IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

AN INTRODUCTION TO NEUTRO-Λ-TRI OPEN SETS ON NEUTROSOPHIC- TRI-TOPOLOGICAL SPACES

M.Muthulakshmi[1], P.Yuvashanmuga Sree[2], V.Sukashini[3],R.Vinitha[4]

[1]Department of mathematics, assistant professor of mathematics, Sakthi college of arts and science for women, Oddanchatram, palani main road, Dindigul-626 624,

[2]Department of mathematics, assistant professor of mathematics, Sakthi college of arts and science for women, Oddanchatram, palani main road, Dindigul-626 624,

[3]PG scholar Department of Mathematics, Sakthi college of arts and science for women,

Oddanchathram, palani main road, Dindigul-624 624,

[4]PG scholar Department of Mathematics, Sakthi college of arts and science for women,

Oddanchathram, palani main road, Dindigul-624 624,

Abstract

In this study, a new type of open sets is defined on neutrosophic-tri-topological space and named as neutro-Λ-tri-open sets. This type of set is defined with the concept of intersection. Also, defined neutro-Λ-tri-closed sets, neutro-Λ-tri-t-open (closed) sets in this space with suitable example. Later the interior and closure concept also defined. Some of their basic properties and theorems are stated with proof.

Keywords: Neutro-Λ - tri-open (closed) sets, neutron-Λ-tri- t- open (closed) sets, neutro-Λ-tri-interior, neutro-Λ-tri-closure.

1. Introduction

Fuzzy set was introduced by Lotfi A. Zadeh [1] in 1965 as an extension of the classical notion of sets. The neutrosophic set (NS) was introduced by F. Smarandache [2] as an extension of fuzzy sets. The topology was investigated by Leonhard Euler. The motivation insight behind topology is that some geometric problems depend not on the exact shape of the object. From this, the concept of tri topological space was extended by Martin M. Kovar in 2000.

The notion of neutrosophic Topological space (NTS) was developed by Salma and Albolwi [6] in 2012. Thereafter the concept of neutrosophic bi-topological space was presented by Ozturk and Ozkan [4] in 2019. Later, on Das and Tripathy [17] introduced the pairwise neutrosophic b-open set via neutrosophic bi-topological spaces. After that, the motivation to do research on neutrosophic tri topological space to extend the neutrosophic bi- topological space.

The concept of neutrosophic tri-topological space was extended by Suman Das and Surapati Pramanik [16]. They introduced the different types of open sets and closed sets namely, neutrosophic tri- open sets, neutrosophic tri-closed sets, neutrosophic tri- semi- open sets, neutrosophic tri- pre- open sets etc. Via neutrosphic tri topological space.

1

The main purpose of this paper is to introduce a new type of open set on NTTS, so called as neutron- Λ -tri- open sets. Its complement is named as neutron- Λ -tri- closed set. The concept of interior and closure also defined the concept of neutron- Λ -tri-t-open (closed) set. Few fundamental properties and theorems are given with proof.

This paper is divided as follows: The basic definitions are given in part-2. In part-3, neutro- Λ -tri-open (closed) sets are defined with examples and few properties are given. Also, defined interior and closure. In part-4, neutro- Λ -tri-t-open (closed) sets are defined and few of its properties are given with suitable example. Finally, ended up with a conclusion in part-5.

2. Preliminaries

Definition 2.1. [3] A neutrosophic set M over a universe V is defined as follows:

$$M = \{(n, T_M(n), I_M(n), F_M(n)) : n \in V\}$$

when $T_M(n), I_M(n)$ and $F_M(n)$ are respectively denotes the truth, indeterminacy and falsity membership values of $n \in V$, and such that $^-0 \le T_M(n) + I_M(n) + F_M(n) \le 3^+$ for all $n \in V$.

Definition 2.2. [16] The neutrosophic null set (NSS) (0_N) and the neutrosophic whole set (NWS) (1_N) over a universe V are defined as follows:

i) $0_N = \{(n, 0, 0, 1) : v \in V\}$

ii) $1_N = \{(n, 1, 0, 0) : v \in V\}$. Obviously, $0_N \subseteq 1_N$.

Definition 2.3. [13] Let V be a universe and A_i be a collection of NSs over V. Then, τ is called a neutrosophic topology (NT) on V if the following conditions satisfied:

$i) 0_N, 1_N \in \tau$.

$ii) A_1, A_2 \in \tau \Rightarrow A_1 \cap A_2 \in \tau;$

$iii) \{A_i : i \in \Delta\} \subseteq \tau \Rightarrow \bigcup A_i \in \tau$

In this case the pair (X, τ) is called a neutrosophic topological space.

Definition 2.4.[16] Let V be a universe and τ be some NS over V. Then M an neutrosophic set over V is called **neutrosophic open** if M is open in Neutrosophic topological space.

Definition 2.5.[16] A neutrosophic set M is said to be **neutrosophic closed set** iff M^c is an neutrosophic open set is defined by $M^c = \{n, F_M(n), 1 - I_M(n), T_M(n)\}$.

Definition 2.6.[16] Let V be a universe and τ be a NTSs over V. Then M an NS over V is called neutrosophic tri-open set (N-Tri-OS) if $M \in \tau_1 \cup \tau_2 \cup \tau_3$. A neutrosophic set M is said to be neutrosophic tri-closed set (N-Tri-CS) iff M^c is an N-Tri-OS in $(V, \tau_1, \tau_2, \tau_3)$.

Definition 2.7.[4] Let $(V, \tau_1, \tau_2, \tau_3)$ be a neutrosophic-tri-topological space. The neutrosophic interior of N, denoted by $\text{int}_p^n(N)$, is the neutrosophic union of all neutrosophic open sets of N,

$$\text{i.e., } \text{int}_p^n(N) = \cap \{C \in (\tau_i^n)^c : N \subseteq C\}.$$

It is clear that $\text{int}_p^n(N)$ is the smallest neutrosophic closed set containing N.

Definition 2.8.[4] Let $(V, \tau_1, \tau_2, \tau_3)$ be a neutrosophic-tri-topological space. The neutrosophic closure of N, denoted by $cl_p^n(N)$, is the neutrosophic intersection of all neutrosophic closed sets of N,

$$\text{i.e., } cl_p^n(N) = \cap \{C \in (\tau_i^n)^c : N \subseteq C\}.$$

It is clear that $cl_p^n(N)$ is the smallest neutrosophic closed set containing N.

Definition 2.9.[9] Let V be a non-empty set and τ_1, τ_2 and τ_3 are the three topologies on V. The set V together with three topologies is called a tri-topological space and is denoted by $(V, \tau_1, \tau_2, \tau_3)$.

Definition 2.10.[16] Let $(V, \tau_1), (V, \tau_2)$ and (V, τ_3) be any three different NTSs. Then, the structure $(V, \tau_1, \tau_2, \tau_3)$ is called a neutrosophic tri-topological space (NTTS).

3. Neutro- Λ -tri open sets

Definition 3.1 Let $(V, \tau_1, \tau_2, \tau_3)$ be a NTTS over a universe V. Then a NS M^Λ over V is said to be a neutro- Λ -tri-open set (N$^\Lambda$TOS) if M^Λ belongs to the collection of finite intersection of NOSs $A_{i \in I} \in \tau_{i=1,2,3}$ respectively.

$$\text{(ie.,) } M^\Lambda \in \tau_1 \cap \tau_2 \cap \tau_3.$$

where τ_1, τ_2, τ_3 are three different topologies. A NS M^Λ is said to be neutro- Λ -tri closed set N$^\Lambda$TCS if $(M^\Lambda)^c$ is a N$^\Lambda$TOS

in (V, τ_1, τ_2, τ_3).

Definition3.2 Let $(V, \tau_1, \tau_2, \tau_3)$ be a NTTS over a universe V. Then its union is defined by,

$$M_i^\Lambda = \{(v, T_{A_i}(v), I_{A_i}(v), F_{A_i}(v)) : v \in V\}.$$

where,

$$T_{A_i}(v) = \max\{T_{A_1}(v), T_{A_2}(v), T_{A_3}(v)\},$$

$$I_{A_i}(v) = \min\{I_{A_1}(v), I_{A_2}(v), I_{A_3}(v)\} \text{ and}$$

$$F_{A_i}(v) = \min\{F_{A_1}(v), F_{A_2}(v), F_{A_3}(v)\}.$$

Definition3.3 Let $(V, \tau_1, \tau_2, \tau_3)$ be a NTTS over a universe *V*. Then its intersection is defined by,

$$M_i^{\wedge} = \{(v, T_{A_i}(v), I_{A_i}(v), F_{A_i}(v)) : v \in V\}$$

where,

$$T_{A_i}(v) = \min\{T_{A_1}(v), T_{A_2}(v), T_{A_3}(v)\},$$

$$I_{A_i}(v) = \max\{I_{A_1}(v), I_{A_2}(v), I_{A_3}(v)\} \text{ and}$$

$$F_{A_i}(v) = \max\{F_{A_1}(v), F_{A_2}(v), F_{A_3}(v)\}.$$

Definition 3.4 Let $\{M_i^{\wedge} / i \in I\}$ be a collection of NTTS over a universe *V*. Then,

$$\bigcup_{i \in I} M_i^{\wedge} = \{(v, \max T_{A_i}(v), \min I_{A_i}(v), \min F_{A_i}(v))\},$$

$$\bigcap_{i \in I} M_i^{\wedge} = \{(v, \min T_{A_i}(v), \max I_{A_i}(v), \max F_{A_i}(v))\}.$$

Definition 3.5 In a NS *M* over *V* and the null set (0_{nt}) and the whole set (1_{nt}) is given by

$$0_{nt} = \{(V, 0, 0, 1) : v \in V\}$$

$$1_{nt} = \{(V, 1, 0, 0) : v \in V\}$$

Therefore, $0_{nt} \subseteq 1_{nt}$.

Example 3.6 Let $V = \{r, s, t\}$ be a non-empty set and the NSs A_1, A_2, A_3 be defined as

A_1 = {(r, 0.8, 0.2, 0.5), (s, 0.9, 0.1, 0.1), (t, 1.0, 0.7, 0.2)},

A_2 = {(r, 0.6, 0.9, 0.1), (s, 0.8, 0.9, 0.5), (t, 0.3, 0.2, 0.1)} and

A_3 = {(r, 0.2, 0.6, 0.8), (s, 0.2, 0.3, 0.4), (t, 0.4, 0.9, 1.0)}.

Then $\tau_1 = \{0_{nt}, 1_{nt}, A_1\}$, $\tau_2 = \{0_{nt}, 1_{nt}, A_2\}$, $\tau_3 = \{0_{nt}, 1_{nt}, A_3\}$ are three different NTs over V.

Thus $(V, \tau_1, \tau_2, \tau_3)$ is a NTTS.

Then the collection of $N^{\wedge}$TOS are $M_i^{\wedge} = \{M_1^{\wedge}, M_2^{\wedge}, M_3^{\wedge}\}$,

where

$M_1^{\wedge} = A_1 \cap A_2 = \{(r, 0.6, 0.9, 0.5), (s, 0.8, 0.9, 0.5), (t, 0.1, 0.7, 0.2)\}$,

$M_2^{\wedge} = A_1 \cap A_3 = \{(r, 0.2, 0.6, 0.8), (s, 0.3, 0.3, 0.4), (t, 0.1, 0.9, 0.1)\}$ and

$M_3^{\wedge} = A_2 \cap A_3 = \{(r, 0.2, 0.9, 0.8), (s, 0.2, 0.9, 0.5), (t, 0.3, 0.9, 1.0)\}$.

Thus $M_1^{\wedge}, M_2^{\wedge}$ and $M_3^{\wedge}$ are $N^{\wedge}$TOSs.

Also, the collection of $N^{\wedge}$ TCS are $(M_i^{\wedge})^c = \{(M_1^{\wedge})^c, (M_2^{\wedge})^c, (M_3^{\wedge})^c\}$,

where

$(M_1^{\wedge})^c =$ {(*r*, 0.5, 0.1, 0.6), (*s*, 0.5, 0.1, 0.8), (*t*, 0.2, 0.3, 0.2)},

$(M_2^{\wedge})^c =$ {(*r*, 0.8, 0.4, 0.2), (*s*, 0.4, 0.7, 0.3), (*t*, 0.1, 0.1, 0.1)} and

$(M_3^{\wedge})^c =$ {(*r*, 0.8, 0.1, 0.2), (*s*, 0.5, 0.1, 0.2), (*t*, 1.0, 0.1, 0.3)}.

Thus $(M_1^{\wedge})^c$, $(M_2^{\wedge})^c$, $(M_3^{\wedge})^c$ are $N^{\wedge}$TCSs.

Remark 3.7 Let $(V, \tau_1, \tau_2, \tau_3)$ be a NTTS. Then, the collection of $N^{\wedge}TOS$ does not form a NTS over *V*.

Remark 3.7 is illustrated in the following example.

Example 3.8 Consider the Example 3.6,

Here $M_i^{\wedge}$ does not satisfies the axioms of NTS.

Since $M_1^{\wedge} \cup M_2^{\wedge} =$ {(r, 0.6, 0.6, 0.5), (s, 0.8, 0.3, 0.4), (t, 0.1, 0.7, 0.2)} $\notin M_i^{\wedge}$.

$M_1^{\wedge} \cap M_2^{\wedge} =$ {(r, 0.2, 0.6, 0.8), (s, 0.3, 0.9, 0.5), (t, 0.1, 0.9, 1.0)} $\notin M_i^{\wedge}$.

Hence the collection of $N^{\wedge}TOS$ and $M_i^{\wedge}$ does not form a NTS over *V*.

Theorem 3.9 Let $(V, \tau_1, \tau_2, \tau_3)$ be a NTTS. Then, a neutrosophic null set (0_{nt}) and a neutrosophic whole set (1_{nt}) are always $N^{\wedge}$ TOS.

Proof Let $(V, \tau_1, \tau_2, \tau_3)$ be a NTTS and A_1, A_2 and A_3 be any three NOSs.

Suppose that NNS 0_{nt} is defined as

$$0_{nt} = A_1 \cap A_2 \cap A_3$$

where $A_1 = 0_{nt}, A_2 = 0_{nt}$ and $A_3 = 0_{nt}$.

Then A_1, A_2 and A_3 are NOSs.

Thus $A_1 \in \tau_1, A_2 \in \tau_2, A_3 \in \tau_3$.

Hence 0_{nt} is a $N^{\wedge}$ TOS.

Similarly, Let $(V, \tau_1, \tau_2, \tau_3)$ be a NTTS and A_1, A_2 and A_3 be any three NOSs.

Suppose that NWS 1_{nt} is defined as

$$1_{nt} = A_1 \cap A_2 \cap A_3$$

where $A_1 = 1_{nt}$, $A_2 = 1_{nt}$ and $A_3 = 1_{nt}$.

Then A_1, A_2 and A_3 are NOS.

Thus $A_1 \in \tau_1, A_2 \in \tau_2, A_3 \in \tau_3$.

Hence 1_{nt} is a $N^{\wedge}TOS$.

Preposition 3.10 Let $(V, \tau_1, \tau_2, \tau_3)$ be a NTTS over a universe V and A_1, A_2 and A_3 be any three NOSs. Then,

$i) A_1 \cup A_2 \cup A_3 = [A_1 \cup A_2] \cup A_3$ and

$A_1 \cap A_2 \cap A_3 = [A_1 \cap A_2] \cap A_3$.

$ii) A_1 \cup [A_2 \cap A_3] = [A_1 \cup A_2] \cap [A_1 \cup A_3]$ and

$A_1 \cap [A_2 \cup A_3] = [A_1 \cap A_2] \cup [A_1 \cap A_3]$

$iii) A_1 \cup 0_{nt} = A_1$ and $A_1 \cap 0_{nt} = 0_{nt}$

$iv) A_1 \cup 1_{nt} = 1_{nt}$ and $A_1 \cap 1_{nt} = 0_{nt}$.

Proof Obvious.

Definition 3.11 Let $(V, \tau_1, \tau_2, \tau_3)$ be a NTTS over the universe V. Then the $N^{\wedge}$-interior and $N^{\wedge}$-closure of $M^{\wedge}$ and $(M^{\wedge})^c$ is defined by,

i) Union of all $N^{\wedge}TOS$ contained in $M^{\wedge}$.

ii) Intersection of all $N^{\wedge}TCS$ contained in $(M^{\wedge})^c$.

Theorem 3.12 Every NOS need not be a $N^{\wedge}$ TOS and vice versa.

Theorem 3.12 is illustrated in the following example.

Example 3.13 Consider the Example 3.6

Here, A_1 is NOS but not $N^{\wedge}$ TOS

Here, $M_1^{\wedge}$ is $N^{\wedge}$ TOS but not NOS.

Theorem 3.14 Every NCS need not be an $N^{\wedge}$ TCS and vice versa.

Theorem 3.14 is illustrated in the following example.

Example 3.15 Consider the Example 3.6

Here $(M_1^{\wedge})^c$ is a $N^{\wedge}$ TCS but not a NCS.

Here A_1 is a NCS but not a $N^{\wedge}$ TCS.

Remark 3.16 The union of two $N^{\wedge}$ TOSs need not be a $N^{\wedge}$ TOS.

Example 3.17 Let V={r, s, t} be a non-empty set and NSs A_1, A_2, B_1, C_1 and C_2 can be defined as,

A_1 = {(r, 1.0, 0.2, 0.5), (s, 0.9, 0.4, 0.3), (t, 0.8, 0.2, 0.5)},

A_2 = {(r, 0.7, 0.4, 0.6), (s, 0.8, 0.6, 0.5), (t, 0.8, 0.7, 0.9)},

B_1 = {(r, 0.4, 0.2, 0.2), (s, 0.7, 0.3, 0.1), (t, 0.9, 0.2, 0.4)},

C_1 = {(r, 0.9, 0.3, 0.5), (s, 0.8, 0.2, 0.5), (t, 1.0, 0.1, 0.1)},

C_2 = {(r, 0.8, 0.4, 0.9), (s, 0.4, 0.7, 0.6), (t, 0.1, 0.4, 0.3)} and

C_3 = {(r, 0.9, 0.3, 0.1), (s, 0.9, 0.1, 0.3), (t, 1.0, 0.1, 0.1)}.

Then $\tau_1 = \{0_{nt}, 1_{nt}, A_1, A_2\}, \tau_2 = \{0_{nt}, 1_{nt}, B_1\}, \tau_3 = \{0_{nt}, 1_{nt}, C_1, C_2, C_3\}$ are the three different NTs over V.

Thus $(V, \tau_1, \tau_2, \tau_3)$ is a NTTS.

Thus the collection of N$^\wedge$TOS are $M_i^\wedge = \{M_1^\wedge, M_2^\wedge, M_3^\wedge\}$.

where

$M_1^\wedge$ = {(r, 0.4, 0.2, 0.5), (s, 0.7, 0.4, 0.3), (t, 0.8, 0.2, 0.5)},

$M_2^\wedge$ = {(r, 0.4, 0.4, 0.6), (s, 0.7, 0.6, 0.5), (t, 0.8, 0.7, 0.9)},

$M_3^\wedge$ = {(r, 0.9, 0.3, 0.5), (s, 0.8, 0.4, 0.5), (t, 0.8, 0.2, 0.5)},

$M_4^\wedge$ = {(r, 0.1, 0.4, 0.9), (s, 0.4, 0.7, 0.6), (t, 0.1, 0.2, 0.5)},

$M_5^\wedge$ = {(r, 0.9, 0.3, 0.5), (s, 0.9, 0.4, 0.3), (t, 0.1, 0.2, 0.5)},

$M_6^\wedge$ = {(r, 0.7, 0.4, 0.6), (s, 0.8, 0.6, 0.5), (t, 0.8, 0.7, 0.9)},

$M_7^\wedge$ = {(r, 0.7, 0.4, 0.9), (s, 0.4, 0.7, 0.6), (t, 0.1, 0.7, 0.9)},

$M_8^\wedge$ = {(r, 0.4, 0.3, 0.5), (s, 0.7, 0.3, 0.5), (t, 0.9, 0.2, 0.4)},

$M_9^\wedge$ = {(r, 0.4, 0.2, 0.9), (s, 0.4, 0.7, 0.6), (t, 0.1, 0.4, 0.4)} and

$M_{10}^\wedge$ = {(r, 0.4, 0.3, 0.2), (s, 0.7, 0.3, 0.3), (t, 0.9, 0.2, 0.4)}.

Thus $M_1^\wedge, M_2^\wedge, M_3^\wedge, M_4^\wedge, M_5^\wedge, M_6^\wedge, M_7^\wedge, M_8^\wedge, M_9^\wedge$ and $M_{10}^\wedge$ are N$^\wedge$TOSs.

Also, the collection of N$^\wedge$TCS are,

$$(M_i^\wedge)^c = \{(M_1^\wedge)^c, (M_2^\wedge)^c, (M_3^\wedge)^c\},$$

where

$(M_1^\wedge)^c = \{(r, 0.5, 0.8, 0.4), (s, 0.3, 0.6, 0.7), (t, 0.5, 0.8, 0.8)\}$,

$(M_2^\wedge)^c = \{(r, 0.6, 0.6, 0.4), (s, 0.5, 0.4, 0.7), (t, 0.9, 0.3, 0.8)\}$,

$(M_3^\wedge)^c = \{(r, 0.5, 0.7, 0.9), (s, 0.5, 0.6, 0.8), (t, 0.5, 0.8, 0.8)\}$,

$(M_4^\wedge)^c = \{(r, 0.9, 0.6, 0.1), (s, 0.6, 0.3, 0.4), (t, 0.5, 0.8, 0.1)\}$,

$(M_5^\wedge)^c = \{(r, 0.5, 0.7, 0.9), (s, 0.3, 0.6, 0.9), (t, 0.5, 0.8, 0.1)\}$,

$(M_6^\wedge)^c = \{(r, 0.6, 0.6, 0.7), (s, 0.5, 0.4, 0.8), (t, 0.9, 0.3, 0.8)\}$,

$(M_7^\wedge)^c = \{(r, 0.9, 0.6, 0.7), (s, 0.6, 0.3, 0.4), (t, 0.9, 0.3, 0.1)\}$,

$(M_8^\wedge)^c = \{(r, 0.5, 0.7, 0.4), (s, 0.5, 0.7, 0.7), (t, 0.4, 0.8, 0.9)\}$,

$(M_9^\wedge)^c = \{(r, 0.9, 0.8, 0.4), (s, 0.6, 0.3, 0.4), (t, 0.4, 0.8, 0.9)\}$ and

$(M_{10}^\wedge)^c = \{(r, 0.2, 0.7, 0.4), (s, 0.3, 0.7, 0.7), (t, 0.4, 0.8, 0.9)\}$.

Thus $(M_1^\wedge)^c, (M_2^\wedge)^c, (M_3^\wedge)^c, (M_4^\wedge)^c, (M_5^\wedge)^c, (M_6^\wedge)^c, (M_7^\wedge)^c, (M_8^\wedge)^c, (M_9^\wedge)^c$ and $(M_{10}^\wedge)^c$ are N$^\wedge$TCSs.

Remark 3.18 Union of any two N$^\wedge$TOS need not be a N$^\wedge$TOS.

Remark 3.18 is illustrated in the following Example.

Example 3.19 Consider the Example 3.17.

Here $M_5^\wedge$ and $M_6^\wedge$ are N$^\wedge$TOS.

Then $M_5^\wedge \cup M_6^\wedge = \{(r, 0.6, 0.5, 0.4), (s, 0.7, 0.8, 0.8), (t, 0.2, 0.4, 0.3)\}$.

Thus $M_5^\wedge \cup M_6^\wedge$ is not a N$^\wedge$TOS.

Remark 3.20 Intersection of two N$^\wedge$TOS need not be a N$^\wedge$TOS.

Remark 3.20 is illustrated in the following example.

Example 3.21 Consider the Example 3.17.

Here $M_5^\wedge$ and $M_6^\wedge$ are N$^\wedge$TOS.

Then $M_5^\wedge \cap M_6^\wedge = \{(r, 0.6, 0.7, 0.9), (s, 0.1, 1.0, 0.8), (t, 0.1, 0.8, 0.9)\}$

Then $M_5^\wedge \cap M_6^\wedge$ is not a $N^\wedge$TOS.

Therefore $M_5^\wedge \cap M_6^\wedge$ is not N$^\wedge$TOS.

Theorem 3.22 Every NCS need not be a $N^{\wedge}$ TCS and vice versa.

Theorem 3.22 is illustrated in the following example.

Example 3.23 Consider the Example 3.17.

Here $(M_1^{\wedge})^c$ is a N $^{\wedge}$ TCS but not a NCS.

Here A_1 is a NCS but not a N $^{\wedge}$ TCS

Remark 3.24 The union of any two N $^{\wedge}$ TCSs need not be a N $^{\wedge}$ TCS.

Example 3.25 Consider the Example 3.17.

Here $(M_5^{\wedge})^c$ and $(M_6^{\wedge})^c$ are N $^{\wedge}$ TCSs.

Then $(M_5^{\wedge})^c \cup (M_6^{\wedge})^c = \{$(r, 0.6, 0.6, 0.7), (s, 0.5, 0.4, 0.8), (t, 0.9, 0.3, 0.1)$\}$, is

not a N $^{\wedge}$ TCS.

Remark 3.26 The intersection of any two N $^{\wedge}$ TCSs need not be a N $^{\wedge}$ TCS.

Remark 3.26 is illustrated in the following example.

Example 3.27 Consider the example 3.17.

Here $(M_5^{\wedge})^c$ and $(M_6^{\wedge})^c$ are N $^{\wedge}$ TCSs.

Then $(M_5^{\wedge})^c \cap (M_6^{\wedge})^c = \{$(r, 0.5, 0.7, 0.9), (s, 0.3, 0.6, 0.9), (t, 0.5, 0.8, 0.8)$\}$, is

not a N $^{\wedge}$ TCS.

Preposition 3.28 Let $(V, \tau_1, \tau_2, \tau_3)$ be a NTTS over a universe V and A_1^c and A_2^c be any two NTS. Then,

$$i)\ (A_1 \cup A_2)^c = A_1^c \cap A_2^c$$

$$ii)\ (A_1 \cap A_2)^c = A_1^c \cup A_2^c.$$

Proof i) Let $(V, \tau_1, \tau_2, \tau_3)$ be a NTTS over a universe V. Let $v \in V$. Then,

$$A_1 \cup A_2 = \{\langle v, \max(T_{A_1}(v), T_{A_2}(v)), \min(I_{A_1}(v), I_{A_2}(v)), \min(F_{A_1}(v), F_{A_2}(v))\rangle\}$$

$$(A_1 \cup A_2)^c = \{\langle v, \min(F_{A_1}(v), F_{A_2}(v)), 1 - \min(I_{A_1}(v), I_{A_2}(v)), \max(T_{A_1}(v), T_{A_2}(v))\rangle\}$$

Now,

$$A_1^c = \{\langle v, F_{A_1}(v), 1 - I_{A_1}(v), T_{A_1}(v)\rangle\}$$

$$A_2^c = \{\langle v, F_{A_2}(v), 1 - I_{A_2}(v), T_{A_2}(v)\rangle\}$$

From this we can say that

$$A_1^c \cap A_2^c = \{(v, \min(F_{A_1}(v), F_{A_2}(v)), 1 - \min(I_{A_1}(v), I_{A_2}(v)), \max(T_{A_1}(v), T_{A_2}(v)))\}$$

Hence, $(A_1 \cup A_2)^c = A_1^c \cap A_2^c$.

ii) This can be proved by similar way.

4. Λ -Neutrosophic- t- open sets

Definition 4.1 Let $(V, \tau_1, \tau_2, \tau_3)$ be NTTS over the universe V and $M_1^\Lambda, M_2^\Lambda, M_3^\Lambda$ be any three $N^\Lambda TOS$. Then, P^Λ is said to be Λ - Neutrosophic-tri-t-open set (N$^\Lambda$T-t-OS) if,

$$P^\Lambda = M_1^\Lambda \cup M_2^\Lambda \cup M_3^\Lambda$$

Let $(V, \tau_1, \tau_2, \tau_3)$ be NTTS over the universe V. Then, P^c is said to be Λ -

Neutrosophic-tri-t-closed set (N$^\Lambda$T-t-CS) if P is a N$^\Lambda$T-t-OS in $(V, \tau_1, \tau_2, \tau_3)$.

Definition 4.2 Let $(V, \tau_1, \tau_2, \tau_3)$ be a NTTS over a universe V. Then the union of all $N^\Lambda TOS$ is defined by,

$$P_i^\Lambda = \{(v, T_{M_i^\Lambda}(v), I_{M_i^\Lambda}(v), F_{M_i^\Lambda}(v)) : v \in V\}.$$

11

where

$$T_{M_i^\Lambda}(v) = \max\{T_{M_1^\Lambda}(v), T_{M_2^\Lambda}(v), T_{M_3^\Lambda}\},$$

$$I_{M_i^\Lambda}(v) = \min\{I_{M_1^\Lambda}(v), I_{M_2^\Lambda}(v), I_{M_3^\Lambda}(v)\} \text{ and}$$

$$F_{M_i^\Lambda}(v) = \min\{F_{M_i^\Lambda}(v), F_{M_2^\Lambda}(v), F_{M_3^\Lambda}(v)\}.$$

Let $(V, \tau_1, \tau_2, \tau_3)$ be a NTTS over a universe V. Then the intersection of all $N^\Lambda TCS$ contained in P^c.

Definition 4.3 Let $(V, \tau_1, \tau_2, \tau_3)$ be a NTTS over a universe V. Then the intersection of all $N^\Lambda TCS$ is defined by,

$$P_i^\Lambda = \{v, T_{M_i^\Lambda}(v), I_{M_i^\Lambda}(v), F_{M_i^\Lambda}(v) : v \in V\}.$$

where

$$T_{M_i^\Lambda}(v) = \min\{T_{M_1^\Lambda}(v), T_{M_2^\Lambda}(v), T_{M_3^\Lambda}\},$$

$$I_{M_i^\Lambda}(v) = \max\{I_{M_1^\Lambda}(v), I_{M_2^\Lambda}(v), I_{M_3^\Lambda}(v)\} \text{ and}$$

$$F_{M_i^\Lambda}(v) = \max\{F_{M_i^\Lambda}(v), F_{M_2^\Lambda}(v), F_{M_3^\Lambda}(v)\}.$$

Example 4.4 Consider the Example 3.17, Here $M_1^{\wedge}, M_3^{\wedge}, M_9^{\wedge}$ are N$^{\wedge}$ TOSs.

Then $P^{\wedge} = M_1^{\wedge} \cup M_3^{\wedge} \cup M_9^{\wedge}$

$$= \{(r, 0.9, 0.7, 0.4), (s, 0.6, 0.3, 0.4), (t, 0.5, 0.6, 0.9)\} \text{ is a } N^{\wedge} - T - t - OS.$$

Also, $(P^{\wedge})^c = \{(r, 0.4, 0.3, 0.9), (s, 0.4, 0.7, 0.6), (t, 0.9, 0.4, 0.4)\}$ is a $N^{\wedge} - T - t - CS$.

Theorem 4.5 Let $(V, \tau_1, \tau_2, \tau_3)$ be NTTS over *V*. Then, the union of any two N$^{\wedge}$ T-t-OS is a N$^{\wedge}$ T-t-OS.

Proof Let *P* and *S* be any two N$^{\wedge}$ T-t-OSs in NTTS.

If $P = P_1 \cup P_2 \cup P_2$ and $S = S_1 \cup S_2 \cup S_3$. Then,

$$P \cup S = (P_1 \cup P_2 \cup P_3) \cup (S_1 \cup S_2 \cup S_3)$$

12

$$= (P_1 \cup S_1) \cup (P_2 \cup S_2) \cup (P_3 \cup S_3)$$

Since $(P_1 \cup S_1), (P_2 \cup S_2)$ and $(P_3 \cup S_3)$ are N$^{\wedge}$ T-t-OS.

Hence, $P \cup S$ is a N$^{\wedge}$ T-t-OS.

Remark 4.6 The intersection of any two N$^{\wedge}$ T-t-OS need not be a N$^{\wedge}$ TOS.

Example 4.7 Consider the Example 3.17

Here $P_1^{\wedge}$ and $P_2^{\wedge}$ are N$^{\wedge}$ T-t-OS.

`Let $P_1^{\wedge} = M_1^{\wedge} \cup M_3^{\wedge} \cup M_9^{\wedge}$ and $P_2^{\wedge} = M_2^{\wedge} \cup M_4^{c} \cup M_8^{c}$.

Then $P_1^{\wedge} \cap P_2^{\wedge} = \{(r, 0.4, 0.7, 0.5), (s, 0.6, 0.3, 0.5), (t, 0.5, 0.6, 0.9)\}$ is not a N$^{\wedge}$ TOS.

Theorem 4.8 The union of any N$^{\wedge}$ T-t-CS need not be N$^{\wedge}$ TCS.

Theorem 4.8 is illustrated in the following example.

Example 4.9 Consider the Example 3.17.

Here $(P_1^{\wedge})^c$ and $(P_2^{\wedge})^c$ are N$^{\wedge}$ T-t-CS.

Let $(P_1^{\wedge})^c = M_1^{\wedge} \cup M_3^{\wedge} \cup M_9^{\wedge}$ and $(P_2^{\wedge})^c = M_2^{\wedge} \cup M_4^{c} \cup M_8^{c}$.

Then $(P_1^{\wedge})^c \cup (P_2^{\wedge})^c = \{(r, 0.9, 0.7, 0.4), (s, 0.6, 0.3, 0.4), (t, 0.5, 0.8, 0.8)\}$ is not N$^{\wedge}$ TCS.

Theorem 4.10 The intersection of any N$^{\wedge}$ T-t-CS need not a N$^{\wedge}$ TCS.

Theorem 4.10 is illustrated in following example.

Example 4.11 Consider the Example 3.17.

Here $(P_1^{\wedge})^c$ and $(P_2^{\wedge})^c$ are N^T-t-CS.

Let $(P_1^{\wedge})^c = M_1^{\wedge} \cup M_3^{\wedge} \cup M_9^{\wedge}$ and $(P_2^{\wedge})^c = M_2^{\wedge} \cup M_4^c \cup M_8^c$.

Here $(P_1^{\wedge})^c \cap (P_2^{\wedge})^c = \{(r,0.4,0.7,0.5),(s,0.6,0.3,0.5),(t,0.5,0.6,0.9)\}$ is not a N^TCS.

Theorem 4.12 Every N^OS need not be a N^T-t-OS.

Theorem 4.12 is illustrated in the following example.

Example 4.13 Consider the Example 3.17.

13

Here $P^{\wedge}$ is a N^T-t-OS and $M^{\wedge}$ is a N^TOS.

Then $P^{\wedge} = \{(r,0.9,0.7,0.4),(s,0.6,0.3,0.4),(t,0.5,0.6,0.9)\}$ is not a N^TOS.

Theorem 4.14 Consider the Example 3.17.

Here $(P^{\wedge})^c$ is a N^T-t-CS and $(M^{\wedge})^c$ is a N^TCS.

Then $(P^{\wedge})^c = \{(r,0.4,0.3,0.9),(s,0.4,0.7,0.6),(t,0.9,0.4,0.4)\}$ is not a N^TCS.

5. Conclusion

In this paper, a notion of neutro-Λ-tri open set is introduced with the concept of intersection of NOSs on NTTS. Also, defined its complement as neutro-Λ- tri-closed set, with appropriate examples. Later neutro- Λ-tri- interior and neutro-Λ-tri-closure are defined. Few of their theorems are stated and proved. Further, defined a new class of open sets called neutro- Λ-tri-t-open set. Also, discussed some of its basic properties and theorems with suitable examples.

Reference

[1] Zadeh,L.A.(1965). Fuzzy sets. *Informatiom and control*, 8(3), 338-353.

[2] F. Smarandache, Neutrosophy. *Neutrosophic Probability, Set, and Logic*. Amer. Res. Press, Rehoboth, USA, 105 p., 1998, 2000, 2002, 2005, 2006.

[3] Ozturk, T.Y., & Ozkan, A. (2019). *Neutrosophic bitopological spaces. Neutrosophic sets and systems*, 30, 88-97.

[4]Ozturk, Taha Yasin and Alkan Ozkan. "*Neutrosophic Bitopological Spaces.*" Neutrosophic Sets and Systems 30. (2019).

[5] Arokiarani,I., Dhavaseelan,R., Jafari,S., & Parimala,M.(2017). On some new notations and functions in neutrosophic topological spaces. *Neutrosophic Sets and Systems*, 16,16-19.

[6] 1.Salama.A.A, 2 .Alblowi.S.A. 1Egypt, Port Said University *Neutrosophic Set and Neutrosophic Topological* Spaces., Faculty of Sciences Department of Mathematics and Computer Science 2Department of Mathematics, King Abdulaziz University, Saudi Arabia.

[7] Kelly.J.C (1963). *Bitopological spaces.* Proceeding of the London Mathematical society, 3(1), 71-89.

[8] Shanthi,V.K., Chandrasekar,S., Safina, & Begam,K.(2018). *Neutrosophic generalized semi closed sets in neutrosophic topological space.* International journal of research in Advent Technology, 67,1739-1743.

[9] Palaniammal.S &Somasundaram.S *A study of Tri Topological Space*,2 (2011).

[10] Samarandache, F.(1998). *A unifying field in logics, neutrosophy: neutrosophic probability, set and logic.* Rehoboth, American Research Press.

[11] Ebenanjar, E., Immaculate, J., & Wilfred, C.B. (2018). *On neutrosophic b-open sets in neutrosophic topological space.* Journal of Physics Conference Series, 1139(1), 012062.

[12] Maheswari, C., Sathyabama, M., & Chandrasekar, S.(2018). *Neutrosophic generalized b- closed sets in neutrosophic topological spaces.* Journal

[13] Salama, A.A.., & Alblowi, S.A. (2012). *Neutrosophic set and neutrosophic topological space*. ISOR Journal of mathematics, 3(4), 31-35.

[14] Salama,A.A, Samarandache,F.& Alblowi,S.A (2014). *New neutrosophic crisp topological concepts.* Neutrosophic sets and systems,4, 50-54.

[15] Al-Hamido,R.k., & Gharibah, T.(2018). *Neutrosophic crisp tri-topological spaces.* Jounal of New Theory,23,13-21.

[16] Suman das & Surapati Pramanik , *Neutrosophic Tri-Topological space*, vol.45,2021.

[17] Das and Tripathy (2020), *Pairwise Neutrosophic-b-Open Set in Neutrosophic Bitopological Space*s.