IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

SOME PROPERTIES OF NEUTROSOPHIC REAL NUMBERS

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Abstract:

In this paper, we will define the logarithm form of a neutrosophic real number. We have proven some properties and theories, including the division of neutrosophic real numbers. In addition, we have given the exponential form of a neutrosophic real numbers.

Keywords: Neutrosophic real numbers, the logarithm and the exponential form of a neutrosophic real numbers.

1. Introduction

The American scientist and philosopher F.Smarandache came to place the neutrosophic logic, and this logic is as a generalization of the fuzzy logic, conceived by Lotfi A. Zadeh and Dieter Klaua in 1965. Neutrosophic sets have been introduced to the literature by Smarandache to handle incomplete, indeterminate, and inconsistent information.In neutrosophic sets, indeterminacy is quantified explicitly through a new parameter I. Truth-membership (T), indeterminacy membership (I), and falsity-membership (F) are three independent parameters that are used to define a neutrosophic number. Smarandache proposed the neutrosophic Logic to represent a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, contradiction, where the concept of neutrosophy is a new branch of philosophy introduced by Smarandache found root index $n \geq 2$ of a neutrosophic real and complex number.

Studying the concept of the Neutrosophic probability, the Neutrosophic statistics , and professor Smarandache entered the concept of preliminary calculus of the differential and integral calculus, where he introduced for the first time the notions of neutrosophic mereo-limit, mereo-continuity, mereoderivative, and mereo-integral . Madeleine Al- Taha presented results on single valued neutrosophic (weak) polygroups . Edalatpanah proposed a new direct algorithm to solve the neutrosophic linear programming where the variables and right hand side represented triangular neutrosophic numbers . Chakraborty used pentagonal neutrosophic number in networking problem, and Shortest Path Problem.Professor F.Smarandache presented the definition of the standard form of neutrosophic real number and conditions for the division of two neutrosophic real number to existand also he defined the standard form of neutrosophic complex number.This paper aims to study and define the exponential form of a neutrosophic real number by defining logarithm form of a neutrosophic real number, division of the neutrosophic real number, and properties of neutrosophic real numbers.

2. Preliminaries

Definition 2.1 [6]

A neutrosophic number has the standard form: $a + bI$

where a, b are real coefficients and I = indeterminacy , such $0.I = 0$

$I^n = I$ for all positive integer n.

If the coefficients a and b are real, and then $a + bI$ is called neutrosophic real number.

For example: $2 + 7I$

Definition 2.2

Real numbers can be defined as the union of both rational and irrational numbers. They can be both positive (or) negative and are denoted by the symbol “R”. All the natural numbers, decimals and fractions come under the real numbers.

For example: 27, 22/7, 0.22.

Definition 2.3

R is a neutrosophic Real number,

The general form: $R = a + bI + c/d + eI/f$

Where a,b,c,d,e and f are real coefficients, whenever $d \neq 0$ and $f \neq 0$, and I is indeterminacy

Definition 2.4

Suppose $R = a + bI + c/d + e/f\,I$ is neutrosophic Real numbers , then absolute value of a neutrosophic Real number is

$$|R| = \sqrt{(a + bI)^2 + (c/d + eI/f)^2}$$

3. Properties of Neutrosophic Real numbers

The following are the four main properties of neutrosophic real numbers,

- Commutative property
- Associative property
- Distributive property
- Identity property

Consider $a + bI$, $c + dI$, $e + fI$ are three neutrosophic real number. Then the above properties can be described using $a + bI$, $c + dI$, $e + fI$ as shown below

- **Commutative property**

If $a + bI$ and $c + dI$ are the number, then the general form will be

$(a + bI) + (c + dI) = (c + dI) + (a + bI)$ for addition and

$(a + bI) . (c + dI) = (c + dI). (a + bI)$ for multiplication.

Addition: $(a + bI) + (c + dI) = (c + dI) + (a + bI)$

For example: $(2+ 7I) + (6 + 9I) = (6 + 9I) + (2 + 7I)$

Multiplication: $(a + bI) \times (c + dI) = (c + dI) \times (a + bI)$

For example: $(2+ 7I) \times (6 + 9I) = (6 + 9I) \times (2 + 7I)$

- **Associative property**

If $a + bI$, $c + dI$ and $e + fI$ are the number,

Then the general form will be

$a + bI + ((c + dI) + (e + fI)) = ((a + bI) + (c + dI)) + e + fI$ for addition

$a + bI . ((c + dI). (e + fI)) = ((a + bI). (c + dI)) . e + fI$ for multiplication

Addition: $a + bI + ((c + dI) + (e + fI)) = ((a + bI) + (c + dI)) + e + fI$

An example of additive associative property is

$2+ 7I + ((6 + 9I) + (3 + 8I)) = ((2+ 7I) + ((6 + 9I)) + 3 + 8I$

Multiplication: $a + bI . ((c + dI). (e + fI)) = ((a + bI). (c + dI)) . e + fI$

An example of additive associative property is

$2+ 7I \times ((6 + 9I) \times (3 + 8I)) = ((2+ 7I) \times ((6 + 9I)) \times 3 + 8I$

- **Distributive property**

For three numbers $a + bI$, $c + dI$ and $e + fI$ are neutrosophic real in nature, the distribution property is represented as,

$a + bI . ((c + dI) + (e + fI)) = (a + bI) (c + dI) + (a + bI) (e + fI)$

$((a + bI) + (c + dI)) e + fI = (a + bI) (e + fI) + (c + dI) (e + fI)$

Example of distributive property is

$2+ 7I ((6 + 9I) + (3 + 8I)) = (2+ 7I) \times (6 + 9I) + (2 + 7I)\times (3 + 8I)$

- **Identity property**

There are additive and multiplicative identities

For addition: $(a + bI) + 0 = a + bI$ (0 is the additive identity)

For multiplication: $(a + bI) \times 1 = 1 \times (a + bI) = (a + bI)$

(1 is the multiplication identity)

Theorem 3.1 [1]

For any neutrosophic real numbers a + b I and c + d I , we have

$$(a + bI, 0) + (c + dI, 0) = ((a + bI) + (c + dI), 0),$$

$$(a + bI, 0)\ (c + dI, 0) = ((a + bI)(c + dI), 0)$$

Proof:

Let a + b I and c + d I be neutrosophic real numbers.

Using Distributive property, we get

$$(a + bI, 0) + (c + dI, 0) = ((a + bI) + (c + dI), 0)$$

$$(a + bI, 0)\ (c + dI, 0) = ((a + bI)(c + dI), 0)$$

This completes the proof.

4: Exponential forms of the neutrosophic real number: [1]

Suppose e^x in a power series around the origin and obtain some properties of e^x

Define $E(a + bI) = \sum_{n=0}^{\infty} (a + bI)^n / n!$ and show that $E(a + bI) = e^{(a+bI)}$ for all $a + bI \in \Re$

Theorem 4.1:

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E (a + b I) satisfies the following properties

i. $E(a + bI)$ is differentiable for all $a + bI \in \Re$ and $E'(a + bI) = E(a + bI)$,

$(a + bI) \in \Re$

ii. $E(a + bI + c + dI) = E(a + bI)\, E(c + dI)$, $(a + bI) \in \Re$

iii. $E(a + bI) \to +\infty$ as $a + bI \to \infty$ and $E(a + bI) \to 0$ as $a + bI \to -\infty$

iv. $(a + bI)^n\ E(-(a + bI)) \to 0$ as $a + bI \to \infty$ for $n \in Z$

Proof:

i. We first observe that the radius of convergence of this power series is infinity ,

Hence $E(a + bI)$ is differentiable for all $a + bI \in \Re$ and

$$E'(a + bI) = \sum_{n=1}^{\infty} n(a + bI)^{n-1} / n!$$

$$= \sum_{n=0}^{\infty} (a + bI)^n / n!$$

Hence $E'(a + bI) = E(a + bI)$

ii. $E(a+bI)\,E(c+dI) = \sum_{n=0}^{\infty} (a+bI)^n / n! \; \sum_{m=0}^{\infty} (c+dI)^m / m!$

$$= \sum_{n=0}^{\infty} \sum_{k=0}^{n} (a+bI)^k (c+dI)^{n-k} / \; k!\,(n-k)!$$

$$\text{“}\sum_{n=0}^{\infty} \sum_{k=0}^{n} a^k b^{n-k} = \sum_{n=0}^{\infty} (a^n b^n)\text{”}$$

$$= \sum_{n=0}^{\infty} 1/n! \sum_{k=0}^{n} \binom{n}{k} (a+bI)^k (c+dI)^{n-k}$$

$E(a+bI)\,E(c+dI) = E(a+bI+c+dI)$

iii. $E(a+bI) > 1 + a + bI$ 5

$\Rightarrow E(a+bI) \to +\infty$ as $a+bI \to \infty$ and if $a+bI \to -\infty$

Then $a+bI = -(c+dI)$, where $c+dI \to \infty$ and

$E(a+bI) = 1/\;E(c+dI) \to 0$ as $a+bI \to -\infty$.

Hence $E(a+bI) \to +\infty$ as $a+bI \to \infty$ and $E(a+bI) \to 0$ as $a+bI \to -\infty$

iv. Power series representation

$E(a+bI) > (a+bI)^{n+1} / (n+1)!$ for $a+bI > 0$ and

Hence $(a+bI)^n \; E(-(a+bI)) = (a+bI)^n \, 1/\; E(a+bI)$

$< (n+1)! / a+bI$

This shows that

$(a+bI)^n \; E(-(a+bI)) \to 0$ as $a+bI \to \infty$

When $n>0$, On the other hand if $n<0$,

Then both $(a+bI)^n$ and $E(-(a+bI))$ tend to 0 as $(a+bI) \to \infty$

Hence $(a+bI)^n \; E(-(a+bI)) \to 0$ as $a+bI \to \infty$ for $n \in Z$

Theorem 4.2:

If f: $E \to [-\infty, \infty]$ is neutrosophic measurable and $f \in L'(E)$ .then $|f(a+bI)| < \infty$ a.e on E

Proof:

Let $F = \{a + bI \in E \ / \ \ |f(a+bI)| = \infty\}$

Then F is neutrosophic measurable and

If m (F) >0,

Then $\int_E |f(a+bI)| dm(a+bI) \geq \int_F |f(a+bI)| dm(a+bI)$

$> n.m(F)$ for all n=1,2,…..

This shows that

$$\int_E |f(a+bI)| dm(a+bI) = \infty \text{, a contradiction.}$$

Hence m (F) =0 (or) that $|f(a+bI)| < \infty$ a.e on E.

5. Logarithm forms of the neutrosop 6 mber [1]:

The power series for $e^{(a+bI)}$ for $a + bI \in \Re$ .we develop the same for log (1 +(a + bI)) for $|a+bI < 1|$

Theorem 5.1:

$$\log(1 + (a+b)) = \frac{(a+bI)}{1!} - \frac{(a+bI)^2}{2!} + \frac{(a+bI)^3}{3!} - \ldots\ldots\ldots \ (|a+bI| < 1)$$

Proof:

The geometric series $\sum_{n=0}^{\infty} (-1)^n (a+bI)^n$ whose radius of convergence is 1 , with partial sum

$S_n(a+bI) = 1 - (-(a+bI))^n / 1 + (a+bI)$ converging to $S(a+bI) = 1/1 + (a+bI)$

we now choose r such that 0< r <1 and restrict $a + bI$ to $|a+bI| \leq r$

if $S_n(a+bI) \to S(a+bI)$ uniformly for $|a+bI| \leq r$ as $n \to \infty$

$$\text{“} \int_a^b f d\alpha = \sum_{n=1}^{\infty} \int_a^b f_n d\alpha \text{“}$$

we get log (1 + (a + bI)) $= \int_0^{a+bI} dt / 1 + t$

$$= \int_0^{a+bI} \sum_{n=0}^{\infty} (-1)^n t^n$$

$$= \sum_{n=0}^{\infty} (-1)^n (a+bI)^{n+1} / n+1$$

$$= \frac{(a+bI)}{1!} - \frac{(a+bI)^2}{2!} + \frac{(a+bI)^3}{3!} - \dots\dots\dots\dots$$

Valid for $|a+bI| \le r < 1$. So, $|a+bI| < 1$.

Conclusion:

In this paper, we defined the logarithm and the exponential form of a neutrosophic real numbers with suitable appropriate proof, and many properties were presented to encapsulated the abstraction of this paper.

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