3 (Atanassov [3])

Let U be a universe. Then an Intuitionistic Fuzzy set (IFs), P in U defined as

$$P = \{(u, \omega_p(u), \gamma_p(u))/u \epsilon U\} \tag{3}$$

where $\omega_p(u)$, $\gamma_p(u)$ denote the membership and non-membership in the interval [0,1]. It satisfies the condition $0 \leq \omega_p(u) + \gamma_p(u) \leq 1$, for all $u \in U$.

Definition 7 (Rajalakshmi R and Julia Rose Mary [8])

Let S be a reference set. A Hesitant bipolar-valued Intuitionistic Fuzzy set (HBVIFs) $\widetilde{H}_P$ in S is defined by,

$$\widetilde{H}_P = \{< s, h_M^+(s), h_{NM}^+(s), h_M^-(s), h_{NM}^-(s) > s \in S\} \quad (4)$$

where $h_M^+(s), h_{NM}^+(s): S \to [0,1]$ and $h_M^-(s), h_{NM}^-(s): S \to [-1,0]$. The positive Hesitant bipolar-valued Intuitionistic Fuzzy elements h_M^+ and h_{NM}^+ denote the possible satisfactory degree of membership and non-membership of an element $s \in S$ corresponding to a HBVIFs $\widetilde{H}_P$ respectively. While the negative hesitant bipolar-valued Intuitionistic Fuzzy elements h_M^- and h_{NM}^- denote the possible satisfactory degree of membership and non-membership of an element $s \in S$ to the implicit counter property to the set $\widetilde{H}_P$ respectively. In addition, a HBVIFs $\widetilde{H}_P$ must satisfy the conditions $0 \le h_M^+, h_{NM}^+ \le 1$, $-1 \le h_M^-, h_{NM}^- \le 0$, $0 \le h_M^+ + h_{NM}^+ \le 1$ and $-1 \le h_M^- + h_{NM}^- \le 0$. For definition (6) an example is illustrated is illustrated below.

Example 2

Let S=$\{s_1, s_2, s_3\}$ be the set. Then

$$\widetilde{H}_P = \begin{Bmatrix} < s_1, \{0.3, 0.2\}, 0.1, -0.3, \{-0.1, -0.5\} >, \\ < s_2, 0.5, \{0.2, 0.8\}, -0.7, -0.1 >, \\ < s_3, 0.5, 0.2, -0.6, \{-0.5, -0.7\} > \end{Bmatrix} \quad (5)$$

is a hesitant bipolar-valued Intuitionistic Fuzzy elements in S.

3. Main results

Proposed Hesitant Bipolar Intuitionistic Fuzzy ENTOPY based TOPSIS with GRA Techniques

In this proposed method, m are alternatives $Q_i(i = 1,2, \dots m)$ and n are selection criteria $P_j(j = 1,2, \dots n)$. the process of Hesitant bipolar Intuitionistic fuzzy Entropy based TOPSIS method with GRA techniques is then described below,

Step 1: Construct the Hesitant Bipolar Intuitionistic fuzzy decision matrix

$\breve{H} = \breve{h}_{ij} = [\breve{h}_{ij}]_{m*n}$

$\breve{h}_{ij} = \{\gamma_{ij} \in \breve{h}_{ij}\}$

γ_{ij}=$\{< s, h_M^+(s), h_{NM}^+(s), h_M^-(s), h_{NM}^-(s)/s \in S >\}$

Now the decision matrix becomes,

Table 1. Hesitant Bipolar Intuitionistic Fuzzy Decision matrix

	P_1	P_2	P_3	P_n
Q_1	$< s(\gamma_{11}) >$	$< s(\gamma_{12}) >$	$< s(\gamma_{13}) >$......	$< s(\gamma_{1n}) >$
Q_2	$< s(\gamma_{21}) >$	$< s(\gamma_{22}) >$	$< s(\gamma_{23}) >$......	$< s(\gamma_{2n}) >$
	.	.	.	.
	.	.	.	.
	.	.	.	.
Q_m	$< s(\gamma_{m1}) >$	$< s(\gamma_{m2}) >$	$< s(\gamma_{m3}) >$......	$< s(\gamma_{mn}) >$

Step 2: Determine the score function of Hesitant Bipolar intuitionistic fuzzy decision matrix element

$$s(\breve{h}) = \frac{1}{4}\left(\left(\left(\frac{1}{l_{h_M^+}} \sum_{\lambda_M^+ \in h_M^+} h_M^+\right) - \left(\frac{1}{l_{h_{NM}^+}} \sum_{\lambda_{NM}^P \in h_{NM}^+} h_{NM}^+\right) + \left(\frac{1}{l_{h_M^-}} \sum_{\lambda_M^N \in h_M^-} h_M^-\right) - \left(\frac{1}{l_{h_{NM}^-}} \sum_{\lambda_{NM}^N \in h_{NM}^-} h_{NM}^-\right)\right)\right)$$

Step 3: Construct the normalized score matrix

$$\breve{H}_{ij} = \frac{s_{ij}}{\sum_{i=1}^{n} s_{ij}} \quad (6)$$

Step 4: Calculate the criteria weights by using weighted entropy

$$E(A) = -K\sum_{i=1}^{m} Z_{ij} lnZ_{ij} \tag{7}$$

among them, $K = \frac{1}{\ln m}$ is non-negative constant.

Then new Entropy matrix can be denoted by,

$$E = \begin{bmatrix} e_{11} & e_{12} \cdots & e_{1n} \\ \vdots & \ddots & \vdots \\ e_{m1} & e_{m2} \cdots & e_{mn} \end{bmatrix}$$

Normalization of the hesitant bipolar Intuitionistic Fuzzy Entropy Values are given by

$$E_{ij} = \frac{e_{ij}}{\max\{e_{i1}, e_{i2} \ldots\ldots e_{in}\}}$$

For $1 \leq i \leq m \; and \; 1 \leq j \leq n$.

Calculate the information utility b_j for the j-th indicator by

$$w_{Ej} = \frac{a_j}{\sum_{j=1}^{m} a_j} = \frac{1-e_j}{\sum_{j=1}^{n}(1-e_j)} \quad , a_j = 1 - e_j \; (j = 1,2 \ldots\ldots n) \tag{8}$$

$0 \leq w_j \leq 1$.

Step 5: Calculate the evaluation value U_{ij} of the evaluation plan i by

$$U_{ij} = w_{Ej} * \breve{H}_{ij} \tag{9}$$

Step 6: The HBIF positive ideal solution (HBIF-PIS), $\tilde{h}_+$, and the HBIF negative ideal solution (HBIF-NIS), $\tilde{h}_-$, are given by,

$$\breve{h}_+ = \{([\max(\gamma_{ij})]/(\gamma_{ij}) \in \breve{h}_{ij}), ([\min(\gamma_{ij})]/(\gamma_{ij}) \in \breve{h}_{ij})\} \tag{10}$$

$$\breve{h}_- = \{([\min(\gamma_{ij})]/(\gamma_{ij}) \in \breve{h}_{ij}), ([\max(\gamma_{ij})]/(\gamma_{ij}) \in \breve{h}_{ij})\} \tag{11}$$

Step 7: Calculate the Distance Measured from HBIF-PIS and HBIF-NIS.

$$S_i^+ = \sqrt{\sum_{j=1}^{n}\{(U_j^+ - U_{ij})^2\}} \tag{12}$$

$$S_i^- = \sqrt{\sum_{j=1}^{n}\{(U_j^- - U_{ij})^2\}} \tag{13}$$

Step 8: Apply the formula specified in equation (14).

By using the following formula, we evaluate the Grey relational coefficient between each alternative and HBIF-PIS

$$\delta_{ij}^+ = \delta(\breve{h}_j^+, \breve{h}_{ij}) = \frac{\min s(\breve{h}_j^+, \breve{h}_{ij}) + \rho max \; s(\breve{h}_j^+, \breve{h}_{ij})}{s(\breve{h}_j^+, \breve{h}_{ij}) + \rho max \; s(\breve{h}_j^+, \breve{h}_{ij})} \tag{14}$$

where ρ is the recognition coefficient, $\rho \in [0,1]$ generally $\rho = 0.5$ and i=1,2…m and j=1, 2…n.

Use the following formula to calculate the grey relational coefficient between each alternative and HBIF-PIS

$$\delta_{ij}^- = \delta(\breve{h}_j^+, \breve{h}_{ij}) = \frac{\min s(\breve{h}_j^-, \breve{h}_{ij}) + \rho max \; s(\breve{h}_j^-, \breve{h}_{ij})}{s(\breve{h}_j^-, \breve{h}_{ij}) + \rho max \; s(\breve{h}_j^-, \breve{h}_{ij})} \tag{15}$$

Step 9: Use the following formula to calculate the grey relational grade of each alternative.

$$\theta_i^+ = \sum_{j=1}^{n} \delta_{ij}^+ * w_j \text{ and } \sum_{j=1}^{n} w_j = 1 \tag{16}$$

$$\theta_i^- = \sum_{j=1}^{n} \delta_{ij}^- * w_j \text{ and } \sum_{j=1}^{n} w_j = 1 \tag{17}$$

Calculating the Relative Grey Relational Grade with respect to HBIF-NIS.

$$\varphi_i = \frac{\theta_i^+}{\theta_i^+ + \theta_i^-} \text{ where } 0 \leq \varphi_i \leq 1$$

Step 10: Based on the grey relative grade of each alternative, alternatives are ranked based on the descending order of $\varphi_i's$.

4. Numerical example

The objective of this study is to select the most affecting alternatives, among the various types of pollutions such as water pollution, Land/Soil pollution, Air pollution, Light pollution and Noise pollution etc., based on their characteristics and applications. In this selection process, criteria weights play an important role in examining decision maker's decisions on five alternatives. Here criteria such as (Health risk (P_1), climate change (P_2), Global warming (P_3), Extinct animals (P_4) and drought areas (P_5)) are considered. All criteria considered as non-beneficial. From the use of our proposed method the most desirable alternative will be chosen based on the above-mentioned criteria. In this research paper we have concentrated mainly on human health risks, because health risk is one of the most important factors in creating a healthy community. Hence according to the steps discussed in Section 3, the numerical example based on pollutions as alternatives is evaluated.

Step 1: Creating the hesitant bipolar intuitionistic fuzzy decision matrix

Here, we construct the decision matrix of hesitant bipolar intuitionistic fuzzy values. The hesitant bipolar intuitionistic fuzzy values are shown in table 2. We develop the hesitant bipolar intuitionistic fuzzy values for five alternatives and five attributes.

Table 2. Decision matrix with HBIFs

alternatives	Criteria				
	P_1	P_2	P_3	P_4	P_5
Q_1	{0.1, 0.2}, {0.5, 0.6} {−0.82, −0.9}, {0, −0.1}	{0.2, 0.3}, {0.6, 0.7} {−0.74, −0.8}, {-0.1, -0.2}	{0.31, 0.35}, {0.5,0.55} {−0.7, −0.8}, {-0.15, -0.2}	{0.1, 0.25}, {0.5, 0.6}, {-0.45, -0.5}, {-0.4, -0.5}	{0.2, 0.25}, {0.5, 0.55}, {-0.3, -0.4}, {-0.4, -0.45}
Q_2	{0.2, 0.3}, {0.6, 0.7} {−0.7, −0.75}, {−0.2, −0.25}	{0.31,0.5}, {0.4, 0.45} {−0.63, −0.7}, {-0.25, -0.3}	{0.3, 0.35}, {0.4, 0.55}, {−0.5, −0.55}, {-0.4, -0.45}	{0.1, 0.2}, {0.5,0.6} {-0.8, -0.9}, {0.0, -0.1}	{0.15, 0.2}, {0.5,0.6} {-0.85, -0.9}, {0.0, -0.1}
Q_3	{0.4, 0.45}, {0.5,0.55}, {−0.6, −0.75}, {-0.1, -0.2}	{0.3, 0.4}, {0.5, 0.6}, {−0.6, −0.65}, {−0.25, −0.35}	{0.2, 0.3}, {0.5, 0.55}, {−0.4, −0.5}, {-0.4, -0.45}	{0.1, 0.2}, {0.6, 0.7}, {-0.3, -0.45}, {-0.3, -0.5}	{0.2, 0.3}, {0.6, 0.7} {−0.7, −0.75}, {−0.2, −0.25}
Q_4	{0.2, 0.25}, {0.5, 0.7}, {−0.5, −0.6}, {-0.3, -0.4}	{0.3,0.4}, {0.4, 0.6}, {−0.45, −0.5}, {-0.2, -0.3}	{0.1, 0.2}, {0.7,0.8}, {-0.55, -0.6} {−0.3, −0.4},	{0.2,0.3}, {0.6, 0.7} {−0.75, −0.85}, {-0.3, -0.4}	{0.1, 0.2}, {0.7,0.8}, {−0.3, −0.4}, {-0.55, -0.6}
Q_5	{0.3,0.35}, {0.5,0.6}, {−0.4, −0.5}, {-0.4, -0.45}	{0.2,0.3}, {0.6, 0.7}, {-0.5, -0.6}, {−0.33, −0.4},	{0.2,0.3}, {0.65, 0.7} {−0.6, −0.7}, {-0.2, -0.3}	{0.2, 0.3}, {0.6, 0.7} {−0.6, −0.75}, {−0.2, −0.25}	{0.15, 0.2}, {0.6, 0.7}, {-0.2, -0.35} {-0.5, -0.6},

Step 2: Determining the hesitant bipolar intuitionistic fuzzy score matrix

In this step, we evaluate the hesitant bipolar intuitionistic fuzzy score matrix Based on that the HBIFs is calculated and tabulated in table 3.

Table 3. Score matrix with HBIFs

	P_1	P_2	P_3	P_4	P_5
Q_1	-0.3025	-0.255	-0.1925.	-0.1	-0.0562
Q_2	-0.225	-0.1025	-0.0625	-0.3	-0.3
Q_3	-0.1562	-0.1312	-0.075	-0.1187	-0.225
Q_4	-0.1437	-0.0937	-0.2062	-0.2125	-0.0937
Q_5	-0.0625	-0.1462	-0.2062	-0.2125	-0.05

Step 3: Normalizing the Hesitant Bipolar Intuitionistic Fuzzy score matrix

In calculating hesitant bipolar intuitionistic fuzzy ENTROPY method, Eq. (6) is used, then hesitant bipolar intuitionistic fuzzy decision matrix of score matrix is evaluated and tabulated in table 4.

Table 4. Normalize score matrix with HBIFs

	P_1	P_2	P_3	P_4	P_5
Q_1	0.339926	0.349986	0.259294	0.105966	0.077528
Q_2	0.252837	0.140681	0.084186	0.317898	0.41385
Q_3	0.175525	0.180071	0.101024	0.125781	0.310388
Q_4	0.161479	0.128603	0.277748	0.225177	0.129259
Q_5	0.070233	0.200659	0.277748	0.225177	0.068975

Step 4: Calculate the criteria weights by using weighted entropy.

Now, we evaluate the entropy values E_i based on the Eq. (7). Thus, the values of normalized entropy hesitant bipolar intuitionistic fuzzy set take $E_1 = -4.9991$, $E_2 = -3.6232$, $E_3 = -4.4781$, $E_4 = -4.0081$ and $E_5 = -4.3106$.

According to the equation, $a_j = 1 - E_j$. a_j represents for the j-th criterion is calculated and found to be $a_1 = 1 - (-4.9991) = 5.9991$.

similarly, we obtain $a_2 = 4.6232$, $a_3 = 5.4781$, $a_4 = 5.0081$, $a_5 = 5.3106$ are evaluated and found with the aid of a_i's

Now w_{Ej} are calculated by using (8).

(i.e) $w_{Ej} = \frac{a_j}{\sum_{j=1}^{m} a_j} = \frac{1-e_j}{\sum_{j=1}^{n}(1-e_j)}$

Thus, the entropy weight w_j of the j-th criterion is given by,

$\sum_{j=1}^{n}(1-e_j)$=5.9991+4.6232+5.4781+5.0081+5.3106 = 26.4192

Hence, $w_{E1} = 0.2271$, $w_{E2} = 0.1749$, $w_{E3} = 0.2073$, $w_{E4} = 0.1895$ and $w_{E5} = 0.2010$.

Step 5: Obtain the weight normalization decision matrix

The weight normalization decision matrix using equation (9) are evaluated and represented in table.5

Table 5. Weighted matrix with HBIFs

	P_1	P_2	P_3	P_4	P_5
Q_1	0.077189	0.061246	0.053765	0.020087	0.015584
Q_2	0.057413	0.024618	0.017456	0.060262	0.08319
Q_3	0.039858	0.031511	0.020947	0.023844	0.062393
Q_4	0.036668	0.022505	0.057591	0.042685	0.025983
Q_5	0.015948	0.035114	0.057591	0.042685	0.013865

Step 6: Acquire the solutions of HBIF-PIS and HBIF-NIS

Now by TOPSIS method, using equations (10) and (11) calculating the Hesitant bipolar intuitionistic fuzzy positive ideal solution $\check{h}_+$ and Negative ideal solution $\check{h}_-$ found as,

$\check{h}_+ = \{ 0.077189, 0.061246, 0.057591, 0.060262, 0.08319\}$

$\check{h}_- = \{0.015948, 0.022505, 0.017456, 0.020087, 0.013865\}$

Step 7: Calculating the separation measures between HBIF-PIS and HBIF-NIS

Based on the normalized Euclidean distance, distance measurement of both hesitant bipolar intuitionistic fuzzy positive and negative solutions for each alternative is obtained by Equations (12) & (13). The values of HBIF-PIS and HBIF-NIS are calculated and tabulated in table .6

Table 6. Euclidean distance measures from the HBIF-PIS and HBIF-NIS.

Alternatives	Q_1	Q_2	Q_3	Q_4	Q_5
S_i^+	0.078735	0.057823	0.073344	0.082003	0.097715
S_i^-	0.081071	0.090243	0.055082	0.051939	0.047755

Step 8: Calculate Grey Relational Coefficient of Alternatives

Grey relational coefficients are calculated by using equations (14) & (15) between each alternative HBIF-PIS and HBIF-NIS. The calculated values are shown in table 7.

Table 7. Grey Relational Coefficient from the HBIF-PIS and HBIF-NIS.

Alternatives	Q_1	Q_2	Q_3	Q_4	Q_5
δ_{ij}^+	0.836104	1.000000	0.872987	0.815226	0.727835
δ_{ij}^-	0.735991	0.686122	0.926879	0.956893	0.11975

Step 9: Evaluate the Grey Relational Grade of Alternatives.

Now, grey relational grades of alternatives are calculated by equations (16) and (17) are shown in table .8

Table 8. Grey Relational Grade of Alternatives

Alternatives	Q_1	Q_2	Q_3	Q_4	Q_5
θ_i^+	0.189859	0.174994	0.181015	0.154537	0.146306
θ_i^-	0.167125	0.120067	0.19219	0.181392	0.024071

To Calculate the Relative Grey Relational Grade with respect to HBIF-NIS we use of equation (17). Thus, the table .9 becomes

Table 9. Relative grey relational grade of the alternatives

Alternatives	Q_1	Q_2	Q_3	Q_4	Q_5
φ_i	0.239104	0.168363	0.283793	0.282797	0.027268

Step 10. Rank the alternatives

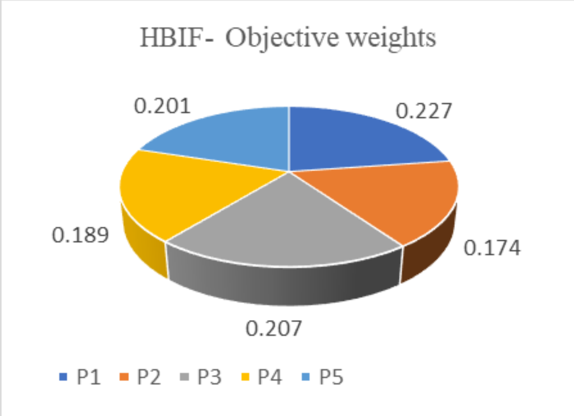
Based on φ_i values now ranking the alternative is carried out.

Table 10. Rank the alternatives

Alternatives	Q_1	Q_2	Q_3	Q_4	Q_5
rank	3	4	1	2	5

5. Results and Discussion

In this section, criteria weights are calculated by HBIF- Entropy method. The obtained value of objective weight vector is $w = (w_{E1}, w_{E2}, w_{E3}, w_{E4}, w_{E5}) = (0.227, 0.174, 0.207, 0.189, 0.201)$. The objective weights of criteria are, $P_1 > P_3 > P_5 > P_4 > P_2$. According to our weight finding technique the decision maker mainly focuses and concentrates on the criteria of people health risk, Decision maker wishes more importance to minimize the health risks. This shown in fig 1. In the next stage, the Hesitant bipolar intuitionistic fuzzy entropy-based TOPSIS method was improved by using the grey relational analysis, and hence hesitant bipolar intuitionistic fuzzy entropy-based TOPSIS-GRA assessment model was derived. In Addition, the principle of entropy-based TOPSIS method was used to calculate the superiority of each alternative based on the evaluation index φ_i. Based on the new multi criteria assessment model, the relative grey relational grade of five alternatives were 23.9%, 16.8%, 28.3%, 28.2% and 2.7% respectively. According to the values high to low, we ranked the alternatives in order and observed as $Q_3 > Q_4 > Q_1 > Q_2 > Q_5$. Finally, we determine Q_3 is the most desirable alternative and is also shown in fig.2



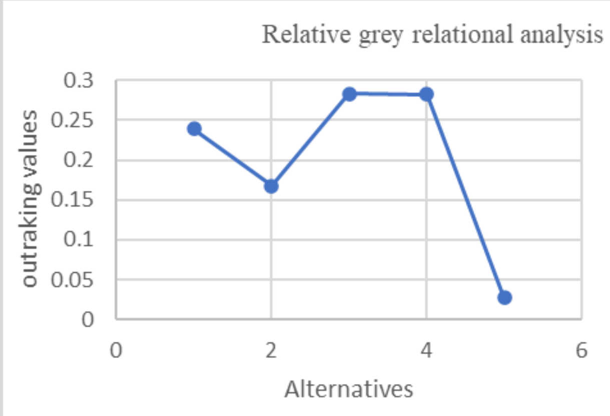


Fig. (1) HBIF- Objective weights by use of Entropy method Fig. (2) Outranking values of φ_i

6. Conclusion:

The main contributions of this paper can be stated as follows:

The first contribution is to combine the Hesitant bipolar intuitionistic fuzzy set with the TOPSIS method to obtain an appropriate MCDM solution. The second contribution is the objective weight value calculated by the Hesitant bipolar intuitionistic fuzzy entropy to change the subjective weight that decision makers set directly in the TOPSIS method. In other words, the objective weight of the Hesitant bipolar intuitionistic fuzzy entropy is estimated instead of the subjective weight, thus reducing the dependence that can be caused by subjective judgment. The third contribution is to extend the TOPSIS method to Hesitant bipolar intuitionistic fuzzy sets with GRA techniques. This, in turn, transformed the weighted matrix into a grey relational coefficient matrix. By introducing the grey relational coefficient, and the closeness between each alternative, the negative-ideal solution is also calculated. In terms of proximity, we found that Air pollution is the most affected pollution.

References

[1] Bellman, R.E; Zadeh, L.A, Decision making in a fuzzy Environment. Manag. Sci. (1970), 4, 141-164.
[2] W.R. Zhang, Bipolar fuzzy sets. IEEE International Conference on Fuzzy Systems. (1998),1:835–840.
[3] K.T. Atanassov, Intuitionistic fuzzy sets. Fuzzy Sets and Systems. (1986), 20:87-96.
[4] V. Torra, Hesitant fuzzy sets. International Journal of Intelligent Systems. (2010),25:529–539.
[5] D. Ezhilmaran & K. Sankar, Morphism of bipolar intuitionistic fuzzy graphs. Journal of Discrete Mathematical Sciences and Cryptography. (2015),18:5, 605-62.
[6] Muhammad Sajjad Ali Khana, Saleem Abdullahb, Asad Alia, Nasir Siddiquic and Fazli Amind, Pythagorean hesitant fuzzy sets and their application to group decision making with incomplete weight information. Journal of Intelligent & Fuzzy Systems. (2017),33. 3971–3985.
[7] Decui Liang, Zeshui Xu, The New Extension of TOPSIS Method for Multiple Criteria Decision Making with Hesitant Pythagorean Fuzzy Sets. Applied Soft Computing. (2017), S1568-4946(17)30377.
[8] R. Rajalakshmi, K. Julia Rose Mary, Hesitant Bipolar-Valued Intuitionistic Fuzzy set in Multi Attribute Group decision making. Strad Research. (2021), vol 8, issue 5, pp.704-711.
[9] Sorin Borza, Inta .M, Serbu R, Marza.B., Multi-Criteria Analysis of Pollution Caused by Auto Traffic in a Geographical Area Limited to Applicability for an Eco-Economy Environment. Sustainability. (2018),10(11), 4240.
[10] Ferhat Duran, İlke Bereketli Zafeirakopoulos. Environmental Risk Assessment of E-waste in Reverse Logistics Systems Using MCDM Methods. (2019),590-603. https://doi.org/10.1007/978-3-319-92267-6_49
[11] Kazancoglu, Yigit; Ozkan-Ozen, Yesim Deniz; Mangla, Sachin Kumar; Ram, Mangey, Risk assessment for sustainability in e-waste recycling in circular economy. Clean Technologies and Environmental Policy, (2020), doi:10.1007/s10098-020-01901-3
[12] Hameem Bin Hameed, Yousaf Ali, Antonella Petrillo, Environmental risk assessment of E-waste in developing countries by using the modified-SIRA method, science of the total environment. (2020),733. 138-525.
[13] Hiral Jariwala, Huma S Syed, Minarva J Pandya, Yogesh M Gajera, Noise pollution & human health: a review, Conference: Noise and Air Pollution: Challenges and Opportunities. (2017), PP.1-4.

[14] Vlachokostas, C.; Achillas, C.; Moussiopoulos, N.; Banias, G. Multicriteria methodological approach to manage urban air pollution. Atmos. Environ. (2011), 45, 4160–4169.

[15] Lizhong Tong, Zhongmin Pu and Jizheng Ma, Maintenance Supplier Evaluation and Selection for Safe and Sustainable Production in the Chemical Industry: A Case Study. (2019), Sustainability, 11 1-15.