ISSN: 2320-2882

IJCRT.ORG



# **INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)**

An International Open Access, Peer-reviewed, Refereed Journal

# Supra-Tri-Semi-Pre Open Sets and t-open sets in Supra Tri-Topological Space

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#### Abs<mark>tract</mark>

The purpose of this paper is to introduce a new type of topological space by combining supra topological space and tri-topological space called supra-tri-topological space. Also, some types of open sets are defined in this space such as semi-open sets, pre-open sets, semi-pre-open sets and t-open sets. The concept of interior and closure are defined all these types of sets. Later, few basic theorems are proved and given suitable examples.

**Keywords** Supra tri-topological space, Supra-tri-open (closed) sets, supra-tri-interior (closure), supra-tri-semi-open (closed) sets, supra-tri-semi-pre-open (closed) sets, supra-tri-tri-topen (closed) sets.

#### 1. Introduction

The concept of bitopological spaces was introduced by J.C.Kelly [12].Tri topological space is a generalization of bi-topological space. The study of tri-topological space was first initiated by Martin M. Kovar[14]. S. Palaniammal [6] study tri topological space. U. D. Tapi [11] introduced semi open sets and pre open set in tri-topological space. Levin (1963) [9] introduced the notion of semi-open set [resp. pre-open (Mashhour et al. 1981a)], the complement of a semi-open set (resp. pre-open (Mashhour et al. 1981a)]. Andrijevi´c (1986) [8] introduced the class of semi-pre-open sets in topological spaces. Since then many authors including Andrijevi´c have studied this class of sets by defining their neighbourhoods, separation axioms and functions. The study of Supra topological space was first initiated by A.S. Mashhour, A.A. Allam, F.S. Mahmoud and F.H.Khedr,[7], on supra topological space(1983). Al-shami introduced and studied supra open (supra closed, supra homeomorphism) maps in supra topological ordered space.In 2017, Al-Shami [3] introduced the semi open sets in supra

topology named supra semi open sets. In 2010, O.R. Sayed [5] introduced supra pre-open sets and supra pre-continuity on topological space.

The objective of this study is to introduce a new topological space named as supra-tri-topological space, which is a combination of supra-topological space and tri-topological space. Later the idea of open sets like semi, pre, semi-pre and t-open sets are defined in this space. The approach on interior and closure are also studied. Further, the essential results are proved with appropriate examples.

The following are the separated sections of this paper.

In section-2, the necessary definitions are given. In section-3, the supra-tri-topological space is introduced with examples. In section-4, semi, pre and semi-pre open sets are studied in this space. In section 5, supra-tri-t-open set is examined with basic theorems. At last, the conclusion is given in section-6.

#### 2. Preliminaries

**Definition 2.1[6]** Let X be a non-empty set and  $\tau_1, \tau_2$  and  $\tau_3$  are three topologies on X. The set X together with three topologies is called a tri-topological space (TTS) and is denoted by  $(X, \tau_1, \tau_2, \tau_3)$ .

**Definition 2.2[1]** A family  $\mu$  of subsets of a non-empty set *X* is called a supra topology (ST) provided that the following two conditions hold,

- (1) *X* and  $\phi \in \mu$
- (2)  $\mu$  is closed under arbitrary union.

Then the pair  $(X, \mu)$  is called a supra topological space(STS). Every element of  $\mu$  is called a supra open set and its complement is called a supra closed set.

**Definition 2.3[3]** A subset A of  $(X, \tau)$  is called supra semi open set (SSOS) if  $E \subseteq cl(int(E))$ . The complement of a SSOS is supra semi closed set (SSCS).

**Definition 2.4[5]** A set A is supra pre-open if  $A \subseteq Int^{\mu}(Cl^{\mu}(A))$ . The complement of supra pre-open is called supra pre-closed. Thus A is supra pre-closed if and only if  $A \subseteq Cl^{\mu}(Int^{\mu}(A)) \subseteq A$ .

**Definition 2.5[8]** A subset A of a topological spaces is called semi-pre-open set if A is a subset of cl(int(cl(A))). The complement of semi-pre-open set is semi-pre-closed set.

**Definition 2.6[2]** A subset of a topological space (TS)  $(X, \tau)$  is called pre-semi open set (PSOS) if  $A \subseteq int (cl(int(A)))$ . The complement of a PSOS is called pre-semi closed set (PSCS).

**Definition 2.7[4]** Let E be a subset of a supra topological space  $(X, \mu)$ . The supra closure of E (denoted by scl(E)) is the intersection of all supra closed sets containing E and the supra interior of E (denoted by sint(E)) is the union of all supra open sets contained in E.

#### 3. Supra tri-topological space

**Definition 3.1** A space  $(X, \tau_1^{st}, \tau_2^{st}, \tau_3^{st})$  is said to be a supra-tri-topological space (STTS) over the universe X equiped with three supra topological space  $\tau_1^{st}, \tau_2^{st}$  and  $\tau_3^{st}$  are three different STs if it satisfies the following axioms

i) 
$$\phi, X \in \tau_i^{st}$$
  
ii)  $\bigcup A \in \tau_i^{st}$ 

where *A* is a subset of *X*. Then the collection of supra-tri-open sets  $T^{st}$  is defined as  $T^{st} = \tau_1^{st} \cup \tau_2^{st} \cup \tau_3^{st}$ . Then  $\mathfrak{I}^{st}$  is said to be supra tri-open set (STOS) if  $\mathfrak{I}^{st} \in T^{st}$ . The complement of STOS is supra tri-closed set (STCS).

**Example 3.2** Let  $X = \{s, t, u, v\}$ , and

$$\tau_1^{st} = \{\phi, X, \{s, u\}, \{t, v\}\}$$
  
$$\tau_2^{st} = \{\phi, X, \{s, t\}, \{u, v\}\} \text{ and }$$
  
$$\tau_3^{st} = \{\phi, X, \{s, t, v\}\} \text{ be three different STs.}$$

Thus  $(X, \tau_1^{st}, \tau_2^{st}, \tau_3^{st})$  is a STTS over X. Then  $T^{st} = \{\phi, X, \{s, u\}, \{t, v\}, \{s, t\}, \{u, v\}, \{s, t, v\}\}$  is a collection of STOS and a collection of STCS is  $(T^{st})^c = \{\phi, X, \{t, v\}, \{s, u\}, \{u, v\}, \{s, t\}, \{u\}\}$  which is the complement of STCS.

**Definition 3.3** The interior of STTS (int <sup>st</sup>) is defined as the union of all finite STOS contained in  $T^{st}$ . int  ${}^{st}(T^{st}) \subseteq T^{st}$ .

**Definition 3.4** The closure of STTS  $(cl^{st})$  is defined as the intersection of all finite STCS containing in  $(T^{st})^c$ .  $cl^{sr}((T^{st})^c)$ .

**Theorem 3.5** Every empty set  $\phi$  and universal set X is always STOSs in STTS.

**Proof** Let  $T^{st} = \phi$ . Since  $\inf^{st}(T^{st}) = \inf^{st}(\phi) = \phi$ . Also  $T^{st} \subseteq \phi$ .

Similarly, let  $T^{st} = X$  Then int  ${}^{st}(T^{st}) = int {}^{st}(X) = X$ . Also  $T^{st} \subseteq X$ .

Thus every empty set  $\phi$  and universal set X is always STOSs in STTS.

**Theorem 3.6** Every empty set  $\phi$  and universal set X is always STCSs in STTS.

**Proof** Let  $(T^{st})^c = \phi$ 

Since  $cl^{st}((\mathbf{T}^{st})^c) = cl^{st}(\phi) = \phi$ . Also  $(\mathbf{T}^{st})^c \supseteq \phi$ .

Similarly, let  $(T^{st})^c = X$ 

Then  $cl^{st}((T^{st})^c) = cl^{st}(X) = X$ . Also  $(T^{st})^c \supseteq \phi$ .

Thus every empty set  $\phi$  and universal set X is always STCSs in STTS.

**Remark 3.7** Let  $(X, \tau_1^{st}, \tau_2^{st}, \tau_3^{st})$  be a STTS. Then the collection of STOSs  $T^{st}$  does not form a ST on *X*.

Remark 3.7 illustrated in the following example.

**Example 3.8** Let  $X = \{a, b, c\}$  and

$$\tau_1^{st} = \{\phi, X, \{a\}\}$$

$$\tau_2^{st} = \{\phi, X, \{b\}\}$$

 $\tau_3^{st} = \{\phi, X, \{c\}\}$  be a three different STs. Thus  $(X, \tau_1^{st}, \tau_2^{st}, \tau_3^{st})$  is a STTS over X.

Then  $T^{st} = \{\phi, X, \{a\}, \{b\}, \{c\}\}\$  is the collection of STOSs. Since  $\{a, b\} \notin T^{st}$ ,  $T^{st}$  does not satisfy the condition (ii) in definition 3.1.

Hence  $T^{st}$  is not a ST.

#### 3.1 Supra Semi-Open Sets and Supra Pre-Open Sets in STTS

**Definition 3.1.1** Let  $A^{st-s}$  be a subset of X. Then  $A^{st-s}$  is said to be a supra tri semi-open set (STSOS) in STTS if  $T^{st} \subseteq cl^{st}$  (int  ${}^{st}(T^{st})$ ). The complement of a STSOS is a supra tri semi- closed set (STSCS)  $(A^{st-s})^c$  if  $(T^{st})^c \supseteq$  int  ${}^{st}(cl^{st}((T^{st})^c))$ .

**Definition 3.1.2** Let  $A^{st_p}$  be a subset of X. Then  $A^{st_p}$  is said to be supra tri pre-open set (STPOS) in STTS if  $T^{st} \subseteq int^{st} (cl^{st}(T^{st}))$ .

The complement of a STPOS is a supra tri pre-closed set (STPCS)  $(A^{st_p})^c$  if  $(T^{st})^c \supseteq cl^{st} (int^{st} ((T)^c))$ .

**Definition 3.1.3** The interior of supra tri-semi topological space (STSTS) is defined as the union of all STSOS contained in  $A^{st-s}$  and is denoted by int  ${}^{st-s}$ . int  ${}^{st-s}(A^{st-s}) \subseteq A^{st-s}$ .

**Definition 3.1.4** The closure of STSTS is defined as the intersection of all STSCS containing in  $(A^{st_{-}s})^c$ and is denoted by  $cl^{st_{-}s}$ ,  $cl^{st_{-}s}((A^{st_{-}s})^c) \supseteq (A^{st_{-}s})^c$ .

**Definition 3.1.5** The interior of supra tri-pre topological space (STPTS) is defined as the union of all STPOS contained in  $A^{st_p}$  and is denoted by int  ${}^{st_p}$ , int  ${}^{st_p}(A^{st_p}) \subseteq A^{st_p}$ .

**Definition 3.1.6** The closure of STPTS is defined as the intersection of all STPCS containing in  $(A^{st_p})^c$  and is denoted by  $cl^{st_p}$ , int  ${}^{st_p}((A^{st_p})^c) \supseteq (A^{st_p})^c$ .

**Example 3.1.7** Let  $X = \{k, l, m\}$  and

$$\tau_1^{st} = \{ \phi, X, \{l\} \},\$$

 $\tau_2^{st} = \{\phi, X, \{k, l\}\}$  and

 $\tau_3^{st} = \{\phi, X, \{k, m\}\}.$ Then  $T^{st} = \{\phi, X, \{l\}, \{k, l\}, \{k, m\}\}$ . Then, STSOS are  $A^{st-s} = \{\{l\}, \{k, l\}, \{l, m\}, \{k, m\}\}$ STSCS are  $(A^{st_s})^c = \{\{k, m\}, \{m\}, \{k\}, \{l\}\}\}$ STPOS are  $A^{st_p} = \{\{k\}, \{l\}, \{m\}, \{k, l\}, \{l, m\}, \{k, m\}\}$  and STPCS are  $(A^{st_p})^c = \{\{l, m\}, \{k, m\}, \{k, l\}, \{m\}, \{k\}, \{l\}\}.$ Theorem 3.1.8 Every STSOS is a STPOS **Proof** Follows from the definition 3.1.1 and 3.1.2. **Remark 3.1.9** Every STPOS need not be a STSOS. Remark 3.1.9 is illustrated in the following example. Example 3.10 Consider example 3.1.7 Here  $\{k\}$  and  $\{m\}$  are STPOS but not STSOS. **Theorem 3.1.11** Every STSCS is a STPCS. **Proof** Follows from the definition 3.1.1 and 3.1.2. Remark 3.1.12 Every STSCS need not be STPCS. Remark 3.1.12 is illustrated in the following example. Example 3.1.13 Consider example 3.1.7 Here  $\{l, m\}$  and  $\{k, l\}$  are STPCS but not STSCS. **Theorem 3.1.14** Every  $\phi$  and X is always STPOS. **Proof** Let  $A^{st_p}$  be a subset of STPOS and  $A^{st_p} = \phi$ . Since  $cl^{st_{-}p}(A^{st_{-}p}) = cl^{st_{-}p}(\phi) = \phi$ . So int  ${}^{st_p}(cl^{st_p}(\mathbf{A}^{st_p})) = int {}^{st_p}(\phi) = \phi$ . Also  $\phi = \phi$ . Similarly, let  $A^{st_p} = X$ . Since  $cl^{st_p}(A^{st_p}) = cl^{st_p}(X) = X$ . So int  ${}^{st_p}(cl^{st_p}(\mathbf{A}^{st_p})) = \operatorname{int} {}^{st_p}(X) = X$ . Also  $X \subseteq X$ .

Hence  $A^{st_p} \subseteq int^{st_p} (cl^{st_p} (A^{st_p})).$ 

Thus every  $\phi$  and X is always STPOS.

**Theorem 3.1.15** Every  $\phi$  and X is always STPCS.



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**Proof** Let  $(A^{st_p})^c$  is a subset of STPCS and  $(A^{st_p})^c = \phi$ . Since  $cl^{st_p}((A^{st_p})^c) = cl^{st_p}(\phi) = \phi$ . So int  $^{st_p}(cl^{st_p}((A^{st_p})^c)) = int^{st_p}(\phi) = \phi$ . Also,  $\phi \supseteq \phi$ . Similarly, let  $(A^{st_p})^c = X$ Since  $cl^{st_p}((A^{st_p})^c) = cl^{st_p}(X) = X$ So int  $^{st_p}(cl^{st_p}((A^{st_p})^c)) = int^{st_p}(X) = X$ . Hence  $(A^{st_p})^c \supseteq int^{st_p}(cl^{st_p}((A^{st_p})^c))$ 

Thus every  $\phi$  and X is always STPCS.

#### Theorem 3.1.16 The union of any two STPOSs is also a STPOS

#### **Proof.** Let A<sup>*st*-*p*</sup> be a collection of STPOS

Then 
$$A^{st_p} \subseteq \operatorname{int} {}^{st_p}(cl^{st_p}(A^{st_p}))).$$
  
Thus  $\bigcup (\operatorname{int} {}^{st_p}(cl^{st_p}(A^{st_p})))) \subseteq \operatorname{int} {}^{st_p}(\bigcup cl^{st_p}(A^{st_p}))) \subseteq \operatorname{int} {}^{st_p}(cl^{st_p}(\bigcup A^{st_p})))$   
Thus  $\bigcup A^{st_p} \subseteq \bigcup (\operatorname{int} {}^{st_p}(cl^{st_p}(A^{st_p}))))$ 

Since  $\bigcup A^{st_p}$  is a STPOS.

Hence the theorem.

Remark 3.1.17 The intersection of any two STPOS is not STPOS.

Remark 3.1.17 is illustrated in the following example.

**Example 3.1.18** Consider the example 3.2.

Then its STPOS are  $\{\{s,t\}, \{t,u\}, \{u,v\}, \{s,u\} \{s,v\}, \{s,t,u\}, \{s,u,v\}, \{s,t,v\}, \{t,u,v\}\}$ 

Let  $\{s, t\}$  and  $\{t, u\}$  be two STPOS.

Then,  $\{s, t\} \cap \{t, u\} = \{t\}$ 

Since  $\{t\}$  is not STPOS.

Theorem 3.1.19 The intersection of any two STPCS is also a STPCS.

**Proof** Let  $(A^{st_p})^c$  be a collection of STPCS.

Then, 
$$(\mathbf{A}^{st_p})^c \supseteq cl^{st_p} (\operatorname{int}^{st_p} (\mathbf{A}^{st_p})^c))$$
  
 $\cap (\mathbf{A}^{st_p})^c \supseteq \cap cl^{st_p} (\operatorname{int}^{st_p} (\mathbf{A}^{st_p})^c)) \supseteq cl^{st_p} (\cap \operatorname{int}^{st_p} ((\mathbf{A}^{st_p})^c))) \supseteq cl^{st_p} (\operatorname{int}^{st_p} ((\mathbf{A}^{st_p})^c)))$ 

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Since  $\cap (A^{st_p})^c$  is a STPCS.

Hence the theorem.

Remark 3.1.20 The union of any two STPCS is not a STPCS.

Remark 3.1.20 is illustrated in the following example.

Example 3.1.21 Consider example 3.2.

then their STPCS are  $\{\{u, v\}, \{s, v\}, \{s, t\}, \{t, u\}, \{v\}, \{t\}, \{u\}, \{s\}\}$ .

Let  $\{u, v\}$  and  $\{s, v\}$  be a two STPCS.

Then,  $\{u, v\} \cup \{s, v\} = \{s, u, v\}$ 

Thus  $\{s, u, v\}$  is not a STPCS.

**Remark 3.1.22** Let  $A^{st_p}$  is subset of STPOS.

Then,  $cl^{st_p}(\operatorname{int} st_p(A^{st_p})) \neq \operatorname{int} st_p(A^{st_p})$ .

Remark 3.1.22 is illustrated in the following example.

Example 3.1.23 Consider example 3.2,

Then its STPOS is  $\{\{s,t\}, \{t,u\}, \{u,v\}, \{s,u\}, \{s,v\}, \{s,t,u\}, \{s,u,v\}, \{s,t,v\}, \{t,u,v\}\}$  and its STPCS is  $\{\{s,t\}, \{s,v\}, \{u,v\}, \{t,v\}, \{t,u\}, \{s\}, \{t\}, \{u\}, \{v\}\}$ .

Then int  ${}^{st_{-p}} \{s, u\} = \{s, u\}$  and  $cl^{st_{-p}} \{s, u\} = X$ 

int 
$${}^{st_{-p}}(cl^{st_{-p}}(\mathbf{A}^{st_{-p}})) = \operatorname{int} {}^{st_{-p}}(cl^{st_{-p}}\{s,u\}) = \operatorname{int} {}^{st_{-p}}\{X\} = X$$

Since  $X \neq \{s, u\}$ . Hence int  $s^{t-p}(cl^{st-p}(\mathbf{A}^{st-p})) \neq int^{st+p}(\mathbf{A}^{st-p})$ .

**Remark 3.1.24** Let  $(A^{st-p})^c$  is a subset of STPCS.

Then  $cl^{st_p}\left(\operatorname{int}^{st_p}\left((A^{st_p})^c\right)\right) = cl^{st_p}\left(A^{st_p}\right)^c$ .

Remark 3.1.24 illustrated in the following example.

Example 3.1.25 Consider example 3.2,

then its STPCS is  $\{\{s, t\}, \{s, v\}, \{u, v\}, \{t, v\}, \{t, u\}, \{s\}, \{t\}, \{u\}, \{v\}\}$ .

Then int  ${}^{st_{-}p} \{t, v\} = \phi$  and  $cl^{st_{-}p} \{t, v\} = \{t, v\}$ 

Then  $cl^{st_p}\left(\inf t^{st_p}\left(\left(A^{st_p}\right)^{c}\right)\right) = cl^{st_p}\left(\inf t^{st_p}\left(\left(A^{st_p}\right)^{c}\right)\right) = cl^{st_p}\left\{\phi\right\} = \phi$ . Since  $\phi \neq \{t, v\}$ .Hence  $cl^{st_p}\left(\inf t^{st_p}\left(\left(A^{st_p}\right)^{c}\right)\right) \neq cl^{st_p}\left(\left(A^{st_p}\right)^{c}\right)$ . **Theorem 3.1.26** Every empty set  $\phi$  and universal set X are STSOS.

**Proof** Let  $A^{st_{-}s}$  be a subset of STSOS and  $A^{st_{-}s} = \phi$ . Since  $\inf_{st_s} (A^{st_s}) = \inf_{st_s} (\phi) = \phi$ So,  $cl^{st_s}(\operatorname{int}^{st_s}(A^{st_s})) = cl^{st_s}(\phi) = \phi$ . Also,  $\phi \subseteq \phi$ . Similarly, let  $A^{st_{-}s} = X$ . Since  $\operatorname{int} {}^{st_s}(\mathbf{A}^{st_s}) = \operatorname{int} {}^{st_s}(X) = X$ . So,  $cl^{st_s}(\operatorname{int}^{st_s}(A^{st_s})) = cl^{st_s}(X) = X$ . Also,  $X \subseteq X$ . Hence  $A^{st_s} \subseteq cl(int^{st_s}(A^{st_s}))$ . Thus  $\phi$  and X is are STSOS. **Theorem 3.1.27** Every  $\phi$  and X is are STSCS. **Proof** Let  $(A^{st_s})^c = \phi$ . Since int  ${}^{st-s}\left(\left(\mathbf{A}^{st-s}\right)^{c}\right) = \operatorname{int}^{st-s}\left(\phi\right) = \phi$ . So,  $cl^{st-s}\left(\inf st_{s}^{st-s}\left(\left(A^{st-s}\right)^{c}\right)\right) = cl^{st-s}(\phi) = \phi$ . Also  $\phi \supseteq \phi$ . Similarly let  $(A^{st_s})^c = X$ . Since int  ${}^{st_s}((A^{st_s})^c) = int {}^{st_s}(X) = X$ So  $c^{st-s}\left(\inf^{st-s}\left(\left(A^{sr-s}\right)^{c}\right)\right) = cl^{st-s}(X) = X$ . Also  $X \supseteq X$ . JCRI Hence  $(A^{st-s})^c \supseteq cl^{st-s} (int^{st-s} ((A^{st-s})^c)).$ Thus  $\phi$  and s X is are STSCS. **Theorem 3.1.28** Let  $A^{st}$  be a subset of STSOS.

Then  $cl^{st_{-}s}(int^{st_{-}s}(A^{st_{-}s})) = cl^{st_{-}s}(A^{st_{-}s}).$ 

**Proof.** If  $A^{st_s}$  is a subset of STSOS,

Then  $A^{st_{-}s} \subseteq cl^{st_{-}s}(\operatorname{int}{}^{st_{-}s}(A^{st_{-}s}))$ . Therefore  $cl^{st_{-}s}(A^{st_{-}s}) \subseteq cl^{st_{-}s}(\operatorname{int}{}^{st_{-}s}(A^{st_{-}s}))$ . But  $cl^{st_{-}s}(\operatorname{int}{}^{st_{-}s}(A^{st_{-}s})) \subseteq cl^{st_{-}s}(A^{st_{-}s})$ . Thus  $cl^{st_{-}s}(\operatorname{int}{}^{st_{-}s}(A^{st_{-}s})) = cl^{st_{-}s}(A^{st_{-}s})$ .

**Theorem 3.1.29** Let  $(A^{st-s})^c$  be a subset of STSCS.

Then 
$$cl^{st_{-}s}\left(\inf t^{st_{-}s}\left((A^{st_{-}s})^{c}\right)\right) = cl^{st_{-}s}\left(A^{st_{-}s}\right)^{c}$$
.

**Proof.** If  $(A^{st-s})^c$  is a subset of STSCS,

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Then 
$$(A^{st_s})^c \supseteq cl^{st_s} (\inf st_s (A^{st_s})^c)$$
. Therefore  $\inf st_s (A^{st_s})^c \supseteq \inf st_s (cl^{st_s} (A^{st_s})^c))$ .  
But  $\inf st_s (cl^{st_s} (A^{st_s})^c)) \supseteq \inf st_s (A^{st_s})^c$ . Thus  $\inf st_s (cl^{st_s} (A^{st_s})^c)) = \inf st_s (A^{st_s})^c$ .

Theorem 3.1.30 The union of two any STSOSs is also a STSOS

**Proof.** Let  $A^{st_{-}s}$  be a collection of STSOS Then  $A^{st_{-}s} \subseteq cl^{st_{-}s} (\operatorname{int} {}^{st_{-}s} (A^{st_{-}s}))$   $\bigcup A^{st_{-}s} \subseteq \bigcup cl^{st_{-}s} (\operatorname{int} {}^{st_{-}s} (A^{st_{-}s})) \subseteq cl^{st_{-}s} (\bigcup \operatorname{int} {}^{st_{-}s} (A^{st_{-}s})) \subseteq cl^{st_{-}s} (\operatorname{int} {}^{st_{-}s} (\bigcup A^{st_{-}s}))$ . Thus  $\bigcup A^{st_{-}s} \subseteq \bigcup cl^{st_{-}s} (\operatorname{int} {}^{st_{-}s} (A^{st_{-}s}))$ .

Since  $\bigcup A^{st_s}$  is a STSPOS.

Hence the theorem.

## **Remark 3.1.31**

The intersection of any two STSOS may not always be STSOS.

Remark 3.1.31 is illustrated in the following example.

# Example 3.1.32

Consider the example 3.2.

Then its STSOS are  $\{\{s, t\}, \{u, v\}, \{s, u\}, \{s, t, u\}, \{s, u, v\}, \{s, t, v\}, \{t, u, v\}\}$ 

Let  $\{s, t\}$  and  $\{s, u\}$  be a two STSOS

Then,  $\{s, t\} \cap \{s, u\} = \{s\}$ 

Thus  $\{s\}$  is not a STSOS.

Theorem 3.1.33 The intersection of any two STSCS is also STSCS.

**Proof** Let  $(A^{st-s})^c$  is a collection of STSCS.

Then, 
$$(A^{st_{-}s})^c \supseteq \operatorname{int} {}^{st_{-}s} \left( cl^{st_{-}s} \left( (A^{st_{-}s})^c \right) \right)$$
  
 $\cap (A^{st_{-}s})^c \supseteq \cap \operatorname{int} {}^{st_{-}s} \left( cl^{st_{-}s} \left( (A^{st_{-}s})^c \right) \right) \supseteq \operatorname{int} {}^{st_{-}s} \left( cl^{st_{-}s} \left( (A^{st_{-}s})^c \right) \right) \supseteq \operatorname{int} {}^{st_{-}s} \left( cl^{st_{-}s} \left( (A^{st_{-}s})^c \right) \right) \right)$ 

$$\cap (A^{st_{-}s})^c \supseteq \cap \operatorname{int} {}^{st_{-}s} \left( cl^{st_{-}s} \left( (A^{st_{-}s})^c \right) \right).$$
Since  $\cap (A^{st_{-}s})^c$  is a STSCS.

Hence the theorem.

**Remark 3.1.34** The union of any two STSCS may not always be STSCS.

Remark 3.1.34 is illustrated in the following example.

Example 3.1.35 Consider example 3.2,

Then its STSCS are  $\{\{u, v\}, \{s, t\}, \{t, v\}, \{v\}, \{t\}, \{u\}, \{s\}\}$ .

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Let  $\{u, v\}$  and  $\{t, v\}$  be a two STSCS.

Then,  $\{u, v\} \cup \{t, v\} = \{t, u, v\}$ 

Here  $\{t, u, v\}$  is not a STSCS.

### 3.2 Supra Semi-Pre Tri-Topological space

**Definition 3.2.1** A subset  $A^{st_{-}sp}$  of topological space is said to be supra tri-semi-pre topological space (STSPTS) if  $A^{st_{-}sp} \subseteq cl^{st_{-}s} (\operatorname{int} {}^{st_{-}p} (A^{st_{-}p})))$ . The collection of  $A^{st_{-}sp}$  is called supra tri-semi-pre open set(STSPOS). The complement of STSPOS is known as supra semi-pre closed set (STSPCS)  $(A^{st_{-}sp})^{c}$ .

**Definition 3.2.2** The interior of the SSPTTS is defined as the union of all open sets in STSPOS contained in  $A^{st_-sp}$  and is denoted by int  ${}^{st_-sp}(A^{st_-sp})$ .  $A^{st_-sp} \subseteq int {}^{st_-sp}(A^{st_-sp})$ .

**Definition 3.2.3** The closure of the SSPTTS is defined as the intersection of all closed sets in STSPCS containing in  $(A^{st-sp})^c$  and is denoted by  $cl^{st-sp}((A^{st-sp})^c)$ .  $(A^{st-sp})^c \supseteq cl^{st-sp}(A^{st-sp})^c$ 

**Example 3.2.4** let  $X = \{k, l, m\}$ . Then its

STSPCS are  $\{\{l\}, \{k, l\}, \{l, m\}, \{k, m\}\}$ 

STSPCS are  $\{\{k, m\}, \{m\}, \{k\}, \{l\}\}$ 

**Theorem 3.2.5** Every empty set  $\phi$  and universal set X is always SSPOS in STTS.

**Proof** Let 
$$A^{st_{-}sp} = \phi$$
. Since  $cl^{st_{-}sp}(A^{st_{-}sp}) = cl^{st_{-}sp}(\phi) = \phi$  and  
int  ${}^{st_{-}sp}(cl^{st_{-}sp}(A^{st_{-}sp})) = int^{st_{-}sp}(\phi) = \phi$ .  
So,  $cl^{st_{-}sp}(int^{st_{-}sp}(cl^{st_{-}sp}(A^{st_{-}sp}))) = cl^{st_{-}sp}(\phi) = \phi$ . Also,  $\phi \subseteq \phi$   
Similarly, let  $A^{st_{-}sp} = X$   
Since  $int^{st_{-}sp}(cl^{st_{-}sp}(A^{st_{-}sp})) = int(X) = X$   
So,  $cl^{st_{-}sp}(int^{st_{-}sp}(cl^{st_{-}sp}(A^{st_{-}sp}))) = cl^{st_{-}sp}(X) = X$  Also  $X \subseteq X$ .  
Hence  $A^{st_{-}sp} \subseteq cl^{st_{-}sp}(int^{st_{-}sp}(cl^{st_{-}sp}(A^{st_{-}sp}))))$ 

Thus the empty set  $\phi$  and universal set X is always SSPOS in STT.

**Theorem 3.2.6** Every empty set  $\phi$  and universal set X is always SSPCS is STTS.

**Proof** Let  $(A^{st_sp})^c = \phi$ 

Since, 
$$cl^{st_sp}\left(\left(\mathbf{A}^{st_sp}\right)^c\right) = cl^{st_sp}\left(\phi\right) = \phi$$
 and int  $st_sp\left(cl^{st_sp}\left(\left(\mathbf{A}^{st_sp}\right)^c\right)\right)$   
So,  $cl^{st_sp}\left(\operatorname{int} st_sp\left(cl^{st_sp}\left(\left(\mathbf{A}^{st_sp}\right)^c\right)\right)\right)$ .

Hence, every empty set  $\phi$  is always SSPCS in STT.

Similarly, let 
$$A^{st_-sp} = X$$
  
Since,  $cl^{st_-sp} \left( \left( A^{st_-sp} \right)^c \right) \right) = cl^{st_-sp} (X) = X$  and  
int  ${}^{st_-sp} \left( cl^{st_-sp} \left( A^{st_-sp} \right)^c \right) \right) =$  int  ${}^{st_-sp} (X) = X$ .  
So,  $cl^{st_-sp} \left($  int  ${}^{st_-sp} \left( cl^{t_-sp} \left( A^{st_-sp} \right)^c \right) \right) = cl^{s_-sp} (X) = X$   
Hence,  $A^{st_-sp} \supseteq cl^{st_-sp} \left($  int  ${}^{st_-sp} \left( cl^{st_-sp} \left( A^{st_-sp} \right)^c \right) \right)$ 

Thus the empty set  $\phi$  and the universal set X is always SSPCS in STTS.

Theorem 3.2.7 The union of any two STSPOSs is also a STSPOS

**Proof.** ;Let  $A^{st_sp}$  be a collection of STSPOS

Then 
$$A^{st\_sp} \subseteq cl^{st\_sp}(\operatorname{int} {}^{st\_sp}(cl^{st\_sp}(A^{st\_sp})))) \subseteq \bigcup cl^{st\_sp}(\operatorname{int} {}^{st\_sp}(cl^{st\_sp}(A^{st\_sp}))))$$
  
 $\subseteq cl^{st\_sp}(\bigcup \operatorname{int} {}^{st\_sp}(cl^{st\_sp}(A^{st\_sp})))) \subseteq cl^{st\_sp}(\operatorname{int} {}^{st\_sp}(\bigcup cl^{st\_sp}(A^{st\_sp}))))$   
 $\subseteq cl^{st\_sp}(\operatorname{int} {}^{st\_sp}(cl^{st\_sp}(\bigcup A^{st\_sp})))).$   
Thus  $\bigcup A^{st\_sp} \subseteq \bigcup cl^{st\_sp}(\operatorname{int} {}^{st\_sp}(cl^{st\_sp}(A^{st\_sp}))))$ 

Since  $\bigcup A^{st-sp}$  is a STSPOS

Hence the theorem.

**Remark 3.2.8** The intersection of any two STSPOS may not always be STSPOS.

Remark 3.2.8 is illustrated in the following example.

**Example 3.2.9** Consider the example 3.2. Then its STSPOS are

$$\{\{s,t\},\{u,v\},\{s,u\},\{s,t,u\},\{s,u,v\},\{s,t,v\},\{t,u,v\}\}$$

Let 
$$A = \{s, t\}$$
 and  $B = \{s, u\}$ .

Then,  $\{s, t\} \cap \{s, u\} = \{s\}$ 

Thus  $\{s\}$  is not a STSPOS.

Theorem 3.2.10 The intersection of any two STSPCS is also STSPCS.

**Proof** Let  $(A^{st_sp})^c$  be a collection of STSPCS.

Then, 
$$(A^{st} - sp)^{c} \supseteq \operatorname{int} {}^{st} - sp \left( cl^{st} - sp \left( \operatorname{int} {}^{st} - sp \left( (A^{st} - sp \right)^{c} \right) \right) \right)$$
  

$$\supseteq \cap \operatorname{int} {}^{st} - sp \left( cl^{st} - sp \left( \operatorname{int} {}^{st} - sp \left( (A^{st} - sp \right)^{c} \right) \right) \right) \supseteq \operatorname{int} {}^{st} - sp \left( \cap cl^{st} - sp \left( \operatorname{int} {}^{st} - sp \left( (A^{st} - sp \right)^{c} \right) \right) \right)$$

$$\supseteq \operatorname{int} {}^{st} - sp \left( cl^{st} - sp \left( \cap \operatorname{int} {}^{st} - sp \left( (A^{st} - sp \right)^{c} \right) \right) \right) \supseteq \operatorname{int} {}^{st} - sp \left( cl^{st} - sp \left( \cap (A^{st} - sp \right)^{c} \right) \right)$$

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Thus 
$$\bigcap (A^{st_sp})^c \supseteq \bigcap int^{st_sp} (cl^{st_sp} (int^{st_sp} (A^{st_sp})^c)))$$

Since  $\supseteq (A^{st_sp})^c$  is a STSPCS.

Hence the theorem.

Remark 3.2.11 The union of any two STSPCS may not always be STSPCS.

Remark 3.2.11 is illustrated in the following example.

Example 3.2.12 consider example 3.2. the STSPCS are

 $\{\{u, v\}, \{s, t\}, \{t, v\}, \{v\}, \{t\}, \{u\}, \{s\}\}.$ 

Let  $A = \{u, v\}$  and  $B = \{s, v\}$ . Then,

 $A \cup B = \{u, v\} \cup \{t, v\} = \{t, u, v\}$ 

Here is not a STSPCS.

**Theorem 3.2.13** Every STSOS is a STSPOS

**Proof** Follows from the example 3.1.7 and example 3.2.4

**Theorem 3.2.14** Every STSCS is a STPCS.

**Proof** Follows from the example 3.1.7 and example 3.2.4.

**Theorem 3.2.15** Let  $A^{st-sp}$  be a subset of STSPOS. Then

 $cl^{st} e^{sp} \left( \operatorname{int} {}^{st} e^{sp} \left( cl^{st} e^{sp} \left( A^{st} e^{sp} \right) \right) \right) = cl^{st} e^{sp} \left( A^{st} e^{sp} \right)$ 

**Proof.** If  $A^{st-sp}$  is a STSPOS,

Then, 
$$A^{st\_sp} \subseteq cl^{st\_sp} (int^{st\_sp} (cl^{st\_sp} (A^{st\_sp}))))$$
.  
But  $cl^{st\_sp} (int^{st\_sp} (cl^{st\_sp} (A^{st\_sp})))) \supseteq cl^{st\_sp} (A^{st\_sp})$ .  
Thus  $cl^{st\_sp} (int^{st\_sp} (cl^{st\_sp} (A^{st\_sp})))) = cl^{st\_sp} (A^{st\_sp})$ .

Hence the theorem.

**Theorem 3.2.16** Let  $(A^{st_sp})^c$  be a subset of STSPCS. Then

int 
$${}^{st_sp}\left(cl^{st_sp}\left(\operatorname{int}{}^{st_sp}\left(\left(\mathbf{A}^{st_sp}\right)^c\right)\right)\right) = \operatorname{int}{}^{st_sp}\left(\left(\mathbf{A}^{st_sp}\right)^c\right)$$

**Proof.** If  $(A^{st_sp})^c$  is a STSPCS

Then 
$$(A^{st_sp})^c \supseteq \operatorname{int} {}^{st_sp} \left( cl^{st_sp} \left( \operatorname{int} {}^{st_sp} \left( (A^{st_sp})^c \right) \right) \right)$$
.  
But  $\operatorname{int} {}^{st_sp} \left( cl^{st_sp} \left( \operatorname{int} {}^{st_sp} \left( (A^{st_sp})^c \right) \right) \right) \subseteq \operatorname{int} {}^{st_sp} \left( (A^{st_sp})^c \right)$   
Thus  $\operatorname{int} {}^{st_sp} \left( cl^{st_sp} \left( \operatorname{int} {}^{st_sp} \left( (A^{st_sp})^c \right) \right) \right) = \operatorname{int} {}^{st_sp} \left( (A^{st_sp})^c \right)$ 

Hence the theorem.

#### 3.3 t-open sets in supra-tri-topological space

**Definition 3.3.1** Asssume that  $(X, \tau_1^{st}, \tau_2^{st}, \tau_3^{st})$  is a supra tri-topological space. Then Z, is an supra tri-set over X is said to be a supra tri-topole set in  $(X, \tau_1^{st}, \tau_2^{st}, \tau_3^{st})$  if and only if there exist supra tri-open set  $Z_1$  in  $\tau_1^{st}, Z_2$  in  $\tau_2^{st}, Z_3$  in  $\tau_3^{st}$  that  $Z = Z_1 \cup Z_2 \cup Z_3$ .

The complement of supra t-closed set is called supra tri-t-closed set.

**Definition 3.3.2** The interior of STTS in t-open sets (*t* int) is defined as the union of all supra-tri-t-open sets contained in  $Z \,.\, Z \subseteq t \, int(Z)$ .

**Definition 3.3.3** The closure of STTS in t-open sets (t cl) is defined as the intersection of all t-closed sets containing in  $Z \cdot Z^c \supseteq t cl(Z^c)$ .

**Example 3.3.4** let  $X = \{a, b, c, d\}$ . Then  $\tau_1^{st} = \{\phi, X, \{a\}\}, \tau_2^{st} = \{\phi, X, \{d\}\}, \tau_3^{st} = \{\phi, X, \{b\}\}$  are three supra topological space.  $T^{st-t} = \{\phi, X, \{a\}, \{b\}, \{d\}\}$ 

 $T^{st_{-}t} = \{a, b, d\}$  is a supra tri-t-open set where  $\{a\} \in \tau_1^{st}, \{d\} \in \tau_2^{st}, \{b\} \in \tau_3^{st}$  such that

 $\mathbf{T}^{st_{-}t} = \{a\} \cup \{d\} \cup \{b\}.$ 

**Theorem 3.3.5** Assume that  $(X, \tau_1, \tau_2, \tau_3)$  be an supra tri-topological space.

Every empty  $(\phi)$  and the universal set (X) are always a supra tri-t-open set in  $(X, \tau_1, \tau_2, \tau_3)$ .

**Proof** we can write the empty set  $(\phi)$  as  $Z = \tau_1^{st} \cup \tau_2^{st} \cup \tau_3^{st}$  where  $\tau_1^{st} = \phi, \tau_2^{st} = \phi, \tau_3^{st} = \phi$  are supra triopen set in  $(X, \tau_1^{st}), (X, \tau_2^{st})$  and  $(X, \tau_3^{st})$  respectively. Hence  $Z = \phi \cup \phi \cup \phi$ . Hence Z is a supra triopen set in  $(X, \tau_1^{st}, \tau_2^{st}, \tau_3^{st})$ .

Similarly, we can write the universal set  $Z = \tau_1^{st} \cup \tau_2^{st} \cup \tau_3^{st}$  where  $\tau_1^{st} = X, \tau_2^{st} = X, \tau_3^{st} = X$  are supra open set in  $(X, \tau_1^{st}), (X, \tau_2^{st})$  and  $(X, \tau_3^{st})$  respectively. Hence  $Z = X \cup X \cup X$  is a supra tri-t-open sets in  $(X, \tau_1^{st}, \tau_2^{st}, \tau_3^{st})$ .

**Theorem 3.3.6** Every supra open set in  $(X, \tau_1^{st}), (X, \tau_2^{st})$  and  $(X, \tau_3^{st})$  are supra tri-t-open set in  $(X, \tau_1^{st}, \tau_2^{st}, \tau_3^{st})$ .

**Proof** Suppose that Z be an supra open set in  $(X, \tau_1^{st})$ . Now, we can write  $Z = Z \cup \phi \cup \phi$  Therefore there exist supra open set  $Z, \phi, \phi$  in  $(X, \tau_1^{st}), (X, \tau_2^{st})$  and  $(X, \tau_3^{st})$  respectively such that  $Z = Z \cup \phi \cup \phi$ . Hence Z is a supra tri-t-open set in  $(X, \tau_1^{st}, \tau_2^{st}, \tau_3^{st})$ .

Suppose that Z be an supra open set in  $(X, \tau_2^{st})$ . Now we can write  $Z = \phi \cup Z \cup \phi$ . Therefore there exist supra open set  $\phi, Z, \phi$  in  $(X, \tau_1^{sr}), (X, \tau_2^{st})$  and  $(X, \tau_3^{st})$  respectively such that  $Z = \phi \cup Z \cup \phi$ . Hence Z is a supra tri-t-open set in  $(X, \tau_1^{st}, \tau_2^{st}, \tau_3^{st})$ .

Suppose that Z be an supra open set in  $(X, \tau_3^{st})$ . Now we can write  $Z = \phi \cup \phi \cup Z$ . Therefore there exist supra open set  $\phi, \phi, Z$  in  $(X, \tau_1^{st}), (X, \tau_2^{st})$  and  $(X, \tau_3^{st})$  respectively such that  $Z = \phi \cup \phi \cup Z$ . Hence Z is a supra tri-t-open set in  $(X, \tau_1^{st}, \tau_2^{st}, \tau_3^{st})$ .

**Theorem 3.3.7** Every supra closed set in  $(X, \tau_1^{st}), (X, \tau_2^{st})$  and  $(X, \tau_3^{st})$  are supra tri-t-closed sets in  $(X, \tau_1^{st}, \tau_2^{st}, \tau_3^{st})$ .

**Proof** Suppose that  $Z^c$  be an supra closed set in  $(X, \tau_1^{st})$ . Now we can write  $Z^c = Z^c \cap \phi \cap \phi$ . Therefore there exist supra closed set  $Z^c, \phi, \phi$  in  $(X, \tau_1^{st}), (X, \tau_2^{st})$  and  $(X, \tau_3^{st})$  respectively such that  $Z^c = Z^c \cap \phi \cap \phi$ . Hence  $Z^c$  is a supra tri-t-closed set in  $(X, \tau_1^{st}, \tau_2^{st}, \tau_3^{st})$ .

Suppose that  $Z^c$  be a supra closed set in  $(X, \tau_2^{st})$ . Now we can write  $Z^c = \phi \cap Z^c \cap \phi$ . Therefore there exist supra closed set  $Z^c, \phi, \phi$  in  $(X, \tau_1^{st}), (X, \tau_2^{st})$  and  $(X, \tau_3^{st})$  respectively such that  $Z^c = \phi \cap Z^c \cap \phi$ . Hence  $Z^c$  is a supra tri-t-closed set in  $(X, \tau_1^{st}, \tau_2^{st}, \tau_3^{st})$ .

Suppose that  $Z^c$  be a supra closed set in  $(X, \tau_3^{st})$ . Now we can write  $Z^c = \phi \cap \phi \cap Z^c$ . Therefore there exist supra closed set  $\phi, \phi, Z^c$  in  $(X, \tau_1^{st}), (X, \tau_2^{st})$  and  $(X, \tau_3^{st})$ . Hence  $Z^c$  is a supra tri-t-closed set in  $(X, \tau_1^{st}, \tau_2^{st}, \tau_3^{st})$ .

**Conclusion** In this study, the idea of supra-tri-topological space is introduced. The interior and closure are also defined in this space. Some types of open sets like semi, pre, semi-pre, t-open sets are defined with interior and closure concept in such space. Also, basic theorems are proved and examples are given.

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