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Bayesian Change Point Estimation Of Burr Type III Distribution Under Squared Error Loss Function

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Abstract

This Paper provides an overview of estimation of change points. The paper discusses the statistical inference problem about a change point model: (1) to determine if any change point should exist in the sequence; and (2) estimate the number and position(s) of change point(s), and other qualities of interest which are related to the change (for example, the magnitude of the jump of the mean). Bayesian methods are proposed to study the estimates of change points in the sequence of Burr Type III Distribution under Squared error Loss function(SELF). A simulation study is done by R-programming.

Keywords: *Change Point Estimation, Burr Type III Distribution. Bayesian method, Natural Conjugate Inverted Gamma Prior, Squared Error Loss Function*

1.1 Introduction

Statistical decision theory is concerned with the making of decisions in the presence of Statistical knowledge which sheds light on some of the uncertainties involved in the decision problem. We consider these uncertainties represented by unknown numerical quantities, say θ (possibly a vector or matrix).

Classical Statistics is directed towards the use of sample information (the data arising from the Statistical investigation) in making inferences about θ .These classical inferences are, mostly, made without considering the true state of parameter .In decision theory, on the other hand, an attempt is made to combine the sample information with other relevant aspects of the problem i.e. by considering the true state of parameter in form of probability model as prior distribution in order to make the best decision.

In addition to the sample information, two other types of information are typically relevant. The first is the knowledge of the possible consequences of the decisions. Often this knowledge can be quantified by determining the loss that would be incurred for each possible decision and for the various possible values of θ. The incorporation of a loss function into Statistical analysis was first studied extensively by Abraham Wald; see Wald (1950),Which also reviews earlier work in decision theory.

The approach to Statistics which formally seeks to utilize prior information is called Bayesian analysis named after Bayes (1763). Bayesian analysis and decision theory go rather naturally together, partly because of their common goal of utilizing non experimental sources of information, and partly because of some deep theoretical ties; thus, we will emphasize Bayesian decision theory in the chapter. There exists, however, an extensively developed non-Bays decision theory and an extensively developed non-decision-theoretic Bayesian viewpoint, both of which we will also cover in reasonable depth.

In decision theory and estimation theory, a Bayes estimator is an estimator or decision rule that maximizes the posterior expected value of a utility function or minimizes the posterior expected value of a Loss Function (also called posterior expected Loss).

1.2 Bayesian Inference

In a Bayesian setup, the unknown parameter is viewed as random variable. The uncertainty about the true value of parameter is expressed by a prior distribution. The parametric inference is made using the posterior distribution which is obtained by incorporating the observed data into the prior distribution using Bayes theorem. Hence we update the prior distribution in the light of observed data. Thus the uncertainty about the parameter prior to the experiment is represented by the prior distribution and the same, after the experiment, is represented by the posterior distribution.

Since that the time of classical inferential procedure interest has alternated between periods of acceptance and rejection of the method as a base for the statistical inference. Often suitable and unsuspected difficulties with alternate methods of statistical inference have been largely responsible for the continued resurgence of the Bayesian method of reasoning and inferences. The method is currently riding a high tide of popularity in virtually all areas of statistical application. Defineti(1937), Good (1950), Jeffrey's(1961), Lindley(1965), Ramsey(1931), Kendall and Stuart (1961), Box and Tiao(1973) and Savage(1954).

The Bayesian method of reasoning is much more direct that is "Deductive". To achieve this direct approach the mean life θ is assumed to be a random variable with a priori or prior p.d.f. $g(\theta)$. This distribution expresses the state of knowledge or ignorance about θ before the sample data is analyzed.

Given a prior distribution, the probability model $f(y|\theta)$ and the data y, Bayes theorem is used to calculate the so called posterior pdf $g(\theta|y)$ of Θ given data y. A distinctive feature of Bayesian Inference is that it takes explicit account of prior information in the analysis.

1.3 Loss Function

Let θ be an unknown parameter of some distribution $f(x|\theta)$ and suppose we estimate θ by some statistic $\hat{\theta}$. Let $L(\hat{\theta}, \theta)$ represent the loss incurred when the true value of the parameter is θ and we are estimating θ by the statistic $\hat{\theta}$.

The most widely used loss function in estimation problems is quadratic loss function given as $L(\hat{\theta}, \theta) = k(\hat{\theta} - \theta)^2$ where $\hat{\theta}$ is the estimate of θ, the loss function is called quadratic weighed loss function if k=1, we have

$$L(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2 \qquad (1.3.1)$$

Known as squared error loss function (SELF).

1.4 Change or Shift Point

Physical systems manufacturing the items are often subject to random fluctuations. It may happen that at some point of time instability in the sequence of lifetimes is observed. Such observed point is known as Change or Shift point inference problem. Such Change or Shift point inference problem is useful in statistical quality control to study the Change or Shifting in process mean, Linear time series models, and models related to econometrics. The monographs, Broemeling and Tsurmi (1987) on structural changes and survey by Zack (1981) are useful references. Bayesian approach may play an important role in the study of such Change or Shift point problem and has been often proposed as a valid alternative in classical estimation procedure. A variety of Change or Shift point problems have studied in Bayesian frame work by many authors like Zellner (1986), Calabria and Pulcini(1994) and Jani and Pandya (1999).

1.5 Burr Type III Distribution

Burr type III distribution with two parameters was first introduced in the literature of Burr (1942) for modelling lifetime data or survival data. It is more flexible and includes a variety of distributions with varying degrees of skewness and kurtosis. Burr type III distribution with two parameters β and θ , which is denoted by (β, θ). Burr III , has also been applied in areas of statistical modelling such as forestry (Gove et al (2008)), meteorology (Mielke (1973)), and reliability (Mokhlis (2005)).

The Probability Density Function and the Cumulative Distribution Function of Burr III are given by, respectively,

$$f(x;\ \theta, \beta) = \theta\,\beta\,x^{-(\beta+1)}\left(1 + x^{-\beta}\right)^{-(\theta+1)};\ x > 0, \theta, \beta > 0 \qquad (1.5.1)$$

And the distribution function

$$F(x;\ \theta, \beta) = \left(1 + x^{-\beta}\right)^{-\theta}; \qquad x > 0, \theta, \beta > 0 \qquad (1.5.2)$$

Reliability function

$$R(t;\,\theta,\beta) = 1 - \left(1 + t^{-\beta}\right)^{-\theta};\, t > 0, \theta, \beta > 0 \qquad (1.5.3)$$

Note that Burr type XII distribution can be derived from Burr type III distribution by replacing X with $\frac{1}{x}$. The usefulness and properties of Burr distribution are discussed by Burr and Cislak (1968) and Johnson et al. (1995). Abd-Elfattah and Alharbey (2012) considered a Bayesian estimation for Burr type III distribution based on double censoring.

In this chapter the Bayes estimates of change point 'm' and scale parameter 'γ' with known α *and* β of three parameter of Generalized Compound Rayleigh Distribution (G.C.R.D.) and also the change point 'm' and scale parameter 'θ' with known β of Exponentiated Inverted Weibull distribution, Burr Type III distribution are obtained by using Squared Error Loss Function(SELF) and Natural conjugate Prior distribution as Inverted Gamma prior distribution. The comparison and sensitivity analysis of Bayes estimates of change point and the parameters of the distributions have been done by using R-programming.

1.6 Bayesian Estimation of Change Point in Burr Type III Distribution under Squared Error Loss Function (SELF)

A sequence of independent life times $x_1, x_2, \dots x_m, x_{(m+1)}, \dots x_n$ $(n \geq 3)$ were observed from Burr Type III Distribution with parameter θ, β. But it was found that there was a change in the system at some point of time 'm' and it is reflected in the sequence after 'x_m' which results change in a sequence as well as parameter value θ. The Bayes estimate of θ and 'm' are derived for symmetric and asymmetric loss function under inverted Gamma prior as natural conjugate prior.

1.6.1 Likelihood, Prior, Posterior and Marginal

Let $x_1, x_2, \dots\dots, x_n$, $(n \geq 3)$ be a sequence of observed discrete life times. First let observations $x_1, x_2, \dots\dots, x_n$ have come from Burr Type III Distribution with probability density function as

$$f(x,\theta,\beta) = \theta\,\beta x^{-(\beta+1)}\left(1 + x^{-\beta}\right)^{-(\theta+1)} \quad (x,\theta,\beta > 0) \qquad (1.6.1.1)$$

Let 'm' is change point in the observation which breaks the distribution in two sequences as $(x_1, x_2, \dots\dots\dots x_m)$ & $x_{(m+1)}, x_{(m+2)}, \dots\dots x_n$

The probability density functions of the above sequences are

$$f_1(x) = \theta_1\, \beta_1\, x^{-(\beta_1+1)}\left(1 + x^{-\beta_1}\right)^{-(\theta_1+1)}; \qquad (1.6.1.2)$$

where $x_1, \dots, x_m > 0; \theta_1, \beta_1, > 0$

$$f_2(x) = \theta_2\, \beta_2\, x^{-(\beta_2+1)}\left(1 + x^{-\beta_2}\right)^{-(\theta_2+1)} \quad ; \qquad (1.6.1.3)$$

where $x_{(m+1)}, x_{(m+2)}, \dots, x_n;\ \theta_2, \beta_2 > 0$

The likelihood functions of probability density function of the sequence are

$$L_1(x|\theta_1,\beta_1) = \prod_{j=1}^{m} f(x_j|\theta_1,\beta_1)$$

$$= \theta_1{}^m \beta_1{}^m \prod_{j=1}^{m} \frac{x_j{}^{-(\beta_1+1)}}{(1+x_j{}^{-\beta_1})} \quad e^{-\theta_1 \sum_{j=1}^{m} \log\left(1 + x_j{}^{-\beta_1}\right)}$$

$$L_1(x|\theta_1,\beta_1,) = (\theta_1\beta_1)^m U_1 e^{-\theta_1 T_{3m}} \qquad (1.6.1.4)$$

Where

$$U_1 = \prod_{j=1}^{m} \frac{x_j{}^{-(\beta_1+1)}}{(1 + x_j{}^{-\beta_1})}$$

$$T_{3m} = \sum_{j=1}^{m} \log\left(1 + x_j{}^{-\beta_1}\right)$$

$$L_2(x|\theta_2,\beta_2) = \prod_{j=(m+1)}^{n} f\left(x_j|\theta_2,\beta_2\right)$$

$$= \theta_2^{(n-m)} \beta_2^{(n-m)} \prod_{j=(m+1)}^{n} \frac{x_j{}^{-(\beta_2+1)}}{(1+x_j{}^{-\beta_2})}\, e^{-\theta_2 \sum_{j=1}^{m} \log\left(1 + x_j{}^{-\beta_2}\right)}$$

$$L_2(x|\theta_2,\beta_2) = (\theta_2\beta_2)^{(n-m)} U_2 e^{-\theta_2 (T_{3n} - T_{3m})} \qquad (1.6.1.5)$$

where

$$U_2 = \prod_{j=m+1}^{n} \frac{x_j{}^{-(\beta_2+1)}}{(1+x_j{}^{-\beta_2})}$$

and $\quad T_{3n} - T_{3m} = \sum_{j=(m+1)}^{n} \log\left(1 + x_j{}^{-\beta_2}\right)$

The joint likelihood function is given by

$$L(\theta_1, \theta_2|\underline{x}) \propto (\theta_1\beta_1)^m U_1\, e^{-\theta_1 T_{3m}} (\theta_2\beta_2)^{n-m} U_2\, e^{-\theta_2 (T_{3n} - T_{3m})} \qquad (1.6.1.6)$$

Suppose the marginal prior distribution of θ_1 and $\theta_2$are natural conjugate prior

$$\pi_1(\theta_1, \underline{x}) = \frac{b_1^{a_1}}{\Gamma a_1} \theta_1^{(a_1-1)} e^{-b_1\theta_1}; \qquad a_1, b_1 > 0, \theta_1 > 0 \qquad (1.6.1.7)$$

$$\pi_2(\theta_2,\underline{x}) = \frac{b_2^{a_2}}{\Gamma a_2}\theta_2^{(a_2-1)}e^{-b_2\theta_2}; \qquad a_2,b_2 > 0,\theta_2 > 0 \qquad (1.6.1.8)$$

The joint prior distribution of θ_1 ,θ_2 and change point 'm' is

$$\pi(\theta_1,\theta_2,m) \propto \frac{b_1^{a_1}}{\Gamma a_1}\frac{b_2^{a_2}}{\Gamma a_2}\theta_1^{(a_1-1)}e^{-b_1\theta_1}\theta_2^{(a_2-1)}e^{-b_2\theta_2} \qquad (1.6.1.9)$$

Where $\theta_1,\theta_2 > 0 \;\&\; m = 1,2,\ldots\ldots(n-1)$

The joint posterior density of θ_1,θ_2 and m say $\rho(\theta_1,\theta_2,m/\underline{x})$ is obtained by using equations $(1.6.1.6)$ & $(1.6.1.9)$

$$\rho(\theta_1,\theta_2\,,m|\underline{x}) = \frac{\mathrm{L}(\theta_1\,,\theta_2/\underline{x})\pi(\theta_1\,,\theta_2,m)}{\sum_m \iint_{\theta_1\theta_2}\mathrm{L}(\theta_1,\theta_2/\underline{x})\pi(\theta_1,\theta_2,m)\,d\theta_1\,d\theta_2} \qquad (1.6.1.10)$$

$$\rho(\theta_1,\theta_2\,,m|\underline{x}) = \frac{\theta_1^{\,(m+a_1-1)}\,e^{-\theta_1(T_{3m}+b_1)}\,\theta_2^{\,(n-m+a_2-1)}\,e^{-\theta_2(T_{3n}-T_{3m}+b_2)}}{\sum_m\int_0^\infty e^{-\theta_1(T_{3m}+b_1)}\,\theta_1^{\,(m+a_1-1)}\,d\theta_1\,\int_0^\infty\theta_2^{\,(n-m+a_2-1)}\,e^{-\theta_2(T_{3n}-T_{3m}+b_2)}\,d\theta_2}$$

Assuming $\theta_1(T_{3m}+b_1) = x$ & $\theta_2(T_{3n}-T_{3m}+b_2) = y$

$$\theta_1 = \frac{x}{(T_{3m}+b_1)} \qquad \& \qquad \theta_2 = \frac{y}{T_{3n}-T_{3m}+b_2}$$

$$d\theta_1 = \frac{dx}{(T_{3m}+b_1)} \qquad \& \qquad \mathrm{d}\theta_2 = \frac{dy}{T_{3n}-T_{3m}+b_2}$$

$$\rho(\theta_1,\theta_2\,,m|\underline{x}) = \frac{e^{-\theta_1(T_{3m}+b_1)}\,\theta_1^{\,(m+a_1-1)}\,e^{-\theta_2(T_{3n}-T_{3m}+b_2)}\,\theta_2^{\,(n-m+a_2-1)}}{\sum_m\frac{\Gamma(m+a_1)}{(T_{3m}+b_1)^{(m+a_1)}}\,\frac{\Gamma(n-m+a_2)}{(T_{3n}-T_{3m}+b_2)^{(n-m+a_2)}}}$$

$$\rho(\theta_1,\theta_2\,,m|\underline{x}) = \frac{e^{-\theta_1(T_{3m}+b_1)}\,\theta_1^{\,(m+a_1-1)}\,e^{-\theta_2(T_{3n}-T_{3m}+b_2)}\,\theta_2^{\,(n-m+a_2-1)}}{\xi(a_1,a_2,b_1,b_2,m,n)} \qquad (1.6.1.11)$$

Where $\xi(a_1,a_2,b_1,b_2,m,n) = \sum_{m=1}^{n-1}\left[\frac{\Gamma(m+a_1)}{(T_{3m}+b_1)^{m+a_1}}\,\frac{\Gamma(n-m+a_2)}{(T_{3n}-T_{3m}+b_2)^{(n-m+a_2)}}\right]$

The Marginal posterior distribution of change point 'm' using the equations $(1.6.1.6)$, $(1.6.1.7)$ & $(1.6.1.8)$

$$\rho(m|\underline{x}) = \frac{\mathrm{L}(\theta_1\,,\theta_2/\underline{x})\,\pi(\theta_1)\,\pi(\theta_2)}{\sum_m \mathrm{L}(\theta_1\,,\theta_2/\underline{x})\,\pi(\theta_1)\,\pi(\theta_2)} \qquad (1.6.1.12)$$

On solving which gives

$$\rho(m|\underline{x}) = \frac{\theta_1^{(m+a_1-1)} e^{-\theta_1(T_{3m}+b_1)} \theta_2^{(n-m+a_2-1)} e^{-\theta_2(T_{3n}-T_{3m}+b_2)}}{\sum_m \theta_1^{(m+a_1-1)} e^{-\theta_1(T_{3m}+b_1)} \theta_2^{(n-m+a_2-1)} e^{-\theta_2(T_{3n}-T_{3m}+b_2)}}$$

$$\rho(m|\underline{x}) = \frac{\int_0^\infty e^{-\theta_1(T_{3m}+b_1)} \theta_1^{(m+a_1-1)} d\theta_1 \int_0^\infty e^{-\theta_2(T_{3n}-T_{3m}+b_2)} \theta_2^{(n-m+a_2-1)} d\theta_2}{\sum_m \int_0^\infty e^{-\theta_1(T_{3m}+b_1)} \theta_1^{(m+a_1-1)} d\theta_1 \int_0^\infty e^{-\theta_2(T_{3n}-T_{3m}+b_2)} \theta_2^{(n-m+a_2-1)} d\theta_2}$$

Assuming $\theta_1(T_{3m}+b_1) = y$ & $\theta_2(T_{3n}-T_{3m}+b_2) = z$

$\theta_1 = \frac{y}{(T_{3m}+b_1)}$ & $\theta_2 = \frac{z}{T_{3n}-T_{3m}+b_2}$

$d\theta_1 = \frac{dy}{(T_{3m}+b_1)}$ & $d\theta_2 = \frac{z}{T_{3n}-T_{3m}+b_2}$

$$\rho(m|\underline{x}) = \frac{\frac{\Gamma(m+a_1)}{(T_{3m}+b_1)^{(m+a_1)}} \frac{\Gamma(n-m+a_2)}{(T_{3n}-T_{3m}+b_2)^{(n-m+a_2)}}}{\sum_m \frac{\Gamma(m+a_1)}{(T_{3m}+b_1)^{(m+a_1)}} \frac{\Gamma(n-m+a_2)}{(T_{3n}-T_{3m}+b_2)^{(n-m+a_2)}}}$$

$$\rho(m|\underline{x}) = \frac{\frac{\Gamma(m+a_1)}{(T_{3m}+b_1)^{(m+a_1)}} \frac{\Gamma(n-m+a_2)}{(T_{3n}-T_{3m}+b_2)^{(n-m+a_2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)} \qquad (1.6.1.13)$$

The marginal posterior distribution of θ_1, using equations $(1.6.1.6)$ and $(1.6.1.7)$

$$\rho(\theta_1|\underline{x}) = \frac{L(\theta_1,\theta_2/\underline{x})\,\pi(\theta_1)}{\int_0^\infty L(\theta_1,\theta_2/\underline{x})\,\pi(\theta_1)d\theta_1}$$

On solving which gives

$$\rho(\theta_1|\underline{x}) = \frac{\sum_m e^{-\theta_1(T_{3m}+b_1)} \theta_1^{(m+a_1-1)} \int_0^\infty e^{-\theta_2(T_{3n}-T_{3m}+b_2)} \theta_2^{(n-m+a_2-1)} d\theta_2}{\sum_m \int_0^\infty e^{-\theta_1(T_{3m}+b_1)} \theta_1^{(m+a_1-1)} d\theta_1 \int_0^\infty e^{-\theta_2(T_{3n}-T_{3m}+b_2)} \theta_2^{(n-m+a_2-1)} d\theta_2}$$

Assuming $\theta_1(T_{3m}+b_1) = y$ & $\theta_2(T_{3n}-T_{3m}+b_2) = z$

$\theta_1 = \frac{y}{(T_{3m}+b_1)}$ & $\theta_2 = \frac{z}{T_{3n}-T_{3m}+b_2}$

$d\theta_1 = \frac{dy}{(T_{3m}+b_1)}$ & $d\theta_2 = \frac{dz}{T_{3n}-T_{3m}+b_2}$

$$\rho(\theta_1|\underline{x}) = \frac{\sum_m e^{-\theta_1(T_{3m}+b_1)} \theta_1^{(m+a_1-1)} \frac{\Gamma(n-m+a_2)}{(T_{3n}-T_{3m}+b_2)^{(n-m+a_2)}}}{\sum_m \frac{\Gamma(m+a_1)}{(T_{3m}+b_1)^{(m+a_1)}} \frac{\Gamma(n-m+a_2)}{(T_{3n}-T_{3m}+b_2)^{(n-m+a_2)}}}$$

$$\rho(\theta_1|\underline{x}) = \frac{\sum_m e^{-\theta_1(T_{3m}+b_1)}\ \theta_1^{(m+a_1-1)}\ \frac{\Gamma(n-m+a_2)}{(T_{3n}-T_{3m}+b_2)^{(n-m+a_2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)} \qquad (1.6.1.14)$$

The marginal posterior distribution of θ_2, using the equation $(1.6.1.6)$ & $(1.6.1.8)$ is

$$\rho(\theta_2|\underline{x}) = \frac{L(\theta_1,\theta_2/\underline{x})\ \pi(\theta_2)}{\int_0^\infty L(\theta_1,\theta_2/\underline{x})\ \pi(\theta_2)\, d\theta_2}$$

$$\rho(\theta_2|\underline{x}) = \frac{\sum_m e^{-\theta_2(T_{3n}-T_{3m}+b_2)}\ \theta_2^{(n-m+a_2-1)} \int_0^\infty e^{-\theta_1(T_{3m}+b_1)}\ \theta_1^{(m+a_1-1)}\, d\theta_1}{\sum_m \int_0^\infty e^{-\theta_1(T_{3m}+b_1)}\ \theta_1^{(m+a_1-1)}\, d\theta_1 \int_0^\infty e^{-\theta_2(T_{3n}-T_{3m}+b_2)}\ \theta_2^{(n-m+a_2-1)}\, d\theta_2}$$

Assuming $\theta_1(T_{3m}+b_1) = y$ & $\theta_1 = \frac{y}{(T_{3m}+b_1)}$

$$\rho(\theta_2|\underline{x}) = \frac{\sum_m \frac{\Gamma(m+a_1)}{(T_{3m}+b_1)^{(m+a_1)}} e^{-\theta_2(T_{3n}-T_{3m}+b_2)}\ \theta_2^{(n-m+a_2-1)}}{\sum_m \frac{\Gamma(m+a_1)}{(T_{3m}+b_1)^{(m+a_1)}}\ \frac{\Gamma(n-m+a_2)}{(T_{3n}-T_{3m}+b_2)^{(n-m+a_2)}}}$$

$$\rho(\theta_2|\underline{x}) = \frac{\sum_m \frac{\Gamma(m+a_1)}{(T_{3m}+b_1)^{(m+a_1)}} e^{-\theta_2(T_{3n}-T_{3m}+b_2)}\ \theta_2^{(n-m+a_2-1)}}{\xi(a_1,a_2,b_1,b_2,m,n)} \qquad (1.6.1.15)$$

1.6.2 Bayes Estimators under Squared Error Loss Function (SELF)

The Bayes estimate of a generic parameter (or function there of) θ based on a SELF is given by

$L_1(\theta,d) = (\theta-d)^2$,where 'd' is a decision rule to estimate θ, is posterior mean .For the change point 'm' which is a non negative integer quantity $m = 1,2,\ldots\ldots(n-1)$ the loss function is defined as

$$L_1(m,\widehat{m}_s) = (m-\widehat{m}_s)^2 \qquad (1.6.2.1)$$

Where $\widehat{m}_s = 1,\ldots\ldots,(n-1)$, is the smallest integer greater than analytical solution .

The Bayes estimate $\widehat{m}_{BS}$ of 'm' under SELF using marginal posterior density equation $(1.8.1.13)$ is given as $\widehat{m}_{BS} = \sum_m m\ \rho(m|\underline{x})$

$$\widehat{m}_{BS} = \frac{\sum_m m \frac{\Gamma(m+a_1)}{(T_{3m}+b_1)^{(m+a_1)}}\ \frac{\Gamma(n-m+a_2)}{(T_{3n}-T_{3m}+b_2)^{(n-m+a_2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)} \qquad (1.6.2.2)$$

Bayes estimate $\hat{\theta}_{1BS}$ of θ_1 under SELF using marginal posterior density equation $(1.6.1.15)$ is given by

$$\hat{\theta}_{1BS} = \int_0^\infty \theta_1\ \rho(\theta_1|\underline{x})\, d\theta_1$$

On simplification which gives

$$\hat{\theta}_{1BS} = \frac{\sum_m \frac{\Gamma(n-m+a_2)}{(T_{3n}-T_{3m}+b_2)^{(n-m+a_2)}} \int_0^\infty e^{-\theta_1(T_{3m}+b_1)}\, \theta_1^{(m+a_1)} d\theta_1}{\xi(a_1, a_2, b_1, b_2, m, n)}$$

Assuming $\theta_1(T_{3m}+b_1) = y$ & $\theta_1 = \frac{y}{(T_{3m}+b_1)}$

$$\hat{\theta}_{1BS} = \frac{\sum_m \frac{\Gamma(n-m+a_2)}{(T_{3n}-T_{3m}+b_2)^{(n-m+a_2)}} \frac{\Gamma(m+a_1+1)}{(T_{3m}+b_1)^{(m+a_1+1)}}}{\xi(a_1, a_2, b_1, b_2, m, n)}$$

$$\hat{\theta}_{1BS} = \frac{\xi[(a_1+1), a_2, b_1, b_2, m, n]}{\xi(a_1, a_2, b_1, b_2, m, n)} \qquad (1.6.1.3)$$

Bayes estimate $\hat{\theta}_{2BS}$ of θ_2 under SELF using marginal posterior density equation (1.6.1.16) is given by

$$\hat{\theta}_{2BS} = \int_0^\infty \theta_2\, \rho(\theta_2 | \underline{x}) d\,\theta_2$$

$$\hat{\theta}_{2BS} = \frac{\sum_m \frac{\Gamma(m+a_1)}{(T_{3m}+b_1)^{(m+a_1)}} \int_0^\infty e^{-\theta_2(T_{3n}-T_{3m}+b_2)}\, \theta_2^{(n-m+a_2)} d\theta_2}{\xi(a_1, a_2, b_1, b_2, m, n)}$$

Assuming $\theta_2(T_{3n} - T_{3m} + b_2) = y$ & $\theta_2 = \frac{y}{(T_{3n}-T_{3m}+b_2)}$

Then $\hat{\theta}_{2BS} = \frac{\sum_m \frac{\Gamma(m+a_1)}{(T_{3m}+b_1)^{(m+a_1)}} \int_0^\infty e^{-y} \frac{y^{(n-m+a_2)}}{(T_{3n}-T_{3m}+b_2)^{(n-m+a_2)}} \frac{dy}{(T_{3n}-T_{3m}+b_2)}}{\xi(a_1, a_2, b_1, b_2, m, n)}$

$$\hat{\theta}_{2BS} = \frac{\sum_m \frac{\Gamma(m+a_1)}{(T_{3m}+b_1)^{(m+a_1)}} \frac{\Gamma(n-m+a_2+1)}{(T_{3n}-T_{3m}+b_2)^{(n-m+a_2+1)}}}{\xi(a_1, a_2, b_1, b_2, m, n)}$$

$$\hat{\theta}_{2BS} = \frac{\xi[a_1, (a_2+1), b_1, b_2, m, n]}{\xi(a_1, a_2, b_1, b_2, m, n)} \qquad (1.6.1.4)$$

Numerical Comparison for Burr Type III Sequences

We have generated 20 random observations from Burr Type III distribution with parameter $\theta = 2$ and $\beta = 0.5$. The observed data mean is 1.8829 and variance is 23.8886. Let the change in sequence is at 11th observation, so the means and variances of both sequences ($x_1, x_2, \ldots, x_m$) and ($x_{(m+1)}, x_{(m+2)}, \ldots, x_n$) are $\theta_1 = 0.8277$, $\theta_2 = 3.2668$ and $\sigma_1{}^2 = 0.7281$and $\sigma_2{}^2 = 51.8509$. If the target value of θ_1 is unknown, its estimating ($\hat{\theta}_1.$) is given by the mean of first m sample observation given m=11, $\theta = 0.827$.

Table 1.1

1	2	3	4	5	6	7	8	9	10
0.0136	1.4266	0.0597	1.6890	2.1583	1.6878	0.0383	0.2314	0.0377	1.5171
11	12	13	14	15	16	17	18	19	20
0.2448	0.3037	0.9397	4.9823	0.0043	22.0095	0.5433	0.0347	0.0001	0.5835

Now again we have generated the random samples of different sizes say 10, 20, 30. Obviously the change or Shift point occurs between 1 to 20 i.e say 11. and calculated the Bayes estimates under squared error loss function by making programs in R-language, again repeating these steps for 1000 times we have calculated the M.S.E. by making program in R and then analyze the data and given the conclusions.

Sensitivity Analysis of Bayes Estimates

In this section we have studied the sensitivity of the Bayes estimates with respect to changes in the parameters of prior distribution a_1, b_1, a_2 and b_2. The means and variances of the prior distribution are used as prior information in computing these parameters. Then with these parameter values we have computed the Bayes estimates of m, θ_1 and θ_2 under squared error loss function (SELF) considering different set of values of (a_1, b_1) and (a_2, b_2).We have also considered different sample sizes n=10(10)30. The Bayes estimates of the change point 'm' and the parameters θ_1 and θ_2 are given in table-1.2 under SELF . Their respective mean squared errors(M.S.E's) are calculated by repeating this process 1000 times and presented in same table in small parenthesis under the estimated values of parameters. All these values appears to be robust with respect to correct choice of prior parameter values and appropriate sample size. From the below table we conclude that -

The Bayes estimates of the parameters θ_1 and θ_2 of Burr Type III obtained with SELF are seems to be efficient as the numerical values of their mse's are very small for $\hat{\theta}_{1BS}$ and $\hat{\theta}_{2BS}$ in comparison with $\hat{m}_{BS}$. The Bayes estimates of the parameters are robust uniformly for all values of prior parameters and all sample size.

Table 1.2

Bayes Estimates of m, θ_1 & θ_2 for Burr Type III and their respective M.S.E.'s Under SELF

(a_1, b_1)	(a_2, b_2)	N	$\hat{m}_{BS}$	$\hat{\theta}_{1BS}$	$\hat{\theta}_{2BS}$
(1.25,1.50)	(1.50,1.60)	10	4.6998 **(11.1087)**	0.5819 **(0.1066)**	0.5953 **(0.1289)**
		20	8.7134 **(64.0405)**	0.5079 **(0.0381)**	0.7758 **(0.0002)**
		30	13.1157 **(126.1777)**	0.8468 **(0.0018)**	0.5808 **(0.0106)**
(1.50,1.75)	(1.70,1.80)	10	4.8593 **(9.9631)**	0.7163 **(0.0326)**	0.6059 **(0.2890)**
		20	9.8093 **(52.2445)**	0.8189 **(0.0908)**	0.7598 **(0.0749)**
		30	12.4380 **(154.8102)**	0.8247 **(0.0092)**	0.6525 **(0.0117)**
(1.75,2.0)	(1.90,2.0)	10	4.8005 **(7.2842)**	0.6870 **(0.3071)**	0.7218 **(0.0500)**
		20	10.4658 **(75.0869)**	0.8321 **(0.0209)**	0.6022 **(0.1025)**
		30	16.3741 **(117.5707)**	0.4882 **(0.0466)**	0.6593 **(0.0389)**
(2.0,2.25)	(2.10,2.20)	10	4.8408 **(4.4745)**	0.7064 **(0.6720)**	0.6081 **(0.1058)**
		20	9.7542 **(63.8403)**	0.9618 **(0.0164)**	1.1311 **(0.1185)**
		30	14.4445 **(203.8680)**	0.6784 **(0.0343)**	0.8011 **(0.0022)**
(2.25,2.50)	(2.30,2.40)	10	4.8219 **(7.2820)**	1.2693 **(0.3476)**	0.8926 **(0.0905)**
		20	9.7935 **(39.5458)**	0.6494 **(0.0393)**	0.6775 **(0.1223)**
		30	13.9288 **(198.0454)**	0.8826 **(0.0041)**	0.6601 **(0.0684)**
(2.50,2.75)	(2.50,2.60)	10	4.9879 **(9.2649)**	0.7849 **(0.0247)**	0.8419 **(0.0978)**
		20	10.0794 **(56.7361)**	0.6852 **(0.2133)**	0.5668 **(0.0451)**
		30	14.6203 **(35.1954)**	0.6186 **(0.2490)**	0.5544 **(0.3407)**

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